

SEISMIC TIME-FREQUENCY ANALYSIS USING AN IMPROVED EMPIRICAL MODE DECOMPOSITION ALGORITHM

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ABSTRACT

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Among the time-frequency analysis approaches, the EMD-based approaches have been proven to show higher spectral-spatial resolution than the traditional approaches. However, the mode mixing problem always exists in these approaches which will affect the subsequent interpretation performance. In this paper, we apply a novel improved complete ensemble empirical mode decomposition (ICEEMD) technique to time-frequency analysis of seismic data. The ICEEMD approach can help decompose a 1D non-stationary signal into intrinsic mode functions with less noise and more physical meaning, and result in a higher frequency resolution in the time-frequency maps. The application of the algorithm to 1D seismic signal can help obtain a more meaningful analysis regarding the non-stationary components. Its application to 2D and 3D seismic data has the potential to enable a better geological and geophysical interpretation. We use a 1D real seismic trace, a 2D seismic section and a 3D seismic cube to show the superior performance of the proposed approach.

KEY WORDS: Empirical Mode Decomposition (EMD), time-frequency analysis, seismic data, Improved Complete Ensemble Empirical Mode Decomposition (ICEEMD), subsurface characterization.

INTRODUCTION

Time-frequency analysis is an important step in seismic data processing and interpretation. It can help characterize the non-stationary relation between time and frequency embedded in the seismic data and can reveal important details of seismic data providing valuable information for reservoir characterization (Zhang et al., 2016; Zhong et al., 2016; Liu et al., 2016a; Wu et al., 2016b). Because of the higher demand in precisely characterizing hydrocarbon reservoirs, more and more difficult situations when exploring oil and gas traps, the requirement for a high-resolution time-frequency analysis technique that is even robust in the case of low signal-to-noise ratio (SNR) (Cai et al., 2011; Lin et al., 2015; Kong et al., 2016; Sun and Wang, 2016; Wu et al., 2016a; Huang et al., 2016) becomes highly attractive.

Short-time Fourier transform (STFT) (Allen, 1977) applied the Fourier transform to an 1D signal over a local time window in order to generate a localized time-frequency map. The fixed window size used in STFT makes it difficult to obtain an appropriate delineation for different frequency components which limits its wide usage (Chakraborty and Okaya, 1995; Cohen, 1995; Sun et al., 2002). Wavelet-based methods are then developed for seismic time-frequency analysis in order to obtain superior spectral resolution. Sinha et al. (2005) developed the time-frequency continuous wavelet transform (TFCWT) which maps the time-scale plane into a time-frequency map directly and obtains a time-frequency map more accurately than the continuous wavelet transform. The S transform was proposed by Stockwell et al. (1996) as an extension to the Morlet wavelet transform. Instead of a fixed time length for each frequency in the window functions chosen for STFT, the S transform analyzes shorter data segments as the frequency becomes higher.

Wigner-Ville distribution (WVD) (Jeffrey and William, 1999; Wu and Liu, 2010) is an important member of the Cohens class time-frequency distribution. It possesses superior time-frequency resolution due to the absence of a window in calculating the time-frequency representation. However, WVD is also limited in a lot of applications because of its cross-term interference and its loss of phase information. Matching pursuit (MP) (Mallat and Zhang, 1993; Wang, 2007; Zhang et al., 2010) is another time-frequency analysis method which decomposes a seismic trace into a series of wavelets that belong to a comprehensive dictionary of functions. The MP method can obtain high resolution when applied to analyze seismic signals but it will cause a heavy computational burden. Besides, the atom library used in the MP method must be carefully chosen. The synchrosqueezing transform (SST) (Daubechies et al., 2011; Herrero et al., 2014; Liu et al., 2016d) is a newly developed method for obtaining high resolution in the time-frequency map. It was originally introduced in the context of audio signal analysis (Daubechies and Maes, 1996). The principle of SST is to reassign frequency components by synchrosqueezing along

the frequency axis of the time-frequency representation from a traditional wavelet transform. SST has been applied to seismic data analysis (Chen et al., 2014a; Herrer et al., 2014; Wang et al., 2014; Xie et al., 2015) and has obtained significantly higher time-frequency resolution than wavelet-based methods.

The empirical mode decomposition (EMD) (Huang et al., 1998; Chen et al., 2014b; Han and van der Baan, 2015; Gan et al., 2016; Chen, 2016; Chen et al., 2017a,b) decomposes a non-stationary signal into different locally stationary components, in a local and fully data-driven manner. In spite of its considerable success, EMD still lacks a solid mathematical foundation and is computationally expensive. In the EMD-based time-frequency analysis approach, the instantaneous frequency attribute is then extracted from each decomposed signal component and then mapped to a 2D time-frequency map. The EMD-based time-frequency analysis approaches are demonstrated to have a much higher resolution than the traditional techniques and thus have been widely investigated in the exploration geophysics community. However, the performance of the EMD-based time-frequency analysis approaches highly depends on the separability of different oscillating components. While the mode-mixing problem exists, the performance of the final time-frequency delineation will be negatively affected. Many noise-assisted versions of EMD approaches have been proposed to alleviate the mode-mixing phenomenon, such as ensemble empirical mode decomposition (EEMD) and complete ensemble empirical mode decomposition (CEEMD). The CEEMD recovered the completeness property of EMD (achieving a negligible reconstruction error), with guaranteed positive and smoothly varying instantaneous frequencies (Han and van der Baan, 2013). However, there are still two drawbacks of CEEMD that deserve an improvement: (1) the decomposed modes contain some residual noise and (2) there are spurious modes in the early stages of CEEMD. The improved complete ensemble empirical mode decomposition (ICEEMD) algorithm was proposed to solve such two problems. The decomposed components using ICEEMD are with less noise and have more physical meanings (Colominas et al., 2014; Chen et al., 2016).

In this paper, we propose an improved time-frequency analysis approach for analyzing seismic data based on the recently developed ICEEMD algorithm. The outline of the paper is summarized as follows. First, we briefly review the traditional EMD-based algorithms and their limitations. Secondly, we introduce the ICEEMD algorithm and the way we can utilize it for seismic time-frequency analysis. Then, we provide several field seismic data examples, in 1D, 2D, and 3D, to show the great potential of the ICEEMD algorithm in probing the subsurface properties and compare the ICEEMD-based performance with the CEEMD-based performance. Finally, some conclusions are drawn at the end of the paper. It is worth mentioning that it is the first time that the ICEEMD algorithm is applied on multidimensional seismic data and is analyzed from the

geoscience point of view since its publication in 2014. The novelty of this paper is not on the algorithmic side, instead, we will focus more on the geophysical applications.

THEORY

EMD, EEMD, and CEEMD

EMD is an adaptive decomposition method for breaking down a non-stationary signal into a set of locally stationary signals. Given a non-stationary input signal, a set of decomposed signals can be obtained via recursively implementing the sifting algorithm:

1. Find the local maxima and minima of the signal.
2. Fit the maxima and minima by cubic spline interpolation in order to generate the upper and lower envelopes.
3. Compute the mean of the upper and lower envelopes and subtract it from the signal.
4. Repeat steps 1-3 until the residual meets the condition of a intrinsic mode function (IMF):
 - the number of extrema and the number of zero crossings cannot differ more than one,
 - the mean value of the upper envelope and lower envelope is zero.
5. Subtract the residual signal obtained from step 3 from the original signal and continue to get other IMFs by recursively doing steps 1 to 3.

EMD has found a great number of applications in different fields of signal analysis. However, when a high-resolution depiction of frequency components is required, EMD cannot obtain successful performance. It is mainly because of the mode-mixing problem of EMD. The mode-mixing problem is defined as the phenomenon in which the IMFs are composed of frequencies of dramatically disparate scales (Kopecky, 2010).

Ensemble empirical mode decomposition (EEMD) was then developed to overcome the mode-mixing problem (Wu and Huang, 2009). In EEMD, several simple modifications are added into the implementation of traditional EMD: (1) a certain percentage of Gaussian white noise is added onto the observed signal and the new signal is decomposed into IMFs via EMD; (2) decompose a noisy

signal with different added Gaussian white noise into IMFs; (3) obtain an ensemble average of the corresponding individual IMFs as the final output IMFs of EEMD. However, because of the added white noise, the reconstruction of original signal after EEMD might not be optimal though the mode-mixing problem is greatly improved.

Complete ensemble empirical mode decomposition (CEEMD) is also a noise-assisted method. The CEEMD technique aims at simultaneously solving the mode mixing problem and maintaining the reconstruction performance by adding appropriate white noise in each stage of sifting process so that the residual signal for computing the following IMFs is unique (Liu et al., 2016b).

Improved complete ensemble empirical mode decomposition algorithm

Though the mode-mixing problem is greatly solved by the EEMD and CEEMD techniques, and the high reconstruction error issue of EEMD can be handled by CEEMD, the CEEMD technique may cause spurious modes that contain a significant amount of residual noise and lack a plausible physical interpretation of each mode. The improved complete ensemble empirical mode decomposition (ICEEMD) technique is recently developed to obtain IMFs with less noise and has more physical meanings.

The k-th IMF IMF_k in CEEMD can be expressed as

$$IMF_k = (1/I) \sum_{i=1}^I E_1[r_{k-1} + \epsilon_{k-1}E_{k-1}(w^i)] \quad (1)$$

where r_k denotes the residue after the k-th iteration: $r_k = r_{k-1} - IMF_k$. When $k = 0$, $r_k = x$. E_k denotes the EMD process to get the k-th component. When $k = 0$, there is no decomposition. w^i denotes the i-th Gaussian white noise realization process. ϵ_k is a parameter chosen to obtain a desired SNR of residue. Detailed studies on the influence of ϵ_k can be found in Colominas et al. (2012).

In a similar way, but introducing a local mean extraction operator M , the k-th IMF can be expressed as

$$IMF_k = r_{k-1} - r_k \quad (2)$$

when $k = 1$, $r_0 = x$. The k-th residue can be obtained by

$$r_k = (1/I) \sum_{i=1}^I M[r_{k-1} + \epsilon_{k-1}E_k(w^i)] \quad (3)$$

The CEEMD estimates the local mean of residue and subtracts it from the averaged residue. However, ICEEMD estimates the local mean and subtracts it from the original signal. In this way, we obtain a reduction in the amount of noise present in the modes. In addition, in order to reduce this scale overlapping that causes the spurious modes, ICEEMD proposes to make no direct use of white noise but use $E_k[w(i)]$ instead to extract the k -th mode.

The instantaneous amplitude $R(t)$ and frequency $F(t)$ for each IMF can be calculated as

$$R(t) = \sqrt{\{x^2(t) + y^2(t)\}} \quad , \quad (4)$$

$$F(t) = (1/2\pi)[x(t)y'(t) - x'(t)y(t)]/[x^2(t) + y^2(t)] \quad ,$$

where $y(t)$ is the Hilbert transform of $x(t)$. Note that the $R(t)$ and $F(t)$ are then mapped to a time-frequency map, followed by convolution with a 2D Gaussian filter, to output the final time-frequency characterization of a given signal.

FIELD DATA EXAMPLES

The first example is a real single-trace seismic data. Fig. 1 shows the data. Fig. 2 shows the time-frequency decomposition performance using four different approaches. The time-frequency decomposition results using EMD, EEMD, CEEMD, and ICEEMD, are shown in Figs. 2a, 2b, 2c, and 2d, respectively. The general performance using the four different approaches are very similar, but detailed comparisons can help us find out some differences that will affect greatly the final interpretation results. Focus on the two areas that are

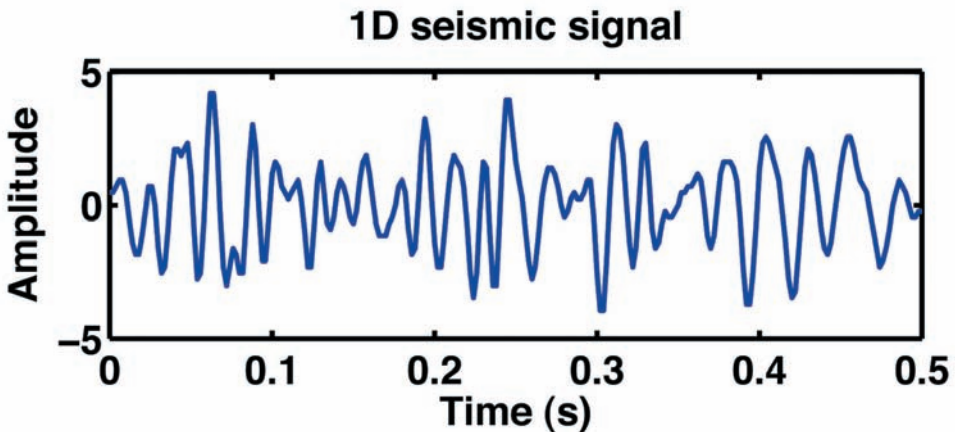


Fig. 1. Real single seismic trace.

emphasized by the green and red frame boxes in Fig. 2. It can be seen that there are three narrow frequency slices in the time-frequency maps shown in each figure in Fig. 2. One is about 45 Hz, one is about 25 Hz, and another one is about 10 Hz. By extracting these three frequency slices from each trace of a 2D or 3D datasets, we can extract significant geological structures or phenomenon from the original seismic profiles, which will facilitate a better decision making in oil & gas production. Because of the most serious mode-mixing problem of EMD, the frequency components in both the green and red boxes are mixing with each other, which will cause a discontinuity or smearing in the finally extracted spectrum-attribute profiles, as introduced above. The EEMD obtains a much better frequency resolution in the green box area by well separating the

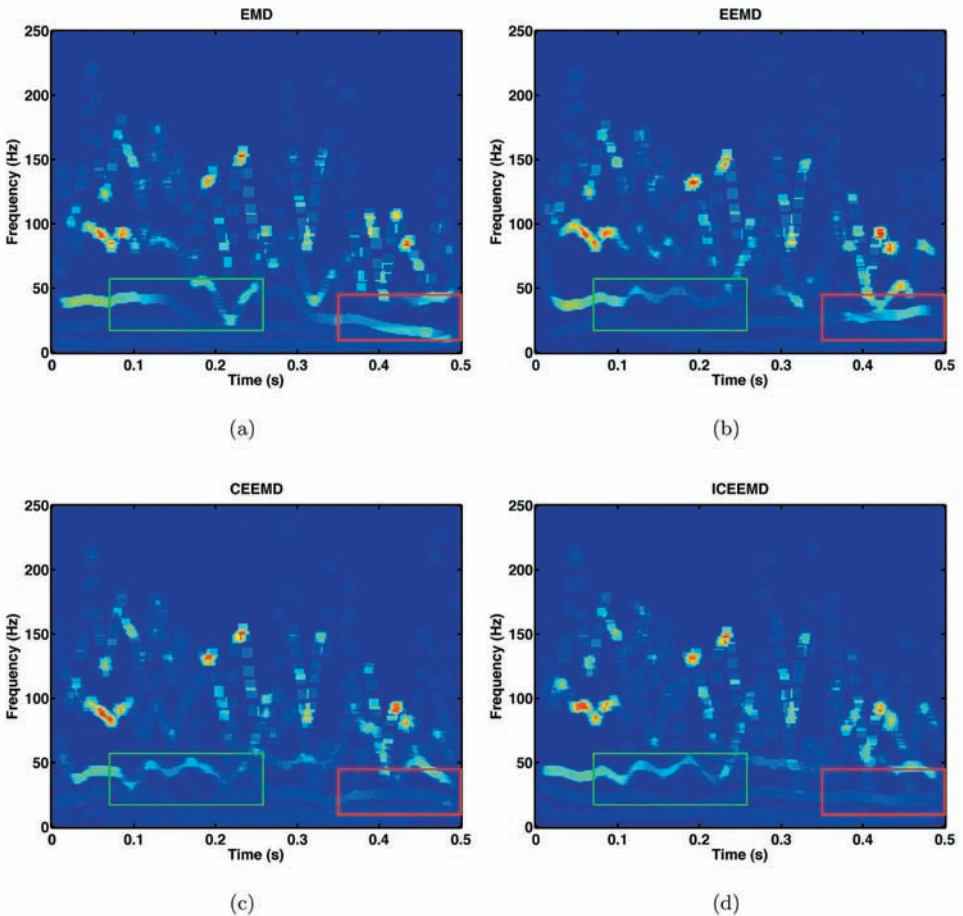


Fig. 2. Time-frequency decomposition performance of (a) EMD, (b) EEMD, (c) CEEMD, (d) ICEEMD.

two frequency slices but still causes a mixed spectrum in the red box area. The time-frequency spectrum from both CEEMD and ICEEMD approaches obtain an obviously better representation in both green and red boxes. There are no mode mixings in the red boxes for both CEEMD and ICEEMD. However, the CEEMD causes two spectrum crossing points (0.1 s and 0.2 s), and the delineation of the frequency slices is not very clear. The ICEEMD, instead, causes no mode mixing and obtains a better frequency representation with higher resolution.

A post-stack seismic section is also used to show the performance of the ICEEMD-based approach in delineating structural and stratigraphic features of the seismic data. Fig. 3a shows a 2D post-stack seismic data that was once used in Fomel (2013); Chen et al. (2014a); Chen and Fomel (2015); Liu et al. (2016c); Chen and Jin (2016). We first apply the CEEMD- and ICEEMD-based time-frequency analysis approaches to all the traces and then select a constant frequency for all the traces to form a separated frequency profile. From the constant frequency slice, we can understand the subsurface structures better. Figs. 3b and 3c show the 30 Hz sections using CEEMD and ICEEMD, respectively. It is clear that the ICEEMD-based approach obtains a much clearer delineation of the main geological features, like the discontinuities. The 50 Hz frequency sections using the two approaches are shown in Figs. 3d and 3e. The performance of ICEEMD is also better than the CEEMD-based approach. From the 50 Hz section using ICEEMD, we can obtain a better delineation of the strata with a potential thin-beds phenomenon. Comparing the low-frequency and the high-frequency slices, we can obtain more abundant information from the section, such as the features indicating the potential reservoirs. The depicted results using the ICEEMD-based approach are consistent with the results from Fomel (2013) and Chen et al. (2014a) in obtaining similar potential reservoir spots but with a higher resolution. The potential discontinuities that indicate the existence of faults are also found in Chen et al. (2014a) and that confirm the previous discovery but are not found in Fomel (2013). The main reason of its absence in Fomel (2013) is that the method proposed in Fomel (2013), though more controllable due to its explicit signal decomposition mechanism, is of much lower resolution compared with the ICEEMD-based approach in this paper and the SST-based approach in Chen et al. (2014a). This observation indicates that higher resolution may facilitate a better depiction of both structural anomaly and low-frequency shadow. The thin beds are not found in both Fomel (2013) and Chen et al. (2014a), which indicates that the ICEEMD-based time-frequency analysis approach may have a even higher resolution than the SST-based approach that is used in Chen et al. (2014a).

We also apply the proposed ICEEMD-based time-frequency approach to the 3D data volume. It is flattened using the plane wave painting approach (Fomel, 2010). The field data is from the Gulf of Mexico (Lomask et al., 2006)

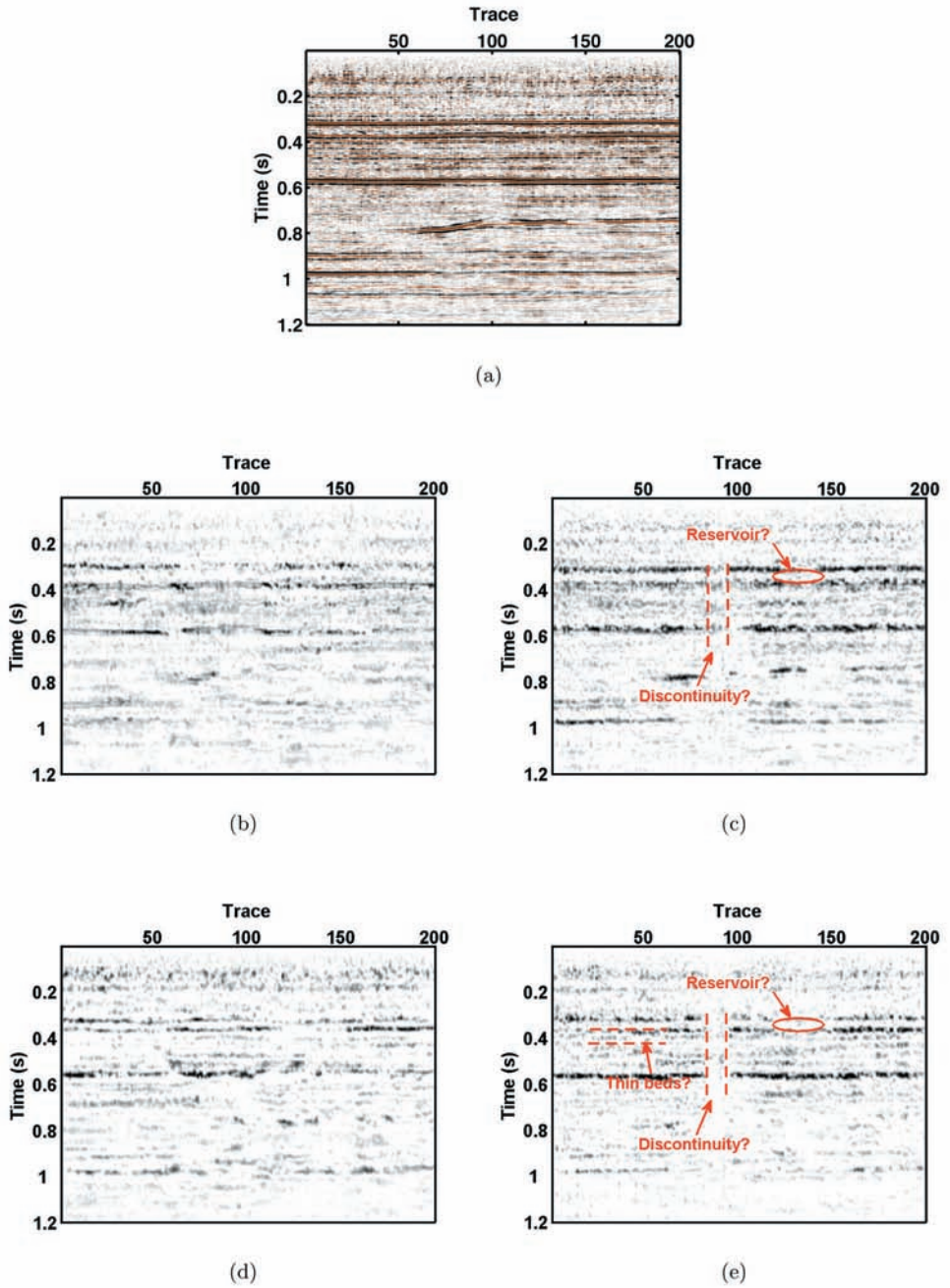


Fig. 3. Time-frequency delineation performance of a real 2D seismic data. (a) Amplitude section. (b) CEEMD (30 Hz). (c) ICEEMD (30 Hz). (d) CEEMD (50 Hz). (e) ICEEMD (50 Hz).

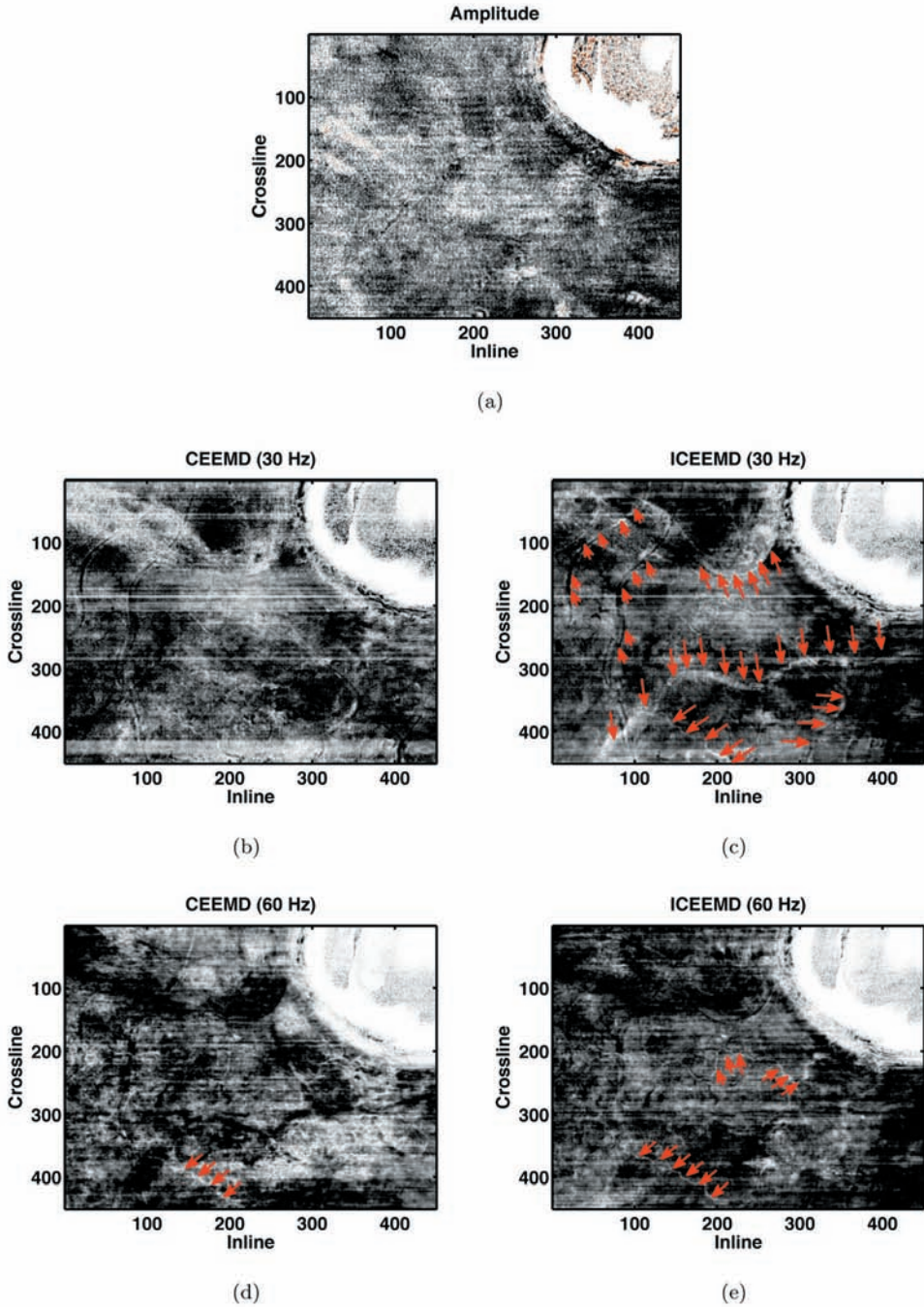


Fig. 4. Time-frequency delineation performance of a real 3D seismic volume. (a) Constant time slice. (b) CEEMD (30 Hz). (c) ICEEMD (30 Hz). (d) CEEMD (60 Hz). (e) ICEEMD (60 Hz). Note the highlighted paleochannels as pointed out by the arrows.

and was previously used in Liu et al. (2011). The constant time slice is shown in Fig. 4a. From the amplitude slice, there is no obvious channel. However, after extracting different frequency slices we can observe encouraging geological phenomena, especially for the paleochannels. Figs. 4b and 4d show the 30 Hz and 60 Hz frequency slices corresponding to the amplitude slice using the traditional CEEMD-based approach. Figs. 4c and 4e show the 30 Hz and 60 Hz frequency slices corresponding to the amplitude slice using the proposed ICEEMD-based approach. Although the CEEMD can show very good channel delineation result, the proposed ICEEMD method can obtain very successful performance. The differences between the two approaches are highlighted by the arrows. It can be clearly shown that, the 30 Hz slice shows the main channel structures while the 60 Hz slice shows subtle features that are not well delineated by the 30 Hz slice. When interpreting this horizon, it is better to utilize multiple frequency slices to have a more comprehensive understanding about the subsurface geological structures. In the 30 Hz slices of two methods, we can also see some acquisition footprints (the white straight lines in Figs. 4b and 4c). The study also indicates that the acquisition footprints may interfere with the accurate interpretation, and should be attenuated before time-frequency decomposition based seismic interpretation.

CONCLUSION

Although the complete ensemble empirical mode decomposition (CEEMD) algorithm can solve the mode-mixing problem of empirical mode decomposition (EMD), there is some residual noise contained in the decomposed modes and there are some spurious modes in the early stages of CEEMD. We introduced the recently developed improved complete ensemble empirical mode decomposition (ICEEMD) algorithm to further improve the EMD-based time-frequency analysis performance of seismic data. Because of the less residual noise left in the decomposed components and less number of spurious modes, the ICEEMD obtains a much better decomposition performance. The resulting time-frequency spectrum has higher resolution and less frequency overlapping. A 1D real seismic trace, a 2D post-stack seismic section, and a 3D seismic cube are all used to demonstrate the superior performance of ICEEMD-based time-frequency analysis to other EMD-based approaches in better delineating time-frequency relation, structural and stratigraphic features, and detecting paleochannels. Since the implementation of ICEEMD is a slight modification of CEEMD, its applications can be used as a direct alternatives to those CEEMD-based applications.

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