

## SEISMIC EVENTS DETECTION IN STRONG LOW-FREQUENCY BACKGROUND NOISE BY COMPLEX SHOCK FILTER

GUANGHUI LI<sup>1</sup>, YUE LI<sup>2</sup> and XUAN LU<sup>1</sup>

<sup>1</sup> Department of Information Engineering, Shanxi University, Taiyuan 030013, P.R. China. [liguanghui0352@163.com](mailto:liguanghui0352@163.com)

<sup>2</sup> Department of Communication and Information Engineering, Jilin University, Changchun 130012, P.R. China. [liyue@jlu.edu.cn](mailto:liyue@jlu.edu.cn)

(Received May 3, 2017; revised version accepted November 23, 2017)

### ABSTRACT

Li, G.-H., Li, Y. and Lu, X., 2018. Seismic events detection in strong low-frequency background noise by complex shock filter. *Journal of Seismic Exploration*, 27: 57-68.

Low-frequency random noise in seismic exploration is difficult to suppress, because it is mixed with seismic events in time and frequency domain. In view of the line-like texture characteristic of seismic exploration and the line structure which seismic events show, we detect seismic events in noisy data by complex shock filter which is generated by incorporating the complex diffusion equation and shock filter. This method can detect seismic events in strong low-frequency random noise, and separate signals from noise effectively. Both the processing results of the synthetical records and the field data show the validity of this algorithm applied in events detection. The SNR (Signal to Noise Ratio) and resolution of seismic data is greatly enhanced.

**KEY WORDS:** complex diffusion equation, shock filter, event detection, SNR, resolution enhancement, low-frequency random noise.

### INTRODUCTION

Seismic events include most useful information, and have important significance in seismic signal analysis. For example, seismic events at time  $t_0$  reflect the interface depth in part, the offset of seismic events means the possibility of fault, and there is some relationship between the dynamic characteristics of seismic waves and stratigraphic characteristics, etc.

The recognition and detection of useful information is an important research field all the time. People use various ways to detect and recognize seismic events for the SNR and resolution enhancement of seismic data, including edge detection (Bondar, 1992), neural network (Liu et al., 1989), wavelet (Li and Zhu, 2000), pattern recognition (Witkin, 1983) and chaos theory (Li et al., 2006), etc. Moreover, 3D structure visualization research, tomography and other image processing methods open avenues for this research. Time-frequency characteristics of low-frequency seismic random noise are similar with seismic signals, and the signal-noise separation is a thorny problem in seismic data processing (Zhang, 2015). There are many methods effective for broadband random noise but ineffective for low-frequency noise.

In recent years, partial diffusion equation (PDE) techniques have been extensively applied to seismic profiling processing, especially which is generated from heat diffusion. The diffusion filtering method is evolving through several levels, from linear to nonlinear, isotropy diffusion to anisotropy. The linear PDE method is introduced to image processing by Witkin (Witkin, 1983) and Koenderink (1984) in the 1980's, then Perona and Malik (1990) addressed this issue by using the general diffusion form to construct a nonlinear isotropy adaptive denoising process. Fhemers and Hocker (2003) applied the diffusion filtering method to seismic data processing for the first time, and proposed the structure-oriented edge-preserving anisotropy diffusion smoothing method which can enhance the structure characteristics of seismic data (Fhemers and Hocker, 2003). Directly inspired by quantum mechanics, Guy Gilboa (Gilboa et al., 2002, 2004) promoted the real anisotropy diffusion equation to complex by combining shock filter which is proposed by Osher and Rudin (1990, 1991).

In this paper, complex shock filter is used to detect seismic events in strong low-frequency random noise, and the detection results are compared with the results of complex diffusion equation. The comparative results show that seismic events detected by complex shock filter can appear clearly in whether the synthetical records or the field data, and the SNR and resolution is improved obviously.

## THEORY FOUNDATION

### Diffusion equation

The diffusion equation is derived from heat equation which describes the distribution characteristics of heat source at different locations and different time. It is written by (Nagasawa, 1993)

$$\begin{cases} I_t = \sigma_p \nabla^2 I, 0 < \sigma_p \in \mathbb{R} \\ I|_{t=0} = I_0 \end{cases}, \quad (1)$$

where  $I_0$  represents the noisy signal,  $\sigma_p$  the diffusion coefficient whose unit is  $m^2 \cdot s^{-1}$ , and  $\nabla$  the gradient operator, respectively. The subscript  $t$  in  $I_t$  denotes the partial derivative  $\partial/\partial t$ .

In eq. (1), the noisy signal  $I_0$  is taken as the initial condition, and the solution  $I$  is the de-noised signal by diffusion in the scheduled time.

### Schrödinger equation

The Schrödinger equation which is a basic equation of quantum mechanics, describes the stationary state of a particle in a three-dimensional potential field. It can be written as

$$i\hbar \frac{\partial \Theta}{\partial t} = -\frac{\hbar^2}{2m_p} \Delta \Theta + V_p(x) \Theta \quad , \quad (2)$$

where  $\Theta = \Theta(t, x)$  denotes the wave function of quantum particle,  $m_p$  the particle mass,  $\hbar$  the reduced Planck constant and  $2\pi\hbar = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$ ,  $V_p(x)$  the external potential field,  $\Delta = (\partial^2/\partial x^2) + (\partial^2/\partial y^2) + (\partial^2/\partial z^2)$  the Laplace operator,  $i = \sqrt{-1}$  the imaginary unit, respectively.

It can be seen that the Schrödinger equation is the diffusion process of a complex wave function. Eq. (1) is a free particle Schrödinger equation when  $V_p(x) = 0$ .

### Complex diffusion equation

Gilboa, Sochen and Zeevi (Gilboa et al., 2002, 2004) expand the diffusion equation from the real number field to the complex number field. The complex linear diffusion equation which is the combination of eq. (2) and the free particle Schrödinger equation can be expressed as

$$\begin{cases} u_\tau = Du_{xx}, D \in \mathbb{C} \\ u(x, 0) = u_0, x \in \mathbb{R} \end{cases} \quad , \quad (3)$$

in which  $x$  denotes the distance variable,  $\tau$  is the time variable of the complex field and  $\tau = c_p t, t > 0$ ,  $t$  is the time variable of the real field and  $c_p, c_p \in \mathbb{C}$  is the constant,  $D$  is the diffusion coefficient and  $D = r_D e^{i\theta}$ ,  $r_D$  is the modulus of  $D$ ,  $i$  is the imaginary unit,  $\theta$  is the phase angle and

$$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), u_0 \text{ is the noisy signal, and } u(x, t) \in \mathbb{C}.$$

It is noticed that when eq. (3) is applied to signal processing in seismic exploration, the physical meaning of  $t$  and  $x$  would change accordingly. The parameter  $t$  is a non-dimensional parameter instead of the time variable, and  $x$  denotes sampling point of a single channel seismic signal in seismic records instead of distance variable.  $u_0$  denotes a noisy signal which is the initial condition of eq. (3), and  $u(x,t)$  denotes the de-noised signal.

The basic solution of eq. (3) is expressed by

$$h(x,t) = \frac{K_p e^{(-x^2/4tc_p)}}{2\sqrt{\pi tc_p}}, \quad (4)$$

where  $K_p (K_p \in \mathbb{C})$  is a constant related to  $u_0$ , and satisfies  $K_p = 1$  when  $c_p \in \mathbb{R}$ .

When the real part and the imaginary part are separated, eq. (4) can be written as

$$\begin{aligned} h(x,t) &= \frac{K_p e^{-i\theta/2}}{2\sqrt{\pi tr_D}} e^{-x^2 \cos\theta/(4\pi r_D)} e^{ix^2 \sin\theta/(4\pi r_D)} \\ &= K_p A_p g_\sigma(x,t) e^{i\alpha(x,t)} \end{aligned} \quad (5)$$

$$g_\sigma(x,t) = \frac{1}{\sqrt{2\pi\sigma(t)}} e^{\left(\frac{-x^2}{2\sigma^2(t)}\right)},$$

$$\sigma(t) = \sqrt{2tr_D / \cos\theta} \quad A = \frac{1}{\sqrt{\cos(\theta)}},$$

$$\alpha(x,t) = \frac{x^2 \sin\theta}{4tr_D} - \frac{\theta}{2},$$

and the other parameters are the same as the above.

There needs to be  $\lim_{t \rightarrow 0} h(x,t) = \delta(x)$ , if eq. (3) satisfies the initial condition  $u(x,t) = u_0$ . It can be known  $K_p = 1/A_p$  in eq. (5) by reason of  $\lim_{t \rightarrow 0} g_\sigma(x,t) e^{i\alpha(x,t)} = \delta(x)$ , and the basic solution of eq. (5) can be written as  $h(x,t) = g_\sigma(x,t) e^{i\alpha(x,t)}$ . When  $t \geq 0$  and  $\theta \rightarrow 0$ , eq. (5) can be written as



$$\begin{cases} \lim_{\theta \rightarrow 0} \operatorname{Re}(u) = g_{\sigma} * u_0 \\ \lim_{\theta \rightarrow 0} \frac{\operatorname{Im}(u)}{\theta} = \operatorname{tr}_D \Delta g_{\sigma} * u_0 \end{cases}, \quad (6)$$

in which  $\operatorname{Re}(u)$  is the real part of  $u(x,t)$ ,  $\operatorname{Im}(u)$  is the imaginary part of  $u(x,t)$ ,  $u_0$  is the noisy data,  $\Delta$  the Laplace operator, respectively.

From eq. (6), it can be seen that the real solution is the convolution between a Gaussian window and noisy data. This can be used to smooth the noisy data. The imaginary solution is the second derivative of the real solution. This can be used for edge detection.

If the imaginary solution of eq. (6) is directly used for seismic events detection in strong low-frequency noise, the detected edge is fairly rough, and could not detail stratum information.

### Shock filter

Shock filter proposed by Osher and Rudin (1991) is a hyperbolic equation. It can be used as a stable sharpening process, and its basic idea is that a dilation operator is used at the maxima of an image while a erosion operator at the minimum. It is estimated that whether a pixel is around the maxima or minimum by Laplace operator. The shock filter equation is expressed as (Gilboa et al., 2002)

$$I_t = -|I_{xx}|F(I_{xx}) \quad , \quad (7)$$

where  $F$  must satisfy  $F(0) = 0$ , and  $F(s)\operatorname{sign}(s) \geq 0$ . When  $F(s) = \operatorname{sign}(s)$ , the typical shock filter equation is written as

$$I_t = -\operatorname{sign}(I_{xx})|I_x| \quad , \quad (8)$$

The extension of eq. (8) from 1-dimensional space to 2-dimensional space can be written as

$$I_t = -\operatorname{sign}(I_{\eta\eta})|\nabla I| \quad , \quad (9)$$

where  $\eta$  denotes the gradient direction.

Contrary to noise smoothing, shock filter can sharpen edges, and it is applied to increasing differences on both sides of grey image corners. This process is sensitive to noise, even though very weak noise would be amplified by shock filter. The combination of shock filter and partial differential equation can control the sensitivity to noise, and have better edge-enhancing effect. The filtering operator  $F(s)$  in eq. (7) has significant

effect on filtering process. In view of the importance of the second derivative, eq. (9) can be expanded in order to enhance edge sharpness, which can be expressed as

$$I_t = -\frac{2}{\pi} \arctan(aI_{xx}t)|I_x| + \lambda I_{xx} \quad , \quad (10)$$

in which  $a$  denotes the sharpening coefficient which controls the sharpening intensities around the gradient zero-crossing. Where  $\lambda$  is the complex diffusion coefficient and satisfies  $\lambda = r \exp(i\theta)$ ,  $|I_x|$  is the gradient of signal in  $x$ -direction. When  $\theta$  is close to zero, the imaginary part is approximated as the Laplacian transformation of Gaussian convolution of an image, and it can be used for edge detection. This operator is second smoothing derivative, it makes scale change with time instead of Gaussian convolution of the image.

From eqs. (9) and (10), the equation of the complex shock filter can be derived, which is expressed as (Gilboa et al., 2002)

$$I_t = -\frac{2}{\pi} \arctan(a \operatorname{Im}\left(\frac{I}{\theta}\right))|I_x| + \lambda I_{xx} \quad . \quad (11)$$

Eq. (11) is expanded to the 2-dimensional space, the equation is written as

$$I_t = -\frac{2}{\pi} \arctan(a \operatorname{Im}\left(\frac{I}{\theta}\right))|\nabla I| + \lambda I_{\eta\eta} + \tilde{\lambda} I_{\xi\xi} \quad , \quad (12)$$

in which  $\tilde{\lambda}$  is the diffusion factor of the real part,  $\eta$  the unit vector in the direction of a gradient,  $\xi$  the unit vector which is perpendicular with  $\eta$ , respectively. Therefore,  $I_{\eta\eta}$ ,  $I_{\xi\xi}$  in eq. (12) is expressed as

$$I_{\eta\eta} = (I_{xx}(I_x)^2 + 2I_xI_yI_{xy} + I_{yy}(I_y)^2)/((I_x)^2 + (I_y)^2) \quad (13)$$

$$I_{\xi\xi} = (I_{xx}(I_x)^2 - 2I_xI_yI_{xy} + I_{yy}(I_y)^2)/((I_x)^2 + (I_y)^2) \quad . \quad (14)$$

Complex shock filters can reduce edge ambiguity, its diffusion process follows the maximum minimum principle, that is the global maximums and minimums are confined to the initial conditions in any time without any new local extrema. There are different weighted values in edge and smoothing zone. In the edge zone, the orders of magnitude of second derivative are greater around the edge zero crossing points, and the sharpness is stronger. It can get smooth results but avoid the need for convolution between noisy signal and Gaussian signal in every iterative process. We introduce the complex shock filter to seismic event detection in strong low frequency background noise. Unlike the image edge detection methods which need realize image grizzled processing, a complex shock filter can directly detect the effective signals of a single channel seismic record. The detected results can stay the way of seismic records are, and detail stratum information.

Fig. 1 shows the detection of a pure rectangular pulse signal by complex shock filter. From Fig. 1, it can be found that the saltation edge of the pulse signal can be detected and expressed accurately. If signal waveforms change gently, accordingly the edge-detection waveforms change gently. For example, a sinusoidal signal and its detection result are shown in Fig. 2, it can be seen the waveform of detection result shown in Fig. 2(b) changes gently just like the original signal shown in Fig. 2(a). Fig. 3 shows a Ricker wavelet and its edge detection result, it can be seen the edge of Ricker wavelet can be detected completely. So when a Ricker wavelet and low-frequency noise mixes to a noisy signal, the effective signal can be detected clearly by complex shock filter, the detection result is shown in Fig. 4.

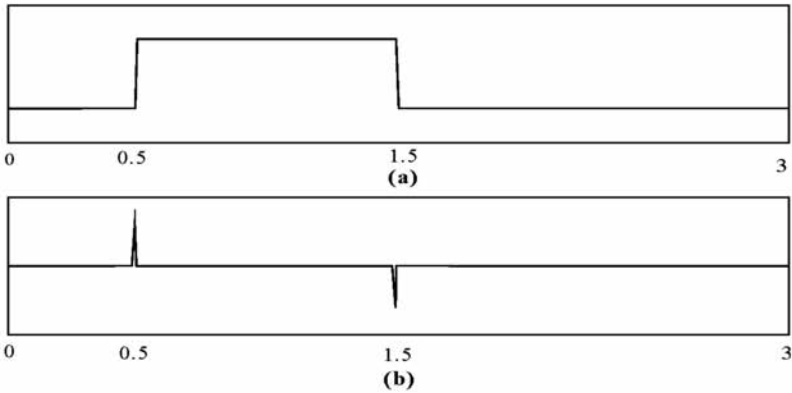


Fig. 1. Complex shock filter detection. (a) Rectangular pulse signal. (b) Edge detection result.

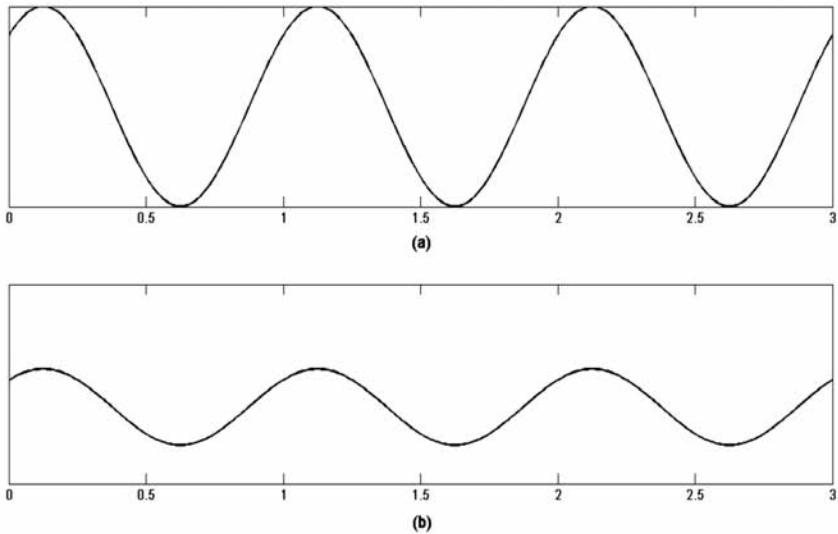


Fig. 2. Complex shock filter detection. (a) Sinusoidal signal. (b) Edge detection result.

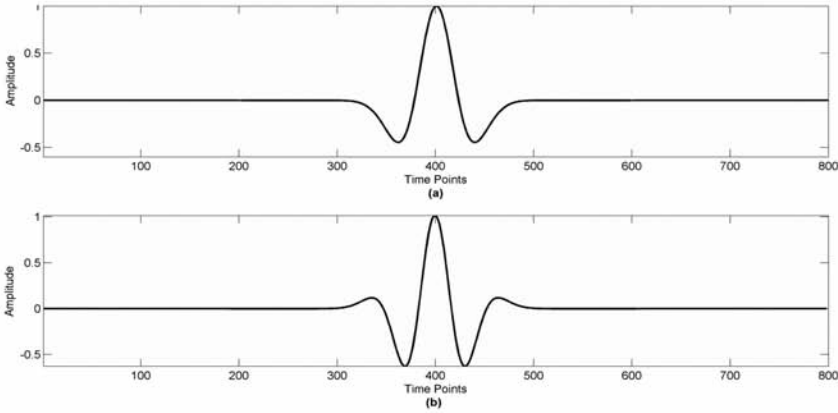


Fig. 3. Complex shock filter detection (a) Ricker wavelet (b) Edge detection result.

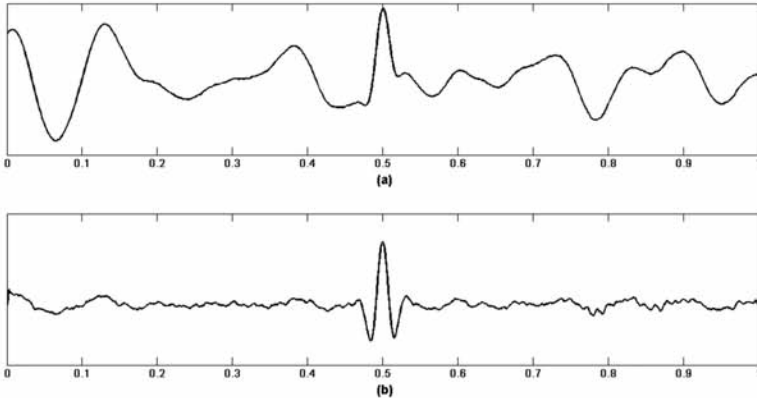


Fig. 4. Complex shock filter detection (a) Noisy seismic signal (b) Edge detection result.

## APPLICATION TO SEISMIC RECORDS

### Synthetic seismic record

The method on a 40-trace synthetic seismic record with strong low frequency is tested to investigate the detection performance of complex shock filter. The processing results of complex shock filter and the conventional complex diffusion equation is compared. Fig. 5(a) shows the noisy seismic synthetic record, the background frequency is intercepted from the first arrival noise of a real seismic record, and its the dominant frequency is 1~20 Hz. The dominant frequency of the three reflection events are 6 Hz, 12 Hz and 18 Hz, respectively. The noise is difficult to separate from effective signals. Fig. 5(b) shows the edge detection result by complex diffusion equation and Fig. 5(c) shows the detection result by complex shock filter. From Fig. 5(c), it can be seen that complex shock filter can detect the



seismic events accurately from the strong low frequency random noise, and the events are clear and continuous. From Fig. 5(b), we can see that complex diffusion equation can detect the events roughly, but the events are not coherent, especially the lower frequency signal.

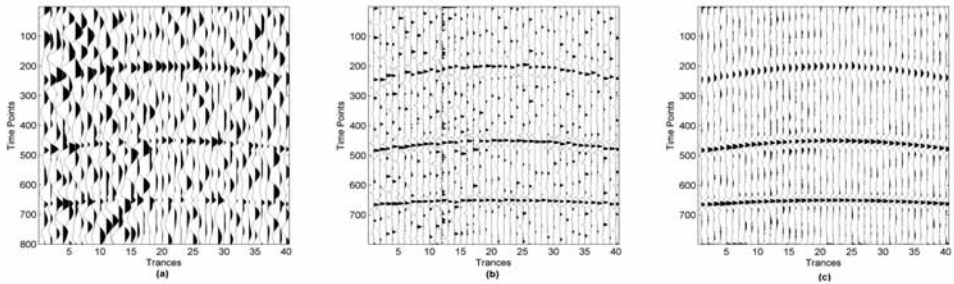


Fig. 5. Detection results of synthetic record (a) Noisy synthetic seismic record. (b) Result of complex diffusion equation (c) Result of complex shock filter.

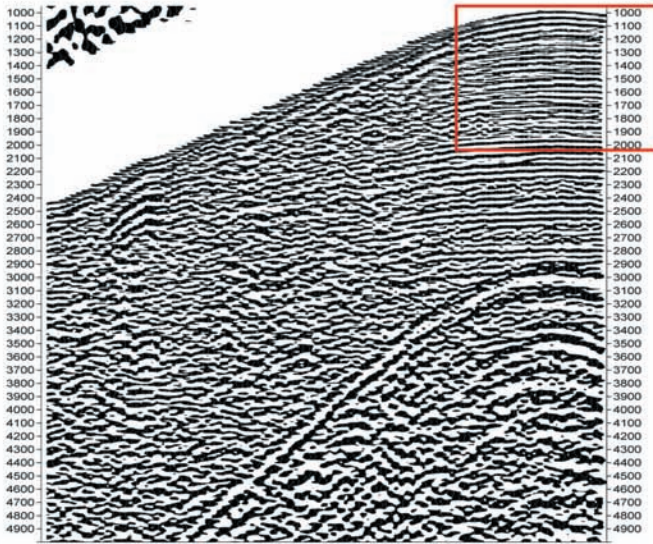
## Field data processing

In order to show the feasibility and effectiveness of the proposed approach in field data processing, we apply it to a 109-channel real seismic record of the desert, in this record, the events are merged by the low-frequency background noise. Figs. 6(a), 6(b) and 6(c) shows the noisy record, the detection result by complex diffusion equation and the detection result by complex shock filter, respectively. Fig. 7 shows the enlarged figures which appear in the boxes in Fig. 6.

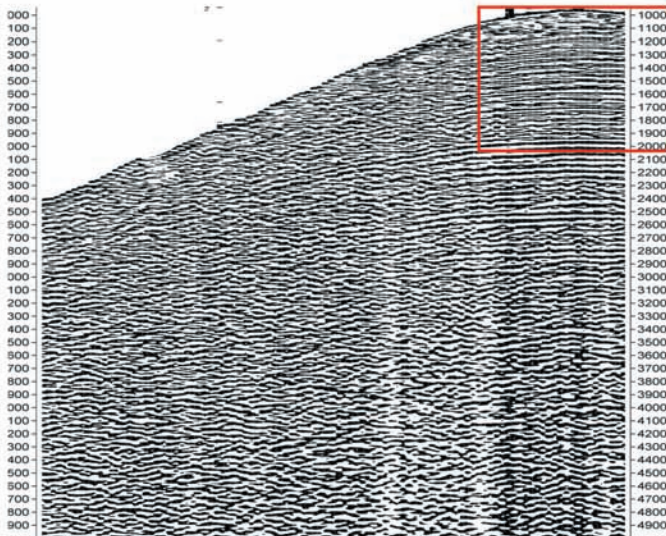
Through the comparison in Fig. 6(b) and Fig. 6(c), it can be seen that complex diffusion equation can detect the events from the background noise roughly, but the reflection events are still a little bit disordered, and cannot be identified clearly. Fig. 6(c) shows the result detected by complex shock filter, some hidden events are revealed, the events are more continuous and smoother, and the resolution of the field data is enhanced obviously. Besides, complex shock filter can effectively suppress surface wave interference.

## CONCLUSION

Complex shock filter which is obtained by combining complex diffusion equation with shock filter is applied to seismic events detection in this paper. It has both good noise immunity and ability of keeping edge. Complex shock filter can detect seismic events clearly from strong low-frequency background noise. The experimental results on seismic synthetic and field data confirmed the effectiveness of complex shock filter. It achieved better performance in detail preservation and resolution enhancement when compared with the complex diffusion processing.

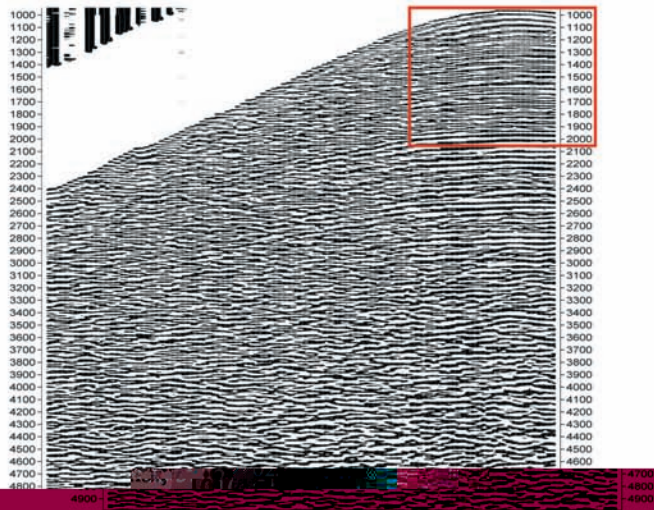


(a)



(b)

Fig. 6. Detection results of field data (a) Noisy seismic data (b) Result of complex diffusion equation.



(c)

Fig. 6. Detection results of field data. (c) Result of complex shock filter.

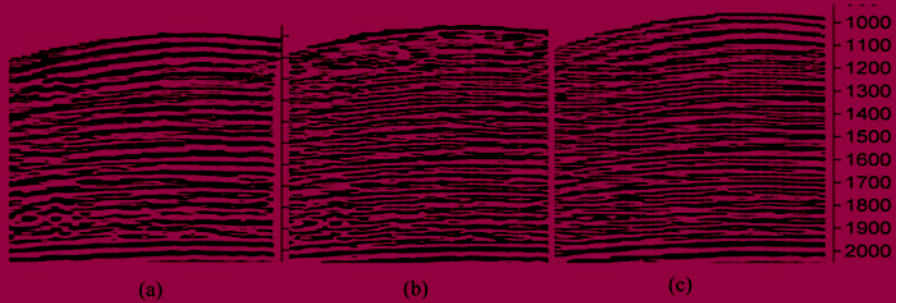


Fig. 7. Enlarged figures in the boxes of Fig.(6), (a) Noisy seismic data (b) Result of complex diffusion equation (c) Result of complex shock filter.

## ACKNOWLEDGEMENT

This research was supported by the National Natural Science Foundation of China (No. 41730422 ) and the Natural Science Foundation of Shanxi Province (No. 201701D221020).



## REFERENCES

- Bondar, I., 1992. Seismic horizon detection using image processing algorithm. *Geophys. Prosp.*, 40: 785-799. DOI: 10.1111/j.1365-2478.1992.tb00552.x
- Fehmers, G.C. and Höcker, C.F.W., 2003. Fast structural interpretation with structure-oriented filtering. *Geophysics*, 68: 1286-1293. DOI: 10.1190/1.1598121
- Gilboa, G., Sochen, N.A. and Zeevi, Y.Y., 2002. Regularized shock filters and complex diffusion. *Proc. European Conf. Computer Vision, ECCV 2002*: 399-313. DOI: 10.1007/3-540-47969-4\_27.
- Gilboa, G., Sochen, N.A. and Zeevi, Y.Y., 2004. Image enhancement and denoising by complex diffusion processes. *IEEE Transact. Patt. Anal. Mach. Intell.*, 26: 1020-1036. DOI: 10.1109/TPAMI.2004.47.
- Koenderink, J., 1984. The structure of images. *Biol. Cyber.*, 50: 363-370. DOI: 10.1007/BF00336961.
- Li, Y., Yang, B.J., Yuan, Y., Zhao, X.P. and Lin, H.B., 2006. Ability to detect weak effective seismic signals by utilizing chaotic vibrator system. *Chin. Sci. Bull.*, 51: 3010-3017. DOI: 10.1007/s11434-006-2191-0.
- Li, Q.C. and Zhu, G.M., 2000. Singularity detection of the thin bed seismic signals with wavelet transform. *Acta Seismol. Sin.*, 13: 61-66. DOI: 10.1007/s11589-000-0082-z.
- Liu, X.W., Xue, P. and Li, Y.D., 1989. Neural network method for tracing seismic events, *Expanded Abstr.*, 59th Ann. Internat. SEG Mtg., Dallas: 719-718. DOI:10.1190/1.1889749.
- Lu, S.Y. and Cheng, Y.C., 1990. An iterative approach to seismic skeletonization. *Geophysics*, 55: 1312-1320. DOI: 10.1190/1.1442778.
- Nagasawa, M., 1993. *Schrödinger Equations and Diffusion Theory*. Monographs in Math, 86. SEG, Tulsa, OK.
- Osher, S.J. and Rudin, L.I., 1990. Feature-oriented image enhancement using shock filters. *SIAM J. Numer. Anal.*, 27: 919-940. DOI: 10.1137/0727053.
- Osher, S. and Rudin, L., 1991. Shocks and other nonlinear filtering applied to image processing. In: Tescher, A.G. (Ed.), *Applications of Digital Image Processing XIV*. *Proc. SPIE*, 1567: 414-431. DOI: 10.1117/12.50835.
- Perona, P. and Malik, J., 1990. Scale-space and edge detection using anisotropic diffusion. *PAMI*, 12: 629-639. DOI: 10.1109/34.56205.
- Witkin, A., 1983. Scale-space filtering. *Internat. Joint Conf. Artificial Intellig.*, Karlsruhe, West Germany: 1019-1021. DOI: 10.1007/978-3-8348-9190-7\_29.
- Zhang, J.H., Zhang, B.B., Zhang, Z.J., Liang, H.X. and Ge, D.M., 2015. Low-frequency data analysis and expansion. *Appl. Geophys.*, 12: 212-220. DOI: 10.1007/s11770-015-0484-2.