APPLICATION OF SYNCHROSQUEEZED WAVE PACKET TRANSFORM IN HIGH RESOLUTION SEISMIC TIME-FREQUENCY ANALYSIS

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ABSTRACT

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Time-frequency (T-F) representation is a cornerstone in the seismic data processing and interpretation. It reveals the local frequency information that is hidden in the Fourier spectrum. The high resolution of the T-F representation is of great significance in depicting subtle geologic structures and in detecting anomalies associated with hydrocarbon reservoirs. The traditional T-F representations include short-time Fourier transform (STFT), continuous wavelet transform (CWT), S-transform (ST) and Wigner-Ville distribution (WVD). However, due to the uncertainty principle and cross-term, these methods suffer from low time-frequency resolution. In this paper, we introduce a new methodology for obtaining a high-quality T-F representation which is termed the synchrosqueezed wave packet transform (SSWPT). It is the first time that SSWPT is applied to multichannel seismic data time-frequency analysis. The SSWPT is a promising tool to provide detailed T-F representation. We validate the proposed approach with a synthetic example and compare the result with existing methods. Two field examples illustrate the effectiveness of SSWPT to identify subtle stratigraphic features for reservoir characterization.

KEY WORDS: time-frequency representation, instantaneous frequency,
synchrosqueezed wave packet transform, seismic data processing.

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INTRODUCTION

Geophysical data often exhibit non-stationary nature. Non-stationary in seismic data processing means that the frequency content of signal varies as a function of time (Cohen, 1989). The Fourier transform (FT) gives the overall frequency information. Therefore, it is inadequate to analysis a non-stationary signal. As a powerful tool for the analysis of non-stationary signals, time-frequency decomposition (also called spectral decomposition) maps a 1D seismic trace into a 2D function of time and frequency (Sinha et al., 2005), thus it has the potential to extract local anomalous behavior that might be buried in a broadband seismic response and to resolve thin layers at high frequencies (Li et al., 2016). There exist various methods for time-frequency analysis. The most commonly used approach is STFT which produces a time-frequency map by applying the Fourier transform over a chosen time window. In STFT, the time-frequency resolution is fixed due to the preselected window length. A longer window leads to poor time resolution while a shorter window improves the time resolution. Therefore, in seismic data analysis, the resolution is dependent on the user-specified window length. To overcome the limitations of STFT, the CWT is developed and applied in many spectral interpretation methods (Chakraborty et al., 1995). From a point view of window analysis, the CWT is full adaptive because the usage of scale factor which is inversely proportional to frequency. However, the CWT offers a time-scale representation and the scale represents a frequency band, this makes it intuitive when we want to interpret the frequency content of the signal. S-transform is proposed by Stockwell et al. (1996), and is conceptually a hybrid of the STFT and the CWT. The S-transform is a fast method exhibiting improved temporal resolution relative to the CWT that directly provides a valid time-frequency analysis (Li et al., 2016). The Wigner-Ville distribution (WVD) (Jeffrey and William, 1999) exhibits the least amount of energy spread in the T-F place, however, the existence of cross or interference term without physical meaning restricts its application. Matching pursuit (MP) (Mallat and Zhang, 1993) decomposes signal into waveforms selected from a dictionary of time-frequency atoms, it gives a higher time-frequency resolution at the expense of computation due to the redundancy of the atom dictionary. Castagna et al. (2003) use MP for seismic spectral analysis to detect the low-frequency shadows associated with hydrocarbon reservoirs. Empirical mode decomposition (EMD) (Huang et al., 1998) is a fully adaptive time-frequency decomposition technique and produces higher T-F resolution than wavelet-based method. Han and van der Baan (2013) investigated the effectiveness of EMD and its extensions ensemble EMD (EEMD) and complete EEMD (CEEMD) in seismic data analysis. In spite of its considerable success in practical application, there is still a lack of solid mathematic foundation for EMD approaches. The synchrosqueezing transform (SST) (Daubechies et al., 2011), originally introduced in the audio signal analysis, combines the classical wavelet transform and a reassignment technique to increase the time-frequency resolution. The SST has been applied to seismic time-frequency analysis (Wang et al., 2014; Herrera et al., 2014) and obtained significantly improved resolution than conventional methods. Following this research, synchrosqueezing short-time Fourier transform (Oberlin et al., 2014) and a generalized synchrosqueezing transform (Li and Liang, 2012) have been proposed, respectively.

We extend the studies of seismic T-F analysis with a recently proposed method called synchrosqueezed wave packet transform (SSWPT) (Yang, 2014). Similar to SST, the SSWPT is also an adaptive method which improves the readability of T-F map using frequency reassignment technique. The wave packet transform (WPT) projects the analyzed signal into a family of wave packets that is obtained through scaling, translating, and modulating a "mother wavelet". By employing an extra geometric parameter, the WPT is able to obtain a flexible and well concentrated time-frequency representation, thus facilitates seismic data processing and further interpretation.

In this paper, we show the desirable performance of SSWPT in seismic time-frequency analysis. Firstly, we briefly describe the theory behind SSWPT. Next, we illustrate the improved T-F resolution of SSWPT over that of the commonly used methods using synthetic example. Finally, we apply SSWPT on 2D and 3D filed data sets to demonstrate its potential in highlighting geologic and stratigraphic feathers with high precision.

SYNCHROSQUEEZED WAVE PACKET TRANSFORM

In the following, $L^2(R)$ denotes the class of real square integrable function. A mother wave packet $\psi(t)\in L^2(R)$ is an analyzing function used to localize a signal in both frequency and time, and its Fourier transform $\hat{\psi}(\xi)$ is a non-negative, real-valued, smooth function with a support $(-d,d)$. By scaling, translating, and modulating this wave packet, we can produce a family of wave packets:

$$\psi^s_{a,b}(t) = a^{s/2}\psi(a^s(t-b))e^{2\pi i(t-b)a} \quad , \tag{1}$$

where $a,b \in R$ and b is translation parameters, a is scale parameter and is not zero. The time support of $\psi^s_{a,b}(t)$ changes for different scales. Different from CWT, the scale in the WPT is proportional to the centre frequency of the wave packet. The geometric parameter $s \in [0,1]$ is a given constant which controls the trade-off between STFT and wavelet transform WT. When s is equal to 1, the family of wave packets would be similar to the standard wavelets. If $s = 0$, the WPT is similar to STFT. Therefore, the geometric parameter can be used to adjust the time and frequency resolution. Because of the three free parameters, the WPT exhibits an additional degree of freedom compared with WT and STFT. Besides, the phase factor $e^{2\pi i(t-b)a}$ in eq. (1) preserves the phase information of each scale.

The WPT is defined as the inner product of signal $f(t)$ with the family of wave packets. This is given by

$$W_f(a,b) = \langle f(t),\ \psi_{a,b}^s(\mathrm{t}) \rangle = a^{s/2}\int_{-\infty}^{+\infty} f(t)\psi^*(a^s(t-b))e^{-2\pi i(t-b)a}dt, \qquad (2)$$

where ψ^* denotes the complex conjugate of ψ and $W_f(a,b)$ is the time-scale map. Eq. (2) can be easily computed in the Fourier domain using the Parseval's theorem, i.e.,

$$W_f(a,b) = \frac{1}{2\pi}a^{-s/2}\int_{-\infty}^{+\infty}\hat{f}(\xi)\hat{\psi}^*(a^{-s}(\xi-a))e^{2\pi ib\xi}d\xi \quad , \qquad (3)$$

where ξ is the frequency variable, $\hat{f}(\xi)$ and $\hat{\psi}(\xi)$ are the Fourier transform of $f(t)$ and $\psi(t)$, respectively. The scale factor a and geometric parameter s adjust the support of the complex wavelet function $\hat{\psi}^*(a^{-s}(\xi-a))$ by squeezing and stretching it. For a purely harmonic signal $f(t) = A\cos(2\pi f_0 t)$ with its Fourier transform

$\hat{f}(\xi) = A\pi[\delta(\xi-f_0)+\delta(\xi-f_0)]$, eq. (3) can be transformed into

$$\begin{aligned} W_f(a,b) &= \frac{A}{2}a^{-s/2}\int_{-\infty}^{+\infty}[\delta(\xi-f_0)+\delta(\xi-f_0)]\hat{\psi}^*(a^{-s}(\xi-a))e^{2\pi ib\xi}d\xi, \\ &= \frac{A}{2}a^{-s/2}\hat{\psi}^*(a^{-s}(f_0-a))e^{2\pi ibf_0}. \end{aligned} \qquad (4)$$

In the frequency plane, if $\hat{\psi}^*(\xi)$ is concentrated around $\xi = 0$, then the energy of WPT $|W_f(a,b)|$ will spread out on a time-frequency strip $Z = \{(a,b): |f_0 - a| \le a^s\}$, and the width of this strip depends on the support of $\hat{\psi}(\xi)$. In order to solve the diffusion problem, we define the reference instantaneous frequency (RIF) of the original signal $f(t)$ by taking the first-order derivative of $W_f(a,b)$ with respect to the time-shifting variable b, i.e.,

$$v_f(a,b) = \frac{-i}{2\pi W_f(a,b)}\frac{\partial W_f(a,b)}{\partial_b}. \qquad (5)$$

For the purely harmonic signal $f(t) = A\cos(2\pi f_0 t)$, we can obtain v_f (a,b), as desired. If we expect no confusion to occur, the TF coefficients which have the same RIF information in the time-scale plane are gathered by transforming the (a,b) plane to the (a,v) plane via

$$T_f(v_l,b) = \sum_{a_k: |v_f(a_k,b)-v_l| \le \Delta v/2} |W_f(a_k,b)|^2(\Delta a)_k \quad , \qquad (6)$$

where a_k is the k-th discrete scale variable that varies in the range specified by $\left|v_f(a_k,b)-v_l\right| \le \Delta v/2$, v_l the l-th discrete frequency variable, $(\Delta a)_k = a_k - a_{k-1}$ and $\Delta f = f_l - f_{l-1}$. Eq. (6) shows that the new T-F representation $T_f(v_l, b)$ is squeezed along the scale (or frequency) axis only. Fig. 1 demonstrates the procedure of SSWPT for a special case with $f(t) = A\cos(40\pi v_0 t)$. If we treat a and v as continuous variables without discretization, then eq. (6) is

$$T_f(v,b) = \int_{A_\varepsilon(b)} |W_f(a,b)|^2 \ \delta(\mathrm{Re}(v_f(a,b)) - v)da \ , \tag{7}$$

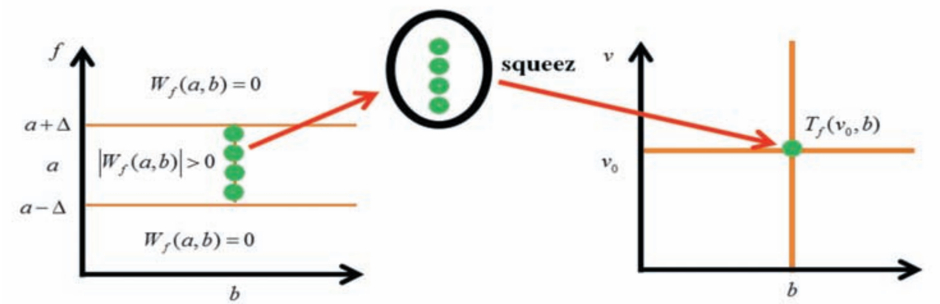

Fig. 1. Procedure in SSWPT for signal $f(t) = A\cos(2\pi v_0 t)$.

where δ is the Dirac delta function and $\mathrm{Re}(v_f(a,b))$ means the real part of $v_f(a,b)$, $v_f(a,b)$ is defined in (5) above, $A_\varepsilon(b) = \{a : |W_f(a,b)| > \varepsilon\}$ and the parameters $\varepsilon > 0$ is a hard threshold on $W_f(a,b)$ to overcome the shortcoming that $W_f(a,b) \approx 0$ is rather unstable when analyzed signal has been contaminated by noise. In (7), the operator T_f is called synchrosqueezing, and the procedure of synchrosqueezing is integrating the TF coefficients around the IF trajectory.

To sum up, the SSWPT is performed in the following steps:

Step 1. Apply the WPT to the analyzed signal $f(t)$, the result is $W_f(a,b)$.

Step 2. Given a threshold $\varepsilon > 0$. At any point (a,b) for which $|W_f(a,b)| > \varepsilon$, calculate the RIF $v_f(a,b)$ by eq. (5).

Step 3. Obtain the SSWPT using eq. (6).

In Fig. 2, the WPT, RIF, SSWPT of $f(t) = \cos(40\pi t)$ are displayed for a better understanding of the process.

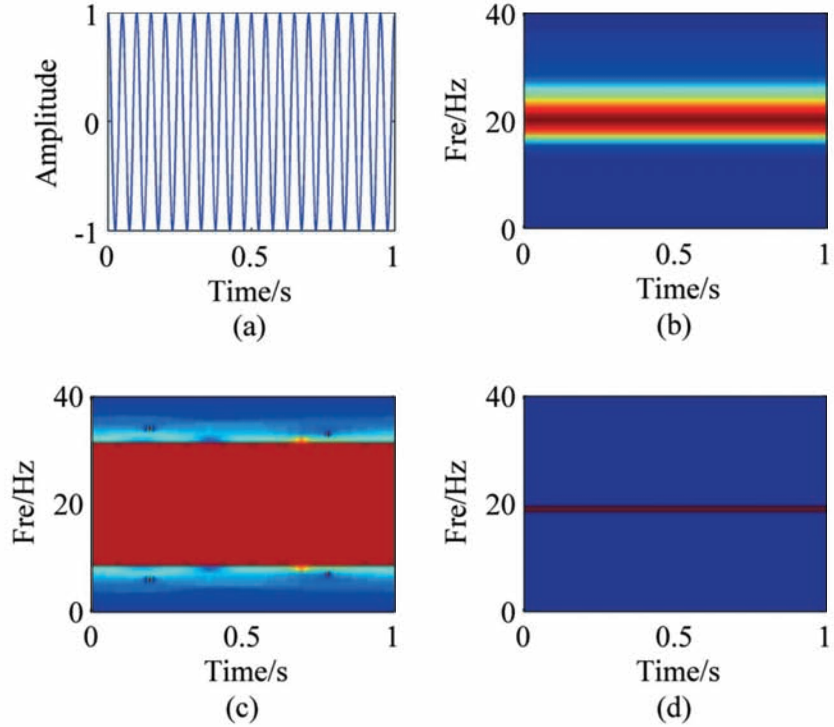

Fig. 2. (a) Synthetic signal $f(t) = \cos(40\pi t)$, (b) WPT, (c) RIF, (d) SSWPT.

In seismic time-frequency analysis, the optimal joint time-frequency concentration is not always required. For example, a better frequency resolution is necessary when we estimate the tuning thickness of fluvial channels, and a better time-localization is required for thin bed detection. Therefore, SSWPT is suitable for the given geologic goals due to its advantage of varying performances.

SYNTHETIC AND REAL DATA EXAMPLE

In this section, synthetic and real data examples are provided to demonstrate the effectiveness of SSWPT. CWT and SST are also presented as comparative methods. In all of these experiments, the CWT and SST use a Morlet wavelet with central frequency and bandwidth estimated from seismic signal. In SSWPT, we chose the parameter pair with $d = 1$ and $s = 2/3$. In order to reduce the noise interference, the threshold parameter used in all methods is $\varepsilon = 10^{-5}$.

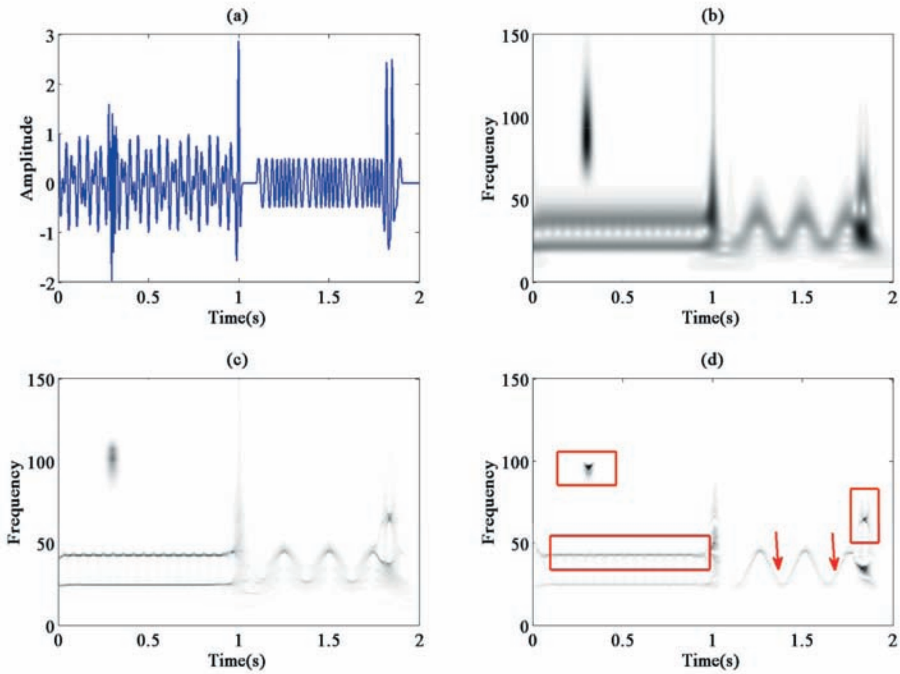

Fig. 3. Time-frequency representation of synthetic signal. (a) Synthetic example, (b) CWT, (c) SST, and (d) SSWPT. The rectangles and arrows show evident in time-frequency resolution of SSWPT with respect to the other approaches.

Firstly, we test the performance of SSWPT with a synthetic signal, which is composed of two initial cosine wave with 25 Hz and 40 Hz, with a 100 Hz Morlet atom at 0.3 s, three 40 Hz zero-phase Ricker wavelets at 1, 1.82 and 1.84 s, and a oscillation component with periodic representations based on CWT, SST and SSWPT, respectively. The CWT is able to distinguish these components but with low resolution, the energy spreads out along the frequency which leads to mixture between the adjacent components. The SST improves the behavior of CWT instantaneous frequency. Fig. 3 shows the time-frequency and gets a sharper time-frequency representation. However, due to its low frequency resolution at high frequencies, the blurring effect still exists at high frequencies. In comparison with CWT and SST, the SSWPT perfectly delineates all these individual components and shows the highest time-frequency resolution.

Next, we apply the SSWPT to a 2D post-stack seismic section (see Fig. 4) from CNOOC (China National Offshore Oil Corporation). We take the 51st seismic trace which is plotted in Fig. 3(a) as an example.

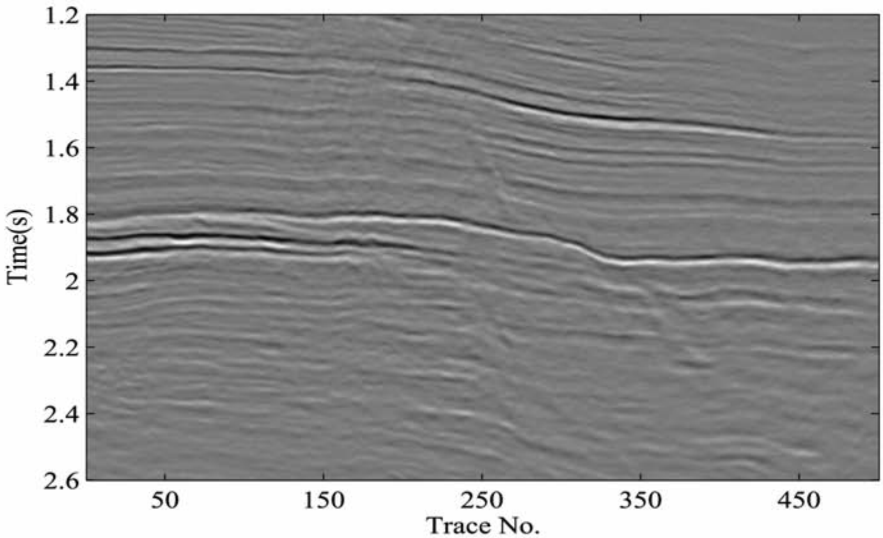

Fig. 4. 2D field seismic data from CNOOC.

The time-frequency spectral based on CWT, SST and SSWPT are shown in Fig. 5 (b)-(d), respectively. We can observe that all methods display a decrease in frequency content over time, most likely caused by attenuation. The SST and SSWPT produce sparer representation and show more features than CWT. In SSWPT image, we can see clear components in the high frequency part and the low frequency components are still accurate, which is crucial for further seismic interpretation and better understanding the subsurface lithology properties, however, in the result of SST (Fig. 5c), the high frequency components would mix together.

The CWT, SST and SSWPT are then applied to the entire section to assess their efficiency at resolving frequency dependent phenomena underground. We extract the 40 Hz frequency slices using CWT, SST and SSWPT, respectively, which are shown in Fig. 6. The slice obtained by CWT is obscured and can not provide useful structural and stratigraphic information due to its low time-frequency resolution. The SSWPT result clearly detects the potential hydrocarbon reservoir that is almost lost in the SST, which demonstrate its effectiveness for identification and interpretation of thin beds.

Finally, we apply these methods to stratigraphic analysis by extracting a horizon slice from a 3D seismic data volume. Visualizing constant-frequency attributes for a horizon from 3D cube can be used to identify geologic structure that otherwise would not be visible on original horizon amplitude map (Sinha et al., 2005). We calculated the 35 Hz frequency volume of the 3D seismic data using CWT, SST and SSWPT, respectively. Then the horizon slices are extracted 38 ms under the picked horizon, as shown in Fig.7. The 3D seismic data are not shown for commercial reasons.

Fig.7(a) shows the original amplitude map of the picked horizon, where not much useful geologic information can be found. From the point of view of interpreter, the extension of channel can be used as an important tool for reservoir characterization. Thus, a precise detection of channels is significant in seismic interpretation. The result based on the CWT blurs the horizon slice, which makes the further interpretation challenging. The SST based horizon slice improves the performance of channel delineation. However, the result based on SSWPT characterize the channels best, and the amplitude variation along the channels is clear.

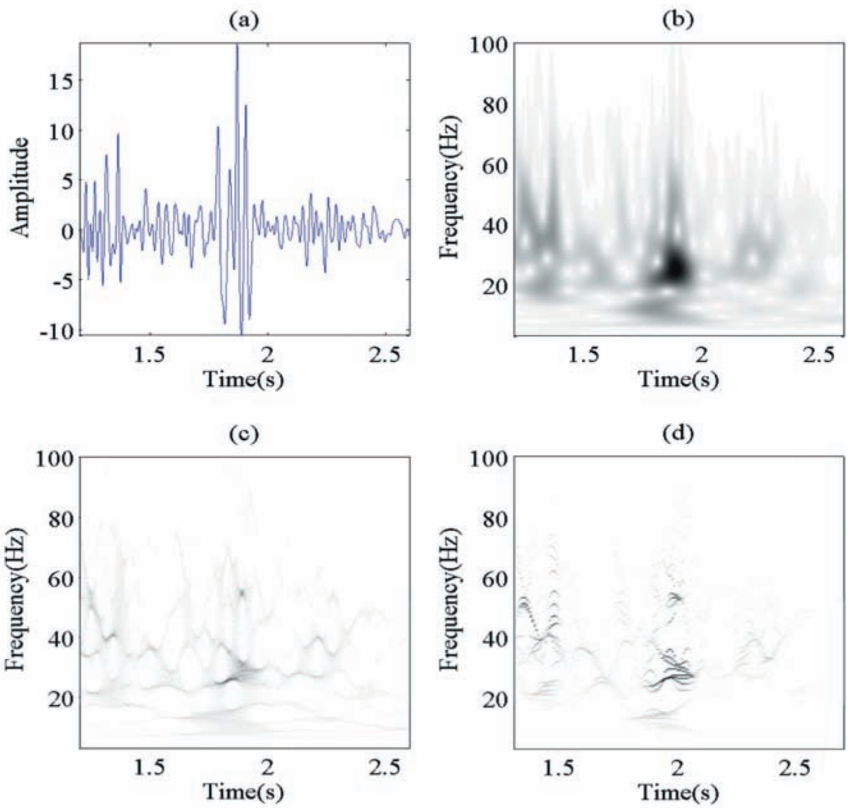

Fig. 5. (a) Individual trace at trace 51 in Fig. 2. Time-frequency representation from (b) CWT, (c) SST and (d) SSWPT. All methods show a decrease in frequency content over time, yet SST and SSWPT produce sparer representation than the CWT. We can see clear components in the high frequency part in SSWPT and the low frequency compoents are still accurate, if SST is applied, it would mix high frequency components together.

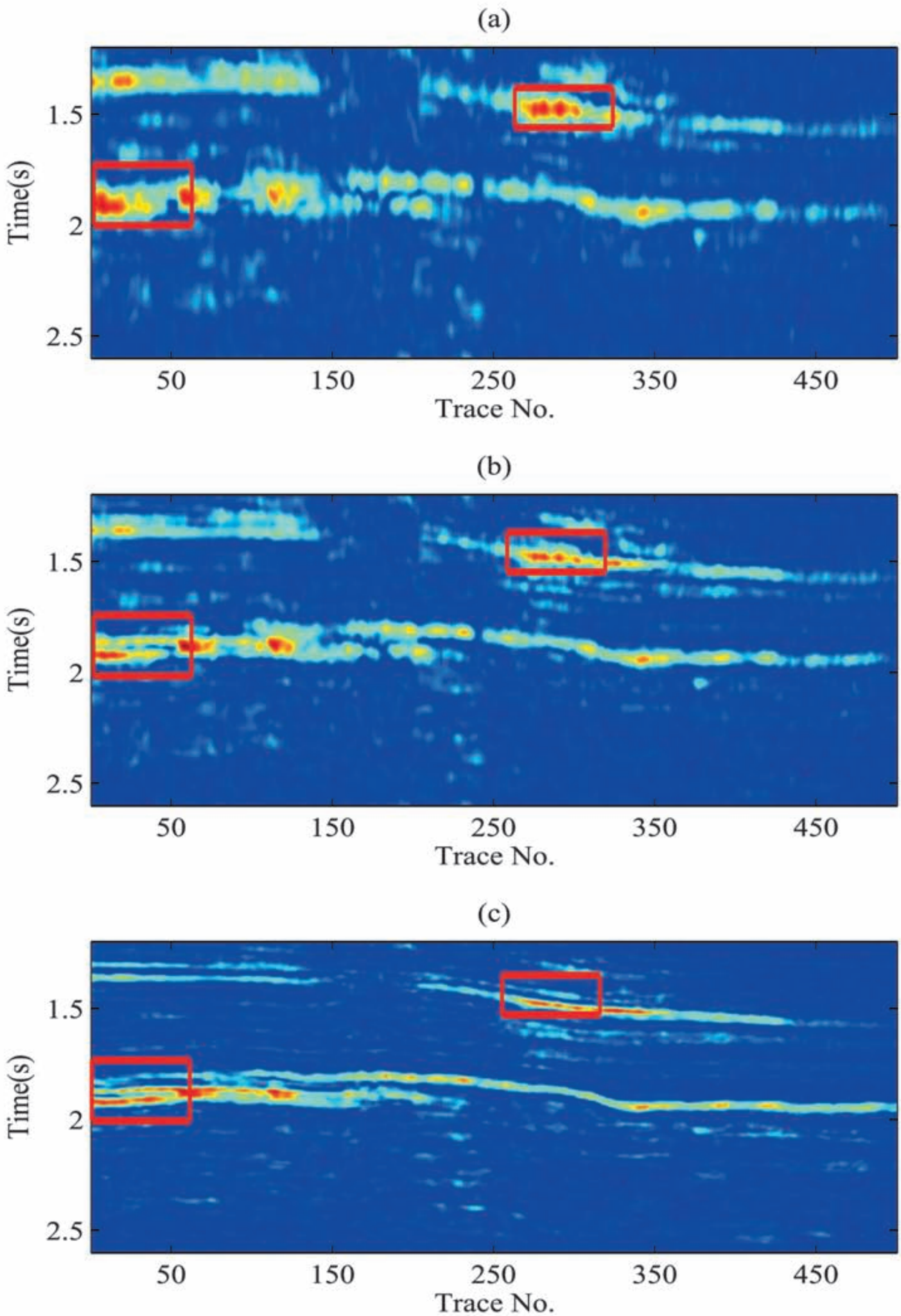

Fig. 6. Constant frequency slices using (a) CWT, (b) SST and (c) SSWPT at 40 Hz. The SSWPT has much greater time-frequency resolution than the CWT and SST, thus better delineating the amplitude variation near the target zones (red rectangle).

The advantage of SSPWT is the higher time-frequency resolution which has been demonstrated in the synthetic and field data examples. However, one potential drawback to this approach is the computation efficiency. We evaluated the computational cost of CWT, SST and SSWPT for both synthetic and real data examples, which is listed in the following Table:

Methods	Synthetic signal	2D field data	3D field data
CWT	0.15s	89.82s	9504.39s
SST	0.23s	123.48s	13261.18s
SSWPT	0.31s	182.06s	19187.94s

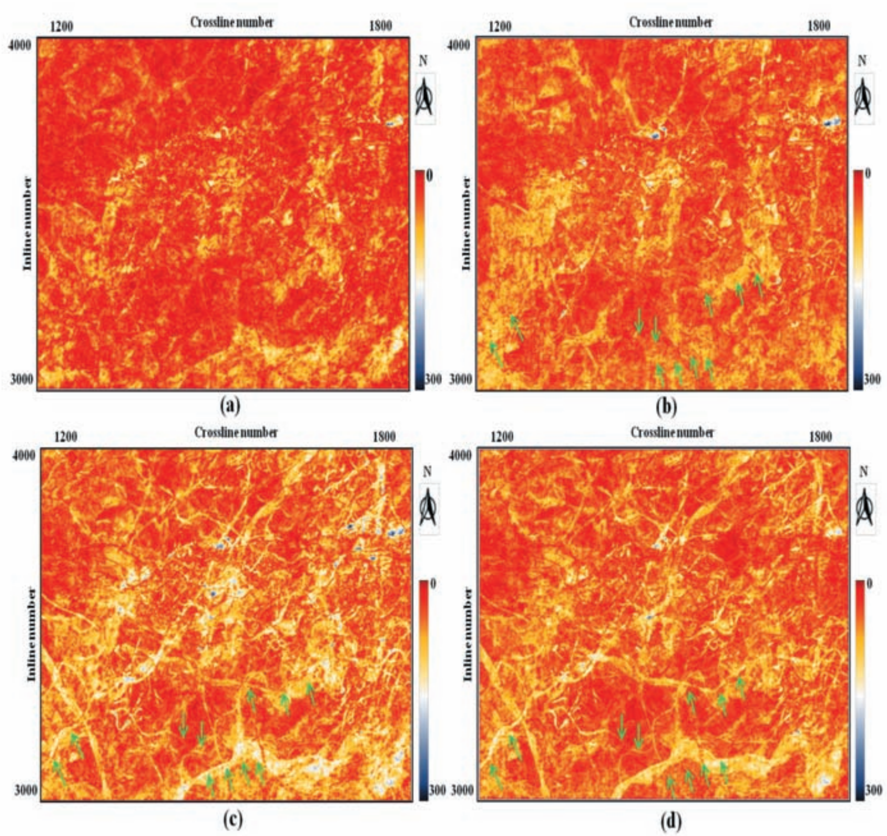

Fig. 7. Channel detection from horizontal slices. (a) Original amplitude map. (b)-(d) 35 Hz horizon slices based on CWT, SST and SSWPT, respectively. The SSWPT result characterizes the channels (indicated by green arrows) more clearly.

All experiments are carried out on a PC station equipped with an Intel core 8-core CPU clocked at 3.6 GHz and 8GB of RAM. Although the SSWPT is slower than the SST, it is acceptable while considering the satisfactory results.

CONCLUSIONS

Spectral decomposition is significant for analyzing the seismic response of subsurface feathers. In this paper, a higher time-frequency resolution and sparser representation, called SSWPT, has been introduced to seismic data processing. This new method incorporates the frequency reassignment technique into the wave packet transform. It performs in two key steps: firstly, estimate the instantaneous frequency from wave packet transform, and then squeeze its value along the frequency axis based upon the instantaneous frequency information. Synthetic experiment shows that the proposed approach is effective in enhancing the T-F resolution. 2D and 3D filed data examples illustrate the superior performance of the SSWPT approach in better depicting the stratigraphic and structural features over traditional approaches.

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