ACOUSTIC-ELASTIC COUPLED FULL-WAVEFORM INVERSION IN THE LAPLACE DOMAIN WITH SCALED GRADIENT FOR IMPROVED DENSITY RECOVERY

Journal of Seismic Exploration

FWI for acoustic-elastic coupled media in the Laplace domain

$$-\frac{s^2}{c^2}\tilde{p} = \frac{\partial^2 \tilde{p}}{\partial x^2} + \frac{\partial^2 \tilde{p}}{\partial z^2} + \tilde{f} \quad , \tag{1}$$

where $\tilde{p} = \tilde{p}(x,z,s)$ is the Laplace domain pressure-field in water column (fluid media); s is the damping coefficient of the Laplace transform, $c(x,z) = \sqrt{k/\rho_A}$ is velocity in water column (fluid media); k and ρ_A are the bulk modulus and density of water column (fluid media), respectively; and $\tilde{f} = \tilde{f}(x,z,s)$ is a source term.

In heterogeneous and isotropic solid media, two-dimensional elastic wave equation in the Laplace domain is written as

$$-\rho_E s^2 \tilde{h} = \frac{\partial}{\partial x}\left((\lambda + 2\mu)\frac{\partial \tilde{h}}{\partial x} + \lambda \frac{\partial \tilde{v}}{\partial z}\right) + \frac{\partial}{\partial z}\left(\mu\left(\frac{\partial \tilde{v}}{\partial x} + \frac{\partial \tilde{h}}{\partial z}\right)\right) \quad , \tag{2}$$

$$-\rho_E s^2 \tilde{v} = \frac{\partial}{\partial x}\left(\mu\left(\frac{\partial \tilde{v}}{\partial x} + \frac{\partial \tilde{h}}{\partial z}\right)\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial \tilde{h}}{\partial x} + (\lambda + 2\mu)\frac{\partial \tilde{v}}{\partial z}\right) \quad , \tag{3}$$

where ρ_E is the density in solid media; λ and μ are the Lamé constants; and $\tilde{h} = \tilde{h}(x,z,s)$ and $\tilde{v} = \tilde{v}(x,z,s)$ are the horizontal and vertical displacements, respectively.

In acoustic-elastic coupled media, pressure fields generated in water column are converted to horizontal and vertical particle displacements at the seafloor, which propagate through the seafloor as the elastic medium. The interface boundaries between fluid and solid media should meet the continuity conditions (Zienkiewicz et al., 2005; Komatitsch et al., 2000) as follows:

$$\nabla \tilde{p} \cdot \mathbf{n} = -\rho_A \ddot{\mathbf{u}}_E \cdot \mathbf{n} \quad , \tag{4}$$

and

$$\sigma \cdot \mathbf{n} = -\tilde{p}\mathbf{n} \quad , \tag{5}$$

where $\ddot{\mathbf{u}}_E$ is the second derivative of the displacement vector with respect to time, $\mathbf{n}$ is the normal vector from the interface and σ is the symmetric, second-ordered stress tensor. When the elastic waves are reflected and return to the seafloor, they are converted to pressure fields and then propagate to the sea surface.

$$\begin{pmatrix} \mathbf{K}^A + {s^2}/{c^2}\,\mathbf{M}^A & \rho_A s^2 \mathbf{Q}^A \sin\theta & \rho_A s^2 \mathbf{Q}^A \cos\theta \\ \left[\mathbf{Q}^E\right]^T \sin\theta & \mathbf{K}^{11} - \rho_E s^2 \mathbf{M}^{11} & \mathbf{K}^{12} \\ \left[\mathbf{Q}^E\right]^T \cos\theta & \mathbf{K}^{21} & \mathbf{K}^{22} - \rho_E s^2 \mathbf{M}^{22} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{p}}(s) \\ \tilde{\mathbf{v}}_x(s) \\ \tilde{\mathbf{v}}_z(s) \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{f}}(s) \\ 0 \\ 0 \end{pmatrix}$$

$\mathbf{K}^A$ $\mathbf{K}^{11}$ $\mathbf{K}^{12}$ $\mathbf{K}^{21}$
$\mathbf{K}^{22}$ $\mathbf{M}^A$
$\mathbf{M}^{11}$ $\mathbf{M}^{22}$
$\mathbf{Q}^A$ $\mathbf{Q}^E$
θ
$\tilde{\mathbf{p}}(s)$
$\tilde{\mathbf{v}}_x(s)$ $\tilde{\mathbf{v}}_z(s)$
$\tilde{\mathbf{f}}(s)$

$\mathbf{S}$

$$\mathbf{S}\tilde{\mathbf{u}} = \tilde{\mathbf{f}}$$

$\tilde{\mathbf{u}}$

λ μ (ρ_E)

l_2

$$E\left(k_{(ix,iz)}\right) = \frac{1}{2}\sum_{i=1}^{n_s}\sum_{j=1}^{n_r}\delta\tilde{r}_{ij}\delta\tilde{r}_{ij}$$

$$\delta\tilde{r}_{ij} = \ln\left(\frac{\tilde{u}_{ij}}{\tilde{d}_{ij}}\right)$$

$\tilde{d}_{ij}$ $\tilde{u}_{ij}$ $\delta\tilde{r}_{ij}$

j

i n_s n_r

$k_{(ix,iz)}$

$\mathbf{S}$

$$\nabla_{k_{(ix,iz)}}\mathbf{E}\left(ix,iz\right) = \mathrm{Re}\left[\sum_{i=1}^{n_s}(\mathbf{v}_{k_{(ix,iz)}})^T\mathbf{S}^{-1}\delta\tilde{\mathbf{r}}_i\right]$$

$k_{(ix,iz)}$

$\mathbf{v}_{k_{(ix,iz)}}$

$$\mathbf{v}_{k_{(ix,iz)}} = -\frac{\partial\mathbf{S}}{\partial k_{(ix,iz)}}\tilde{\mathbf{u}}_i$$

$\tilde{\mathbf{u}}_i$ $\delta\tilde{\mathbf{r}}_i$

i

$$\delta\tilde{\mathbf{r}}_i = \begin{bmatrix} \ln(\tilde{u}_{i1}/\tilde{d}_{i1})/u_{i1} \\ \ln(\tilde{u}_{i2}/\tilde{d}_{i2})/u_{i2} \\ \vdots \\ \ln(\tilde{u}_{i,nr}/\tilde{d}_{i,nr})/u_{i,nr} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$k^{l+1}_{(ix,iz)} = k^{l}_{(ix,iz)} - \alpha^{l} \sum_{s=1}^{ns} NRM \left[\frac{\nabla_{k_{(ix,iz)}} \mathbf{E}(ix,iz)}{\sum_{i=1}^{n_s} (\mathbf{v}_{k_{(ix,iz)}})^{T} \cdot \mathbf{v}_{k_{(ix,iz)}} + \eta} \right]$$

l α^l l-

ns NRM

η

λ μ

$\nabla_{density_{(ix,iz)}} \mathbf{E}_{acc}(ix,iz)$

$$\nabla_{density_{(ix,iz)}} \mathbf{E}_{acc}(ix,iz) = \nabla_{density_{(ix,iz)}} \mathbf{E}_{acc}(ix,iz-1) + \left[\nabla_{density_{(ix,iz)}} \mathbf{E}(ix,iz) \right]^2, \quad ix = 1,\cdots,nx, \quad iz = 2,\cdots,nz$$

$\mathbf{v}_{density_{(ix,iz)}}$

$\nabla_{density_{(ix,iz)}} \mathbf{E}_{acc}(ix,iz)$

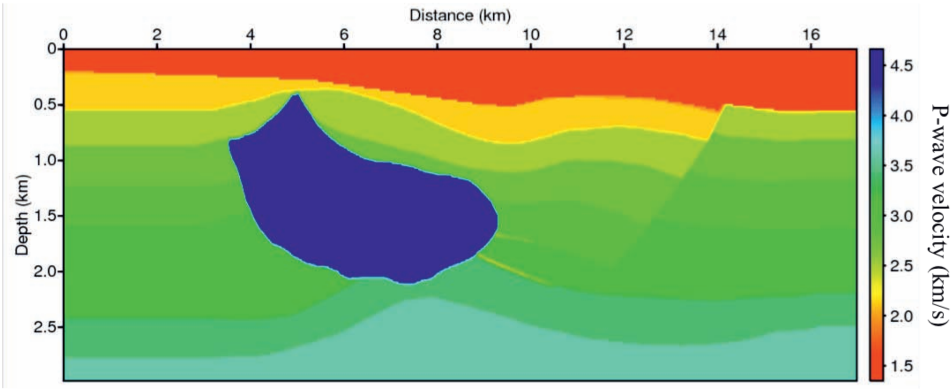
$\nabla_{density_{(ix,iz)}} \mathbf{E}_{\mathbf{L}}(ix,iz)$

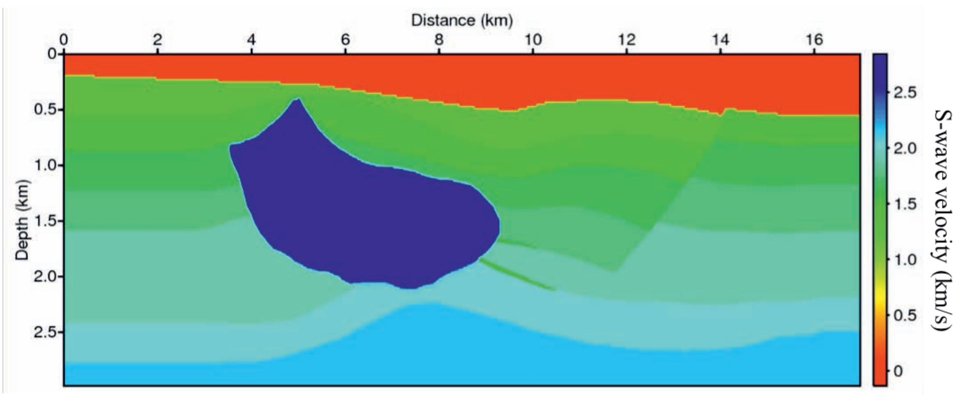
$$\nabla_{density(ix,iz)} \mathbf{E}_L(ix,iz) = \nabla_{density(ix,iz)} \left[\mathbf{E}(ix,iz) \cdot \mathbf{E}_{acc}(ix,iz) \right]$$

$$density_{(ix,iz)}^{l+1} = density_{(ix,iz)}^{l} - \alpha^l \sum_{s=1}^{nf} NRM \left[\frac{\nabla_{density_{(ix,iz)}} \mathbf{E}_{\mathbf{L}}(ix,iz)}{\sum_{i=1}^{n_s} (\mathbf{v}_{density_{(ix,iz)}})^T \cdot \mathbf{v}_{density_{(ix,iz)}} + \eta} \right]$$

Synthetic data example

λ μ


Distance (km)
0 2 4 6 8 10 12 14 16
Depth (km)
0 0.5 1.0 1.5 2.0 2.5
4.5 4.0 3.5 3.0 2.5 2.0 1.5
P-wave velocity (km/s)


Distance (km)
0 2 4 6 8 10 12 14 16
Depth (km)
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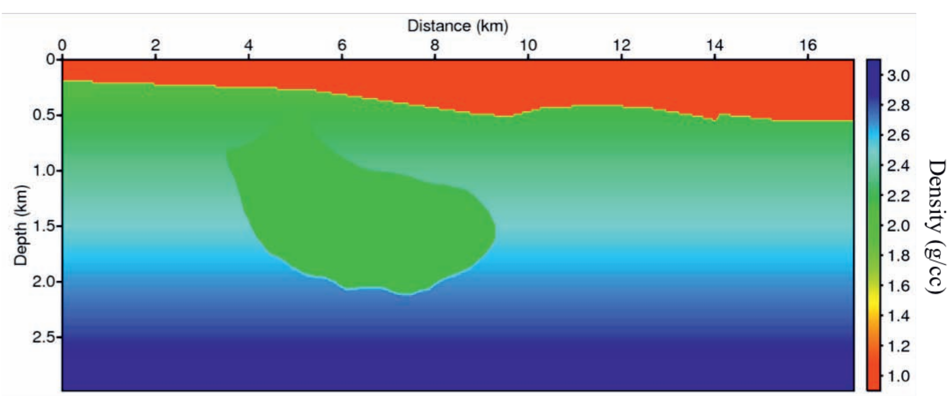
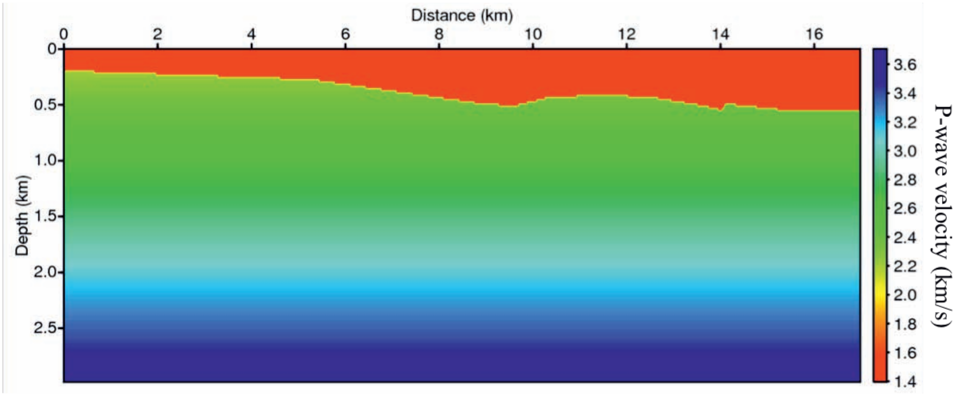
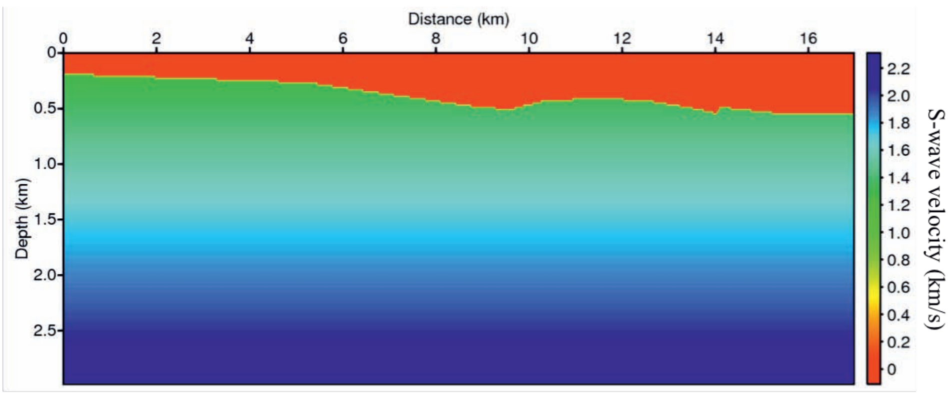
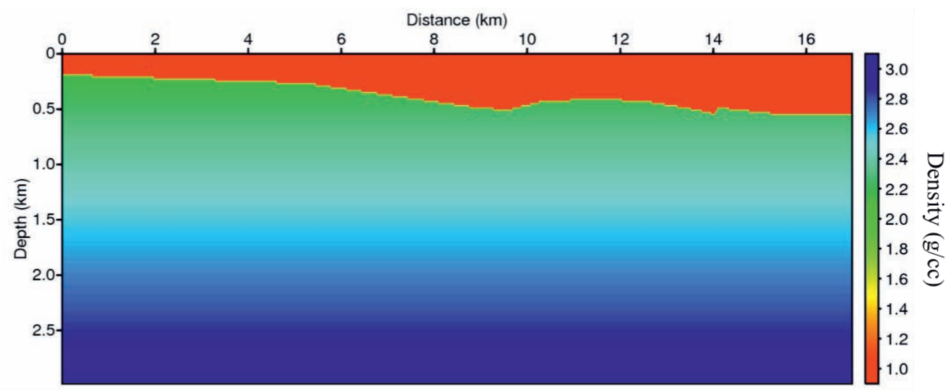
2.5 2.0 1.5 1.0 0.5 0
S-wave velocity (km/s)


Distance (km)
0 2 4 6 8 10 12 14 16
Depth (km)
0 0.5 1.0 1.5 2.0 2.5
3.0 2.8 2.6 2.4 2.2 2.0 1.8 1.6 1.4 1.2 1.0
Density (g/cc)


Distance (km)
0 2 4 6 8 10 12 14 16
Depth (km)
0 0.5 1.0 1.5 2.0 2.5
3.6 3.4 3.2 3.0 2.8 2.6 2.4 2.2 2.0 1.8 1.6 1.4
P-wave velocity (km/s)


Distance (km)
0 2 4 6 8 10 12 14 16
Depth (km)
0 0.5 1.0 1.5 2.0 2.5
2.2 2.0 1.8 1.6 1.4 1.2 1.0 0.8 0.6 0.4 0.2 0
S-wave velocity (km/s)


Distance (km)
0 2 4 6 8 10 12 14 16
Depth (km)
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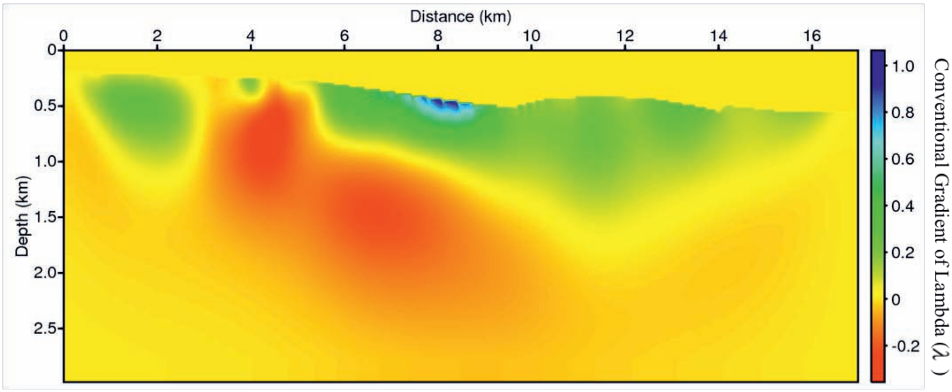
3.0 2.8 2.6 2.4 2.2 2.0 1.8 1.6 1.4 1.2 1.0
Density (g/cc)


Distance (km)
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Depth (km)
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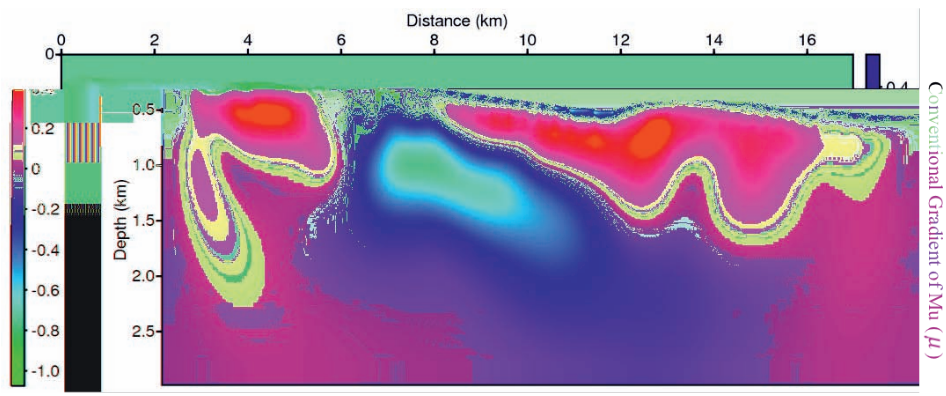
1.0 0.8 0.6 0.4 0.2 0 -0.2
Conventional Gradient of Lambda (λ)


Distance (km)
0 2 4 6 8 10 12 14 16
Depth (km)
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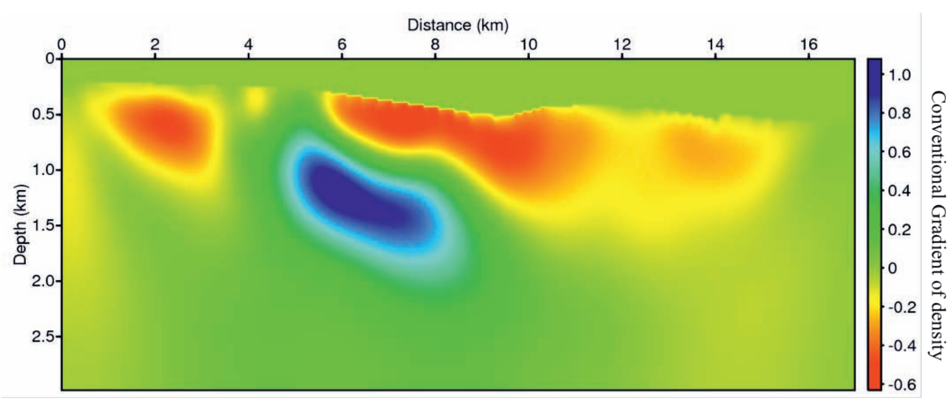
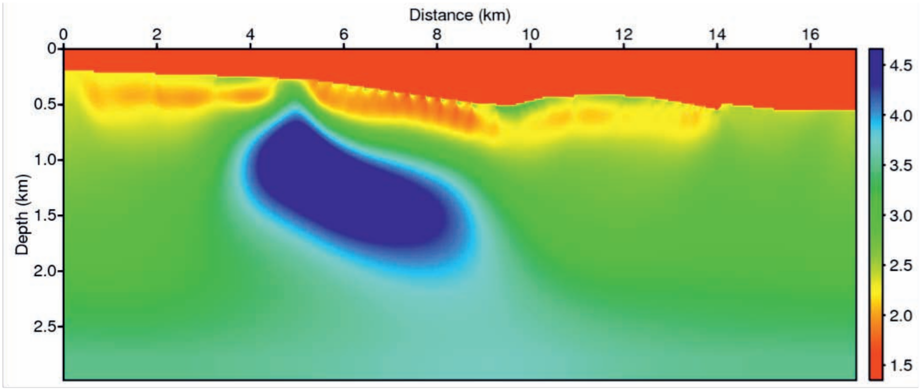
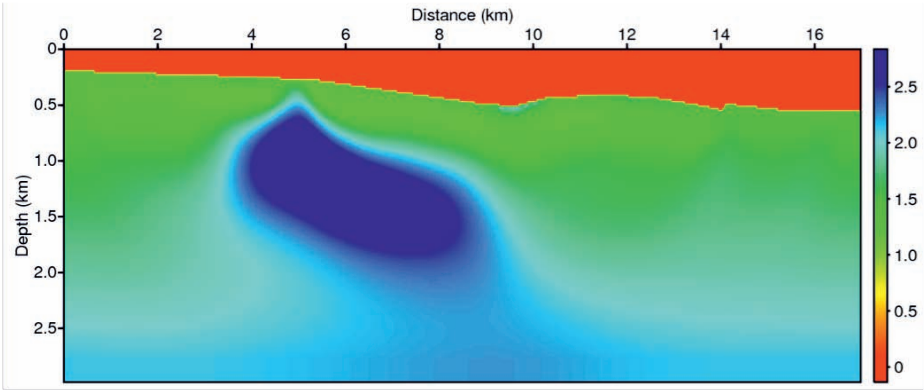
0.2 0 -0.2 -0.4 -0.6 -0.8 -1.0
Conventional Gradient of Mu (μ)


Distance (km)
0 2 4 6 8 10 12 14 16
Depth (km)
0 0.5 1.0 1.5 2.0 2.5
1.0 0.8 0.6 0.4 0.2 0 -0.2 -0.4 -0.6
Conventional Gradient of density

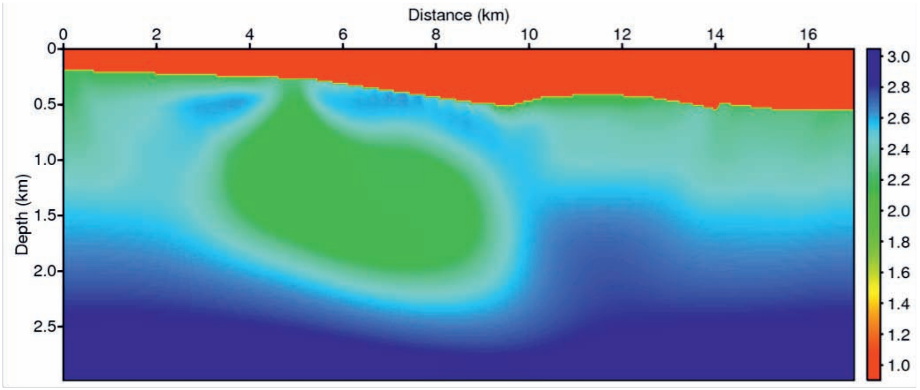
λ



Distance (km)
0
2
4
6
8
10
12
14
16
Depth (km)
0
0.5
1.0
1.5
2.0
2.5
4.5
4.0
3.5
3.0
2.5
2.0
1.5
P-wave velocity (km/s)

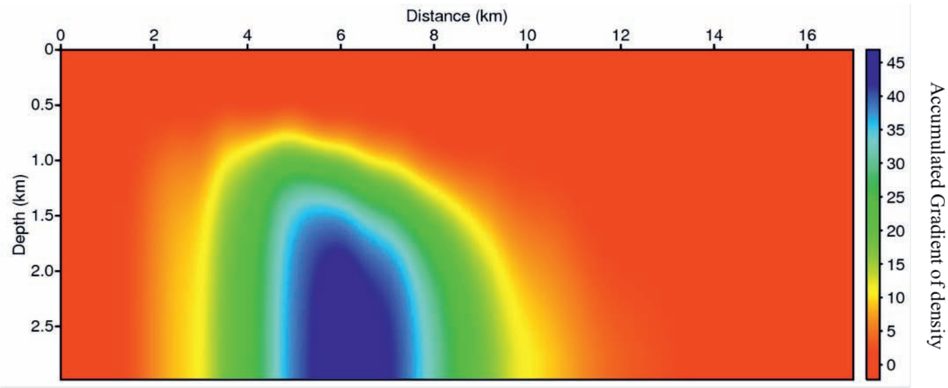


Distance (km)
0
2
4
6
8
10
12
14
16
Depth (km)
0
0.5
1.0
1.5
2.0
2.5
2.5
2.0
1.5
1.0
0.5
0
S-wave velocity (km/s)



Distance (km)
0
2
4
6
8
10
12
14
16
Depth (km)
0
0.5
1.0
1.5
2.0
2.5
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
Density (g/cc)

λ μ


Distance (km)
0 2 4 6 8 10 12 14 16
Depth (km)
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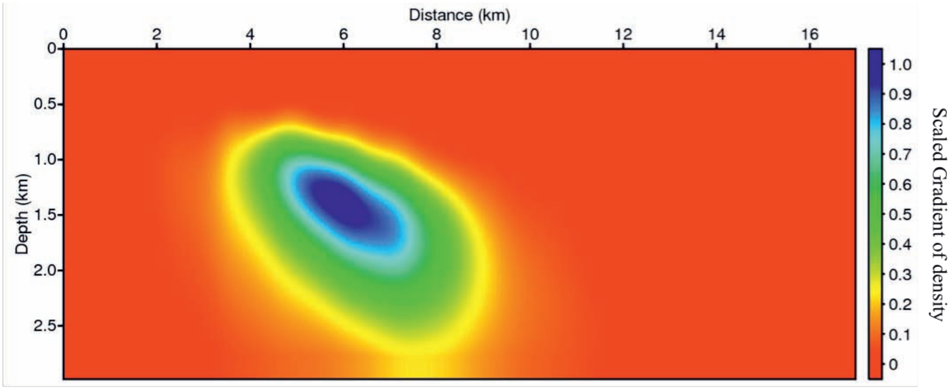
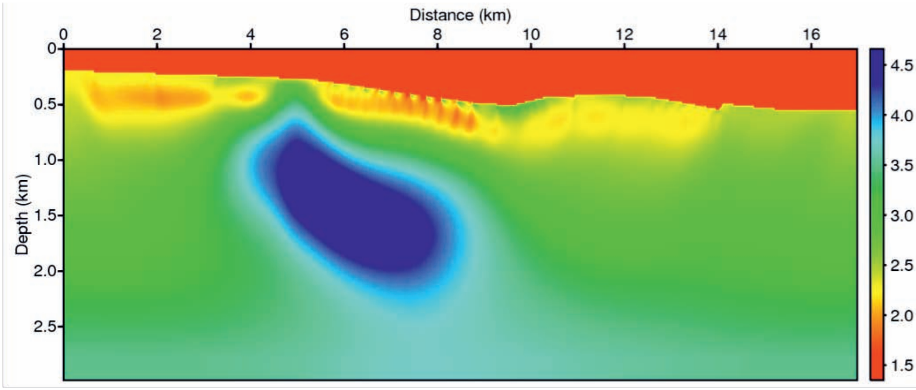
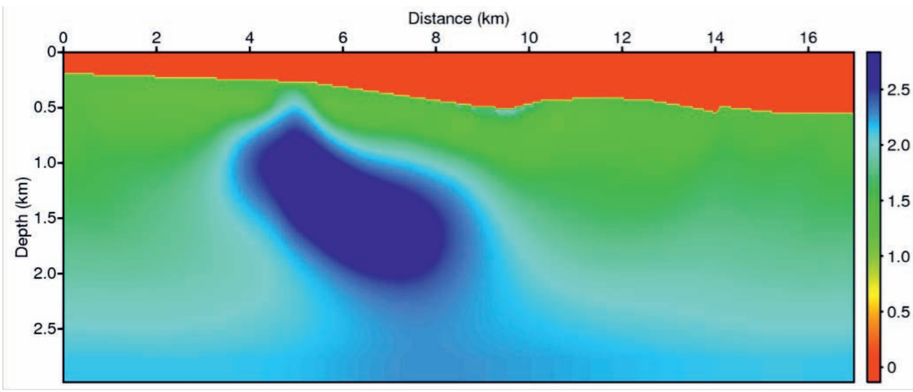
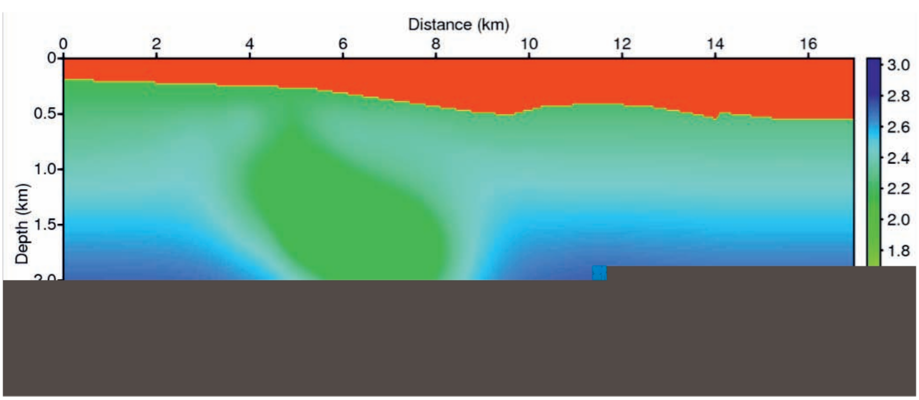
45 40 35 30 25 20 15 10 5 0
Accumulated Gradient of density

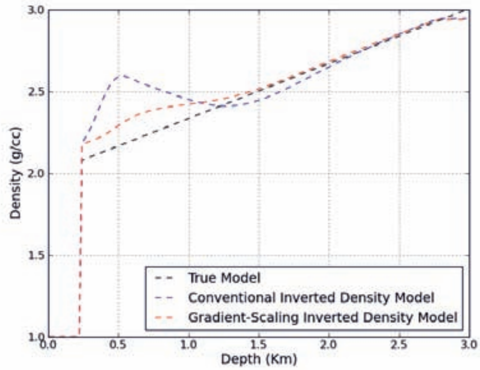

Distance (km)
0 2 4 6 8 10 12 14 16
Depth (km)
0 0.5 1.0 1.5 2.0 2.5
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
Scaled Gradient of density

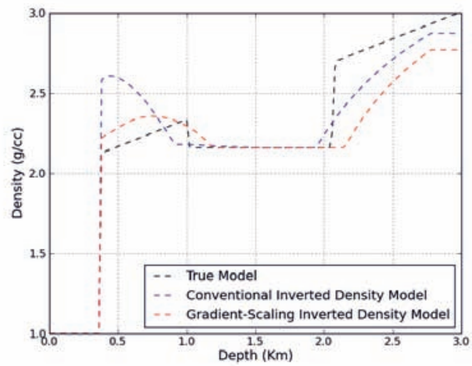
Real data example

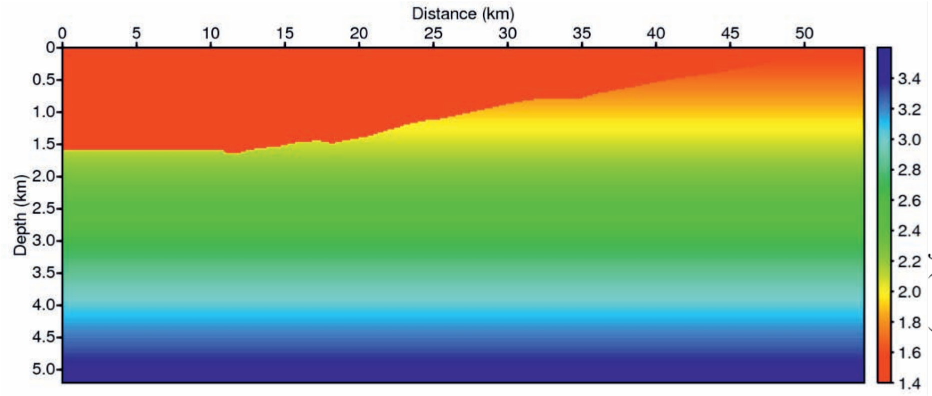

Distance (km)
0
2
4
6
8
10
12
14
16
Depth (km)
0
0.5
1.0
1.5
2.0
2.5
4.5
4.0
3.5
3.0
2.5
2.0
1.5
P-wave velocity (km/s)


Distance (km)
0
2
4
6
8
10
12
14
16
Depth (km)
0
0.5
1.0
1.5
2.0
2.5
2.5
2.0
1.5
1.0
0.5
0
S-wave velocity (km/s)

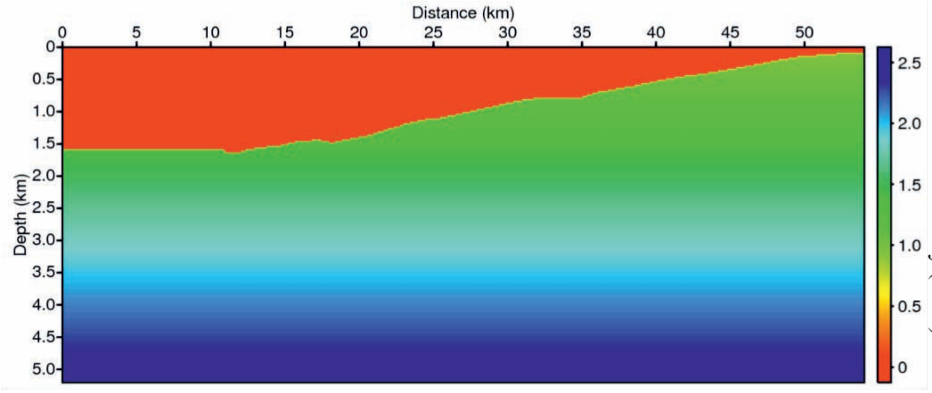

Distance (km)
0
2
4
6
8
10
12
14
16
Depth (km)
0
0.5
1.0
1.5
3.0
2.8
2.6
2.4
2.2
2.0
1.8
Density (g/cc)


Density (g/cc)
Depth (Km)
True Model
Conventional Inverted Density Model
Gradient-Scaling Inverted Density Model

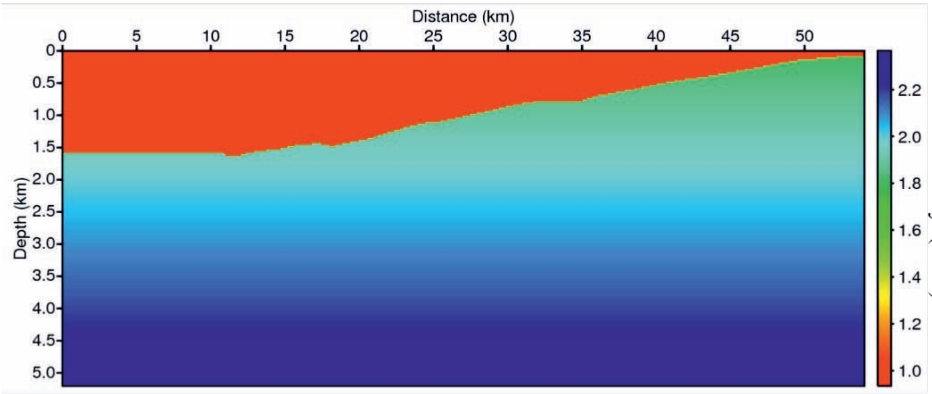

Density (g/cc)
Depth (Km)
True Model
Conventional Inverted Density Model
Gradient-Scaling Inverted Density Model



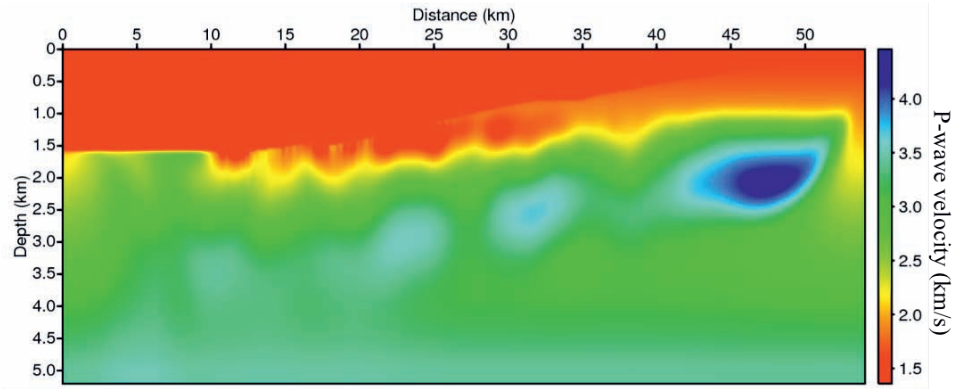
Distance (km)
0
5
10
15
20
25
30
35
40
45
50
Depth (km)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
P-wave velocity (km/s)

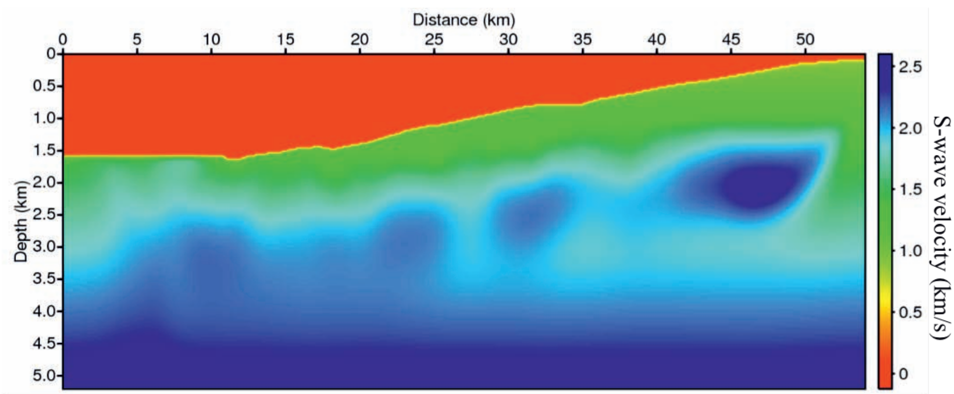


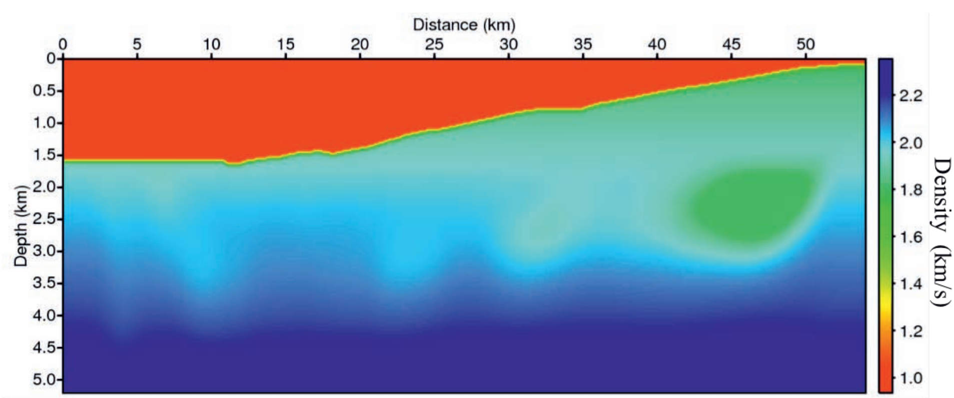
Distance (km)
0
5
10
15
20
25
30
35
40
45
50
Depth (km)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
2.5
2.0
1.5
1.0
0.5
0
S-wave velocity (km/s)



Distance (km)
0
5
10
15
20
25
30
35
40
45
50
Depth (km)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
2.2
2.0
1.8
1.6
1.4
1.2
1.0
Density (km/s)


Distance (km)
0 5 10 15 20 25 30 35 40 45 50
Depth (km)
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
P-wave velocity (km/s)
4.0 3.5 3.0 2.5 2.0 1.5


Distance (km)
0 5 10 15 20 25 30 35 40 45 50
Depth (km)
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
S-wave velocity (km/s)
2.5 2.0 1.5 1.0 0.5 0


Distance (km)
0 5 10 15 20 25 30 35 40 45 50
Depth (km)
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
Density (km/s)
2.2 2.0 1.8 1.6 1.4 1.2 1.0