SEISMIC REFLECTIVITY INVERSION BY CURVELET DECONVOLUTION: A COMPARATIVE STUDY AND FURTHER IMPROVEMENTS

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ABSTRACT

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Curvelet deconvolution refers to seismic deconvolution for reflectivity inversion based on curvelet transform. The curvelet transform is a multi-scale and multi-directional transform that can provide a sparse representation of seismic reflectivity. When using this method to model the reflectivity, the signal is represented effectively by large coefficients and random noise is represented by small ones. In this paper, we conduct a comparative study in the context of reflectivity inversion, to investigate the performance of curvelet deconvolution, least-squares method and L_p-norm deconvolution. It is shown that by using curvelet deconvolution, the inverted reflectivity profiles have a better signal-to-noise ratio (SNR) and a higher resolution than those obtained by the least-squares method. On the other hand, its results excel those obtained by L_p-norm deconvolution in terms of the lateral continuity. Since curvelet deconvolution can offer a trade-off between the sparseness and lateral continuity, we propose an enhanced L_p-norm deconvolution by using the result obtained by curvelet deconvolution as the initial model. Numerical results show that the lateral continuity of the inversed reflectivity profile can be further improved by the proposed method.

KEY WORDS: seismic reflectivity inversion, curvelet transform, L_p-norm deconvolution.

INTRODUCTION

Seismic reflectivity inversion normally applies a sparse constraint to an inversion problem directly. However, in some occasions, the inverted seismic reflectivity is so sparse that the underground structure is destroyed. To tackle this problem, different models have been proposed to represent the reflectivity.

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For example, in the basis pursuit method (Zhang, 2010; Zhang and Castagna, 2011; Zhang et al., 2013), a collection of even and odd dipoles are used to model the reflectivity. Therefore, the sparse constraint is exerted on the corresponding coefficients, reducing the loss of geological structures. However, these methods are normally implemented in a trace-by-trace manner and thus there is a lack of lateral coherency for the retrieved results.

Curvelet transform was introduced as a multi-scale and multi-directional transform (Candès and Donoho, 1999, 2002; Candès et al., 2005). It has been proved that the curvelet transform can provide a sparse representation for the smooth objects with edges such as seismic events. After transforming the data into curvelet domain, the coefficients, which represent the noise and the effective signal, can be separated clearly. Because of this property, the curvelet transform has been used widely in seismic signal processing. Naghizadeh and Sacchi (2010) used this method for the interpolation of aliased regularly seismic data and got similar results as FX and FK interpolation methods. Non-parametric seismic data recovery has been implemented with the curvelet transform (Herrmann and Hennenfent, 2008) and can recover data with up to 80% missing traces. Also, it plays an important role in random, coherent and incoherent noise attenuation (Neelamani et al., 2008; Kumar, 2009), showing a better performance when compared with other methods such as median filter and FX deconvolution.

Furthermore, since the structure of seismic reflectivity along the layers can be seen as curves, the curvelet transform has been applied to model the reflectivity. Non-spiky seismic reflectivity inversion was developed using the curvelet transform (Hennenfent and Herrmann, 2005; Kumar and Herrmann, 2008). It is shown that seismic deconvolution with multichannel curvelet operator can exploit the continuity along the reflectors by promoting the curvelet-domain sparsity. However, since the multichannel curvelet operator cannot provide results that are sparse and spiky, the bandwidth of the inverted seismic reflectivity is not well compensated. Therefore, results with a good resolution cannot be obtained using this method. Furthermore, as this method was originally proposed only for L_1-norm deconvolution, a more general case (L_p-norm) also needs to be proposed.

In this paper, we first implement the curvelet deconvolution by applying the sparse constraint in the curvelet domain. A comparative study is then conducted to investigate the performance of three deconvolution methods, including curvelet deconvolution, the least-squares method and L_p-norm deconvolution. Both synthetic and field data sets have been used for this comparison. Based on the study, we further propose an enhanced L_p-norm deconvolution method by using the result obtained by curvelet deconvolution as the initial model during the inversion process. By showing the residual between the original and inversed data, it is found that the lateral continuity of the spiky reflectivity inversed by the proposed method can be further improved. The proposed method can be used as an alternative to obtain reflectivity results with a good SNR, resolution and lateral continuity.

CURVELET TRANSFORM

The curvelet transform is a multi-scale and multi-directional transform which can decompose the image (data) into harmonic scales. Curvelets can be obtained by rotations and translations of a "mother" curvelet φ_j. There are three parameters which play important roles in the definition of curvelet transform (Candès et al., 2005):

1. Scale parameter j, different scales are corresponding to different frequency bands.

2. Equispaced sequence of rotation angles:

   $\theta_l = 2\pi \cdot 2^{-\lfloor j/2 \rfloor} \cdot l$,

   with $l = 0,1,\ldots$ and $0 \leq \theta_l < 2\pi$.

3. The position:

   $x_k^{(j,l)} = R_{\theta_l}^{-1}(k_1 \cdot 2^{-j}, k_2 \cdot 2^{-j/2})$.

As a result, curvelets are characterized by three indexes: j which represents the scale, l which is associated with angles and k is corresponding to different positions. The formula of curvelet transform based on the "mother" curvelet can be expressed as

$$\varphi_{j,l,k}(x) = \varphi_j[R_{\theta_l}(x - x_k^{(j,l)})] \quad , \tag{1}$$

where R_θ represents the rotation by θ radians and R_θ^{-1} is its inverse:

$$R_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad R_\theta^{-1} = R_\theta^T = R_{-\theta} \ . \tag{2}$$

With the above definitions, curvelet transform has several properties:

1. Tight frame.

   An arbitrary function f can be expanded as a series of curvelets:

$f = \sum_{j,l,k} \langle f, \varphi_{j,l,k} \rangle \varphi_{j,l,k}$, and it fulfills the Parseval relation:

$$\sum_{j,l,k} |\langle f, \varphi_{j,l,k} \rangle| = \| f \|_2^2 \ .$$

2. Parabolic scaling.

   The effective length and width of curvelets obey the anisotropy scaling relation:

   $$\text{width} = \text{length}^2 \ .$$

3. Oscillatory behavior.

   Curvelets are strictly localized in the frequency domain, they can be thought as a pyramid with many directions and positions at each length scale. Fig. 1 shows the tiling of digital curvelet transform in the frequency domain. In the time-space domain, they are needle like with both ends tapering off while smooth along and oscillatory across the ridge, as seen in Fig. 2 (Wu and Hung, 2013).

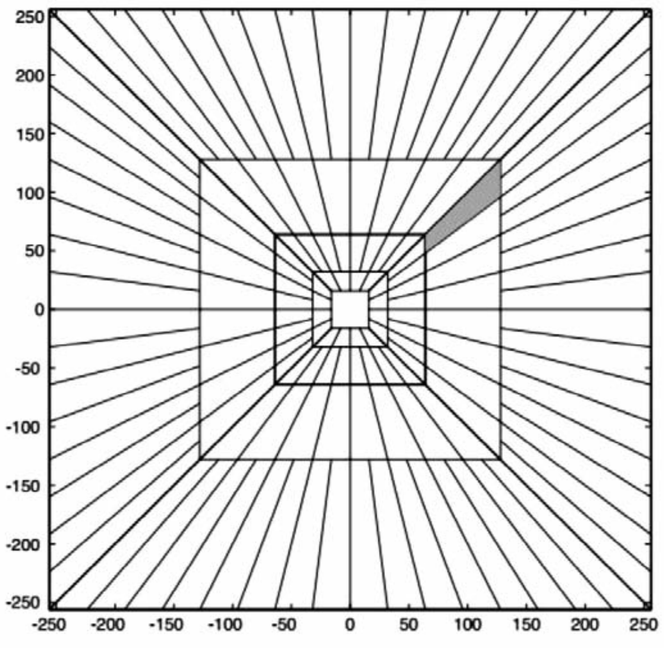


Fig. 1. The tiling of digital curvelet transform in the frequency domain (Adapted from Candès et al., 2005).

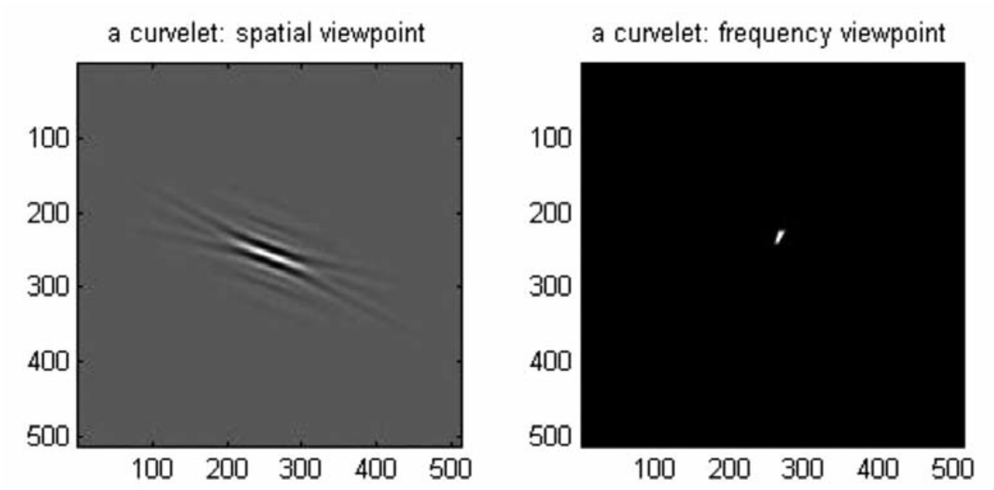


Fig. 2. A curvelet in both spatial and frequency domains.

The curvelet transform can provide a sparse representation of smooth objects with piece-wise discontinuities. After transforming the data into the curvelet domain, only a few curvelet coefficients will be large while the others will decay rapidly. As a result, curvelet transform is a suitable tool for providing a sparse representation of the seismic reflectivity. Wavelets have also been used for signal decomposition, however, in the frequency domain, they are lack of directionality. Only when the orientation is perpendicular to the interfaces, the coefficients of wavelets decay rapidly. So the wavelet coefficients corresponding to the seismic signal is not as sparse as that of the curvelets, which means curvelet transform is more suitable to be used in sparse inversion problem.

The curvelet coefficients c can be obtained by calculating the inner product of r and $\varphi_{j,l,k}$:

$$c_{j,l,k} = \langle r, \varphi_{j,l,k} \rangle \quad . \tag{3}$$

To simplify the form of the equation above, we can re-write (3) into the matrix-vector form:

$$\mathbf{c} = \mathbf{Cr} \quad , \tag{4}$$

where **C** represents the curvelet transform operator.

Since curvelets can provide tight frames, the adjoint operator $\mathbf{C}^T$ is equal to the pseudo-inverse of curvelet operator $\mathbf{C}$ (Candès and Donoho, 2002). Thus we can have the following reconstruction equation:

$$\mathbf{r} = \mathbf{C}^T\mathbf{c} \ . \tag{5}$$

The curvelet transform maps the signal into distinct sets of curvelet coefficients. The coefficients of the effective signal are very large. On the other hand, the energy of random noise is spread out over all frequencies and dips. Therefore the curvelet transform maps random noise into large number of weak amplitude curvelet coefficients. As a result, signal and noise have minimal overlap in the transform domain, and sparse deconvolution method can be used to suppress the noise. In this paper, our algorithm is developed upon the fast discrete curvelet transform via wrapping (Candès et al., 2005).

CURVELET DECONVOLUTION

Seismic reflectivity inversion is based on the convolution model. Conventionally, in order to obtain a sparse reflectivity result, the L_1-norm can be applied to regularize the problem with the objective function (Levy and Fullagar, 1981):

$$J = \|\mathbf{d} - \mathbf{W}\mathbf{r}\|_2^2 + \lambda\|\mathbf{r}\|_1 \ , \tag{6}$$

where $\mathbf{d}$ is the vector of seismic trace, $\mathbf{W}$ is a matrix which represents the discrete convolution operation of seismic wavelet and $\mathbf{r}$ is a vector of reflectivity. λ is a trade-off parameter which reflects the compromise between the accuracy and sparsity of the reflectivity, and the value of λ should be selected accordingly based on different input data sets. Eq. (6) can be solved by using L_1-norm based algorithms; here the L_1-norm solver used in this paper is basis pursuit (Chen et al., 2001). When using basis pursuit, eq. (6) can be transformed into a linear programming problem and solved by primal-dual log-barrier LP algorithm (Kim et al., 2007).

As the structure of reflectors along the layers can be treated as curves, curvelet coefficients can be used to represent seismic reflectivity (Hennenfent and Herrmann, 2005; Kumar and Herrmann, 2008). The effective signal will concentrate on large curvelet coefficients and random noise will be mapped on weak amplitude coefficients. The reflectivity can then be written as: $\mathbf{r} = \mathbf{C}^T\mathbf{c}$, where $\mathbf{c}$ is the curvelet coefficient vector corresponding to seismic reflectivity and $\mathbf{C}$ represents the curvelet transform operator. Since the curvelet coefficients are sparse, the objective function becomes

$$J = \|\mathbf{d} - \mathbf{W}\mathbf{C}^T\mathbf{c}\|_2^2 + \lambda\|\mathbf{c}\|_1 \ . \tag{7}$$

To solve this objective function, first of all, basis pursuit method is used to obtain the result of **c**. And then, the reflectivity is calculated by $\mathbf{r} = \mathbf{C}^T\mathbf{c}$.

It should be noted that the curvelet operator is defined on a data set of size Ntr $*$ Nt, where Ntr is the number of traces and Nt is the number of sample points of the corresponding trace. Therefore, curvelet operator is a multichannel operator which is based on the entire traces of the seismic data rather than a single trace. During the deconvolution process, all traces are processed simultaneously and the curvelet coefficients will be approaching to that of the real earth layers as the iteration continues. Consequently, the inherent continuity of the obtained layers will be preserved.

NUMERICAL RESULTS FOR A COMPARATIVE STUDY

In order to evaluate the performance of the curvelet transform based reflectivity inversion method, a comparative study is conducted and its performance is compared with the least-squares inverse method and L_p-norm deconvolution. The synthetic data set with a flat and a dip structure is first used for the comparison. Fig. 3 illustrates the reflectivity profiles inversed by these three methods. By observing these results, we can come to the conclusion that when the input data is clean, all these three methods can portray the structure of seismic events accurately. The only difference is that the bandwidth of the reflectivity obtained by the curvelet deconvolution is narrower than that by the L_p-norm deconvolution, but wider than that by the least-squares method.

To test these methods' performance under low signal-to-noise (SNR) circumstances, external random noise is added to the original clean synthetic data, and the SNR is set to 2. By applying these three different deconvolution methods on the noisy synthetic data, it can be observed from Fig. 4 that the least-squares inverse method can well preserve the structure, but most of the noise still exists on the reflectivity profile. Furthermore, L_p-norm deconvolution is able to provide a more spiky and sparse reflectivity profile. Unfortunately, some useful structure is also eliminated and the lateral continuity is destroyed during the inversion process. In comparison, the curvelet deconvolution can effectively suppress the noise existed on the seismic profile, while still showing a good lateral continuity of the seismic events. Therefore, it is concluded that the curvelet deconvolution can offer a trade-off between the lateral continuity and the sparseness.

A small stack (Fig. 5a) is further used to test the performance of these three deconvolution methods. The results obtained by the least-squares method, L_p-norm deconvolution and the deconvolution based on curvelet transform are demonstrated in Figs. 5b, 5c and 5d, respectively. From these figures it can be

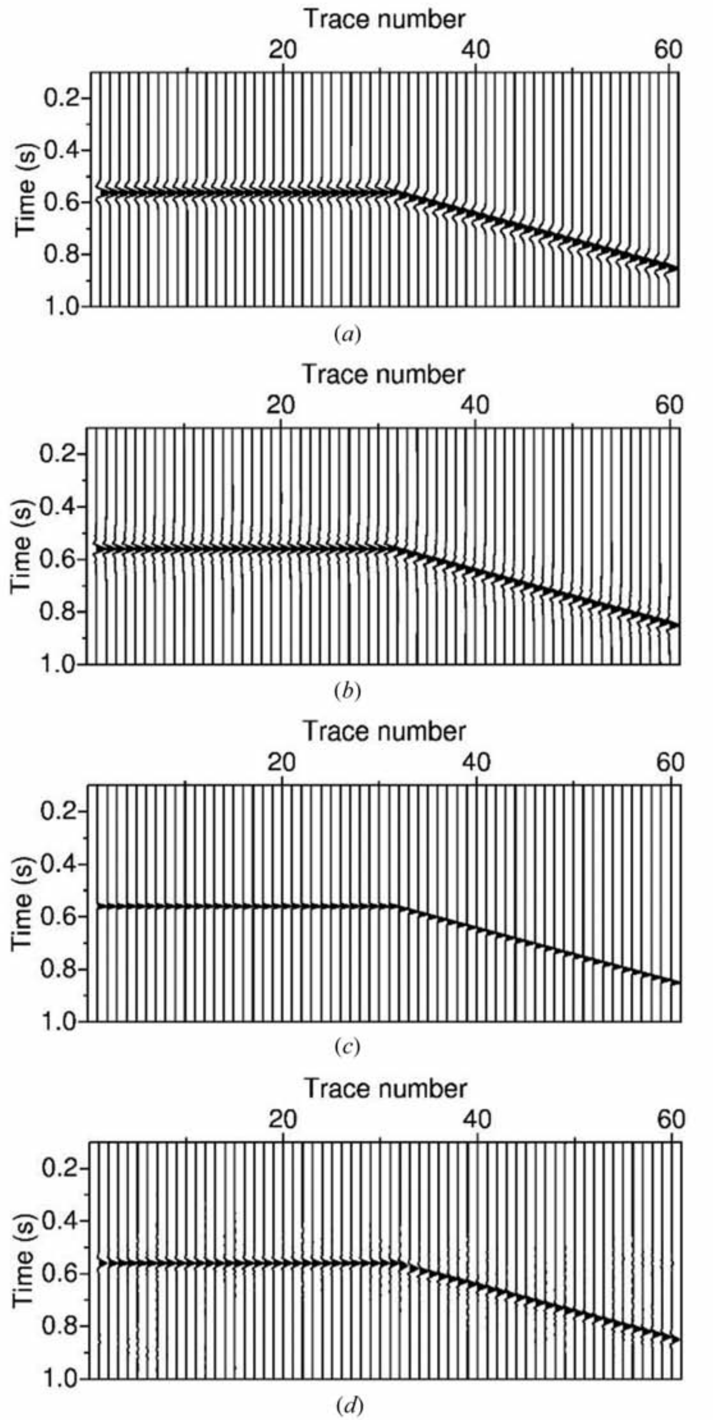


Fig. 3. (a) Synthetic seismogram. Reflectivity obtained by (b) the least-squares inverse method. (c) L_p-norm deconvolution. (d) Curvelet deconvolution.

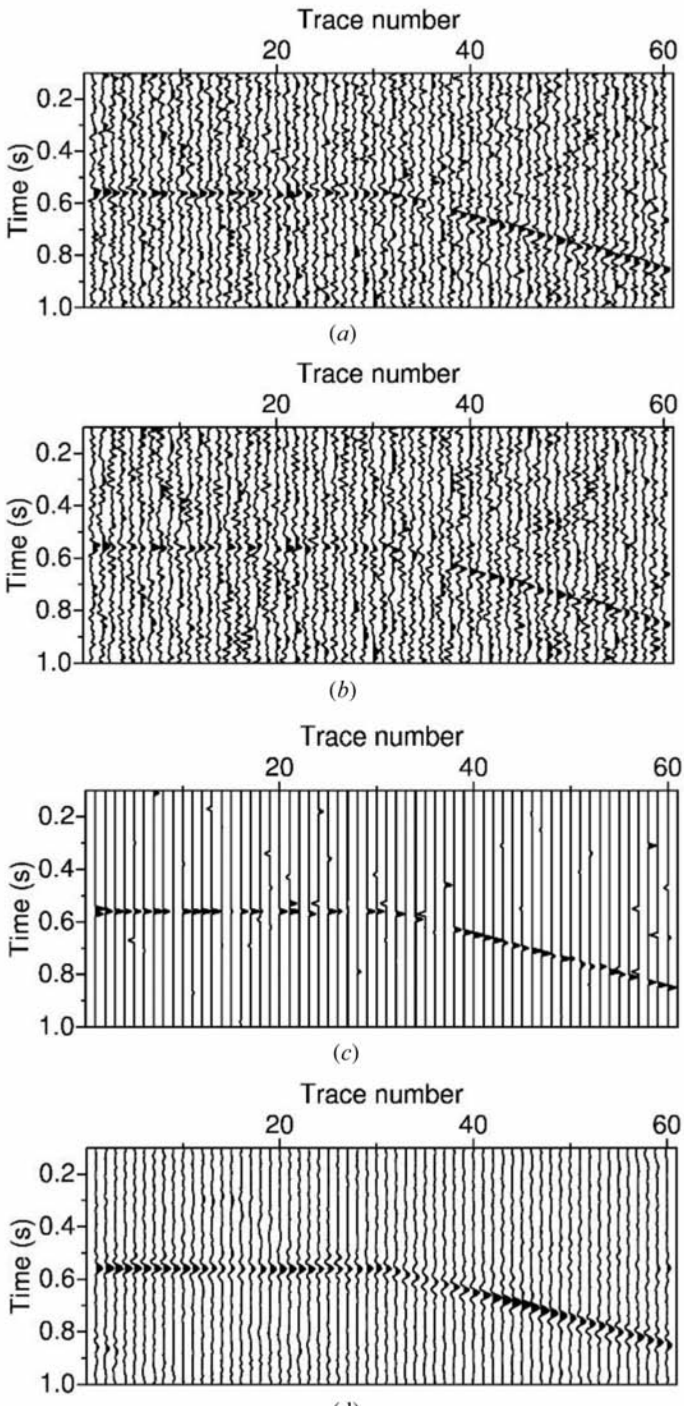


Fig. 4. (a) Synthetic seismogram (SNR=2). Reflectivity obtained by (b) the least-squares inverse method. (c) L_p-norm deconvolution. (d) Curvelet deconvolution.

seen that the stratification of Fig. 5d is better than that of the least-squares method in Fig. 5b, as denoted by the black circle. On the other hand, although the result of the deconvolution based on curvelet transform is not as spiky and sparse as the result of L_p-norm deconvolution shown in Fig. 5c, the lateral continuity for the obtained result is superior, as the structure of the seismic events is well preserved.

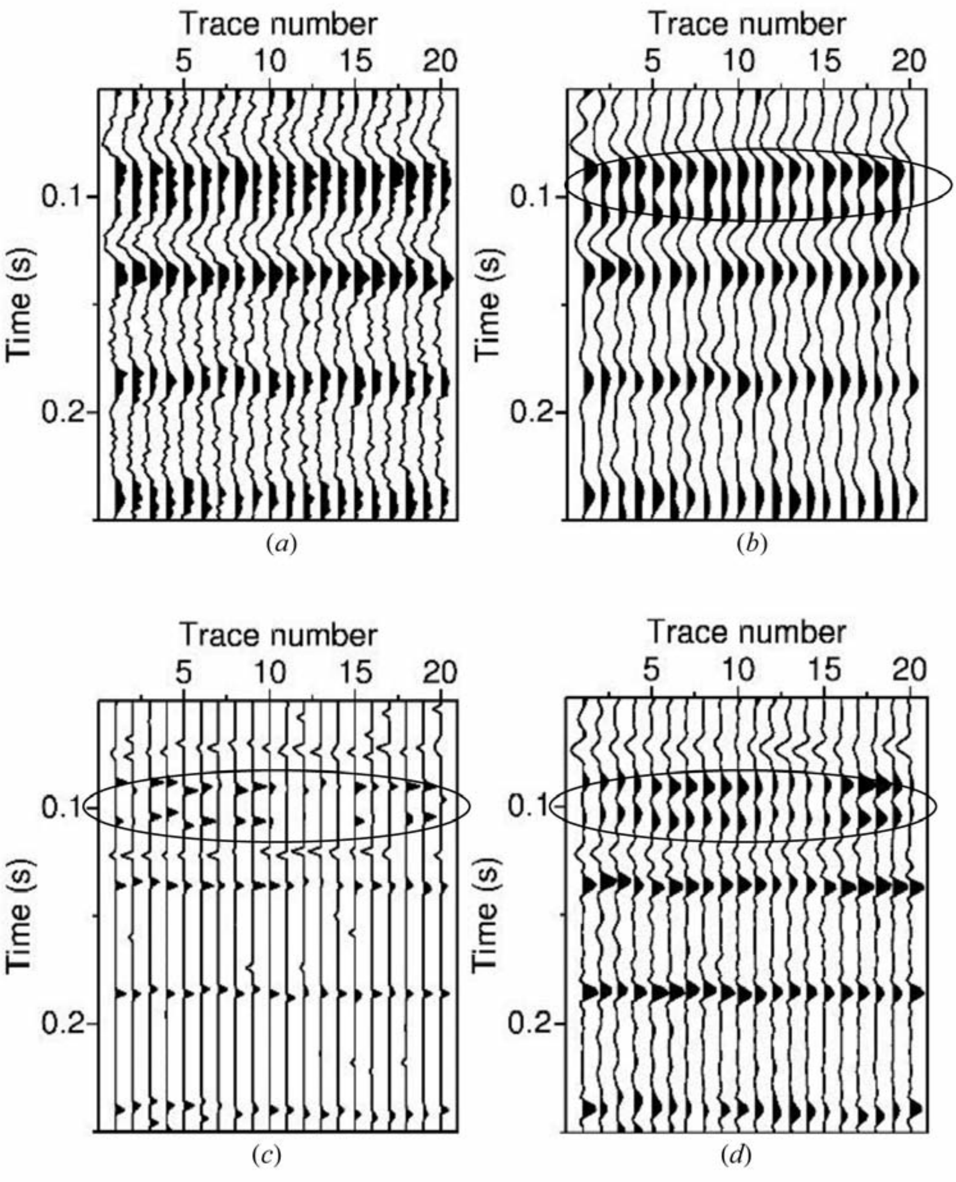
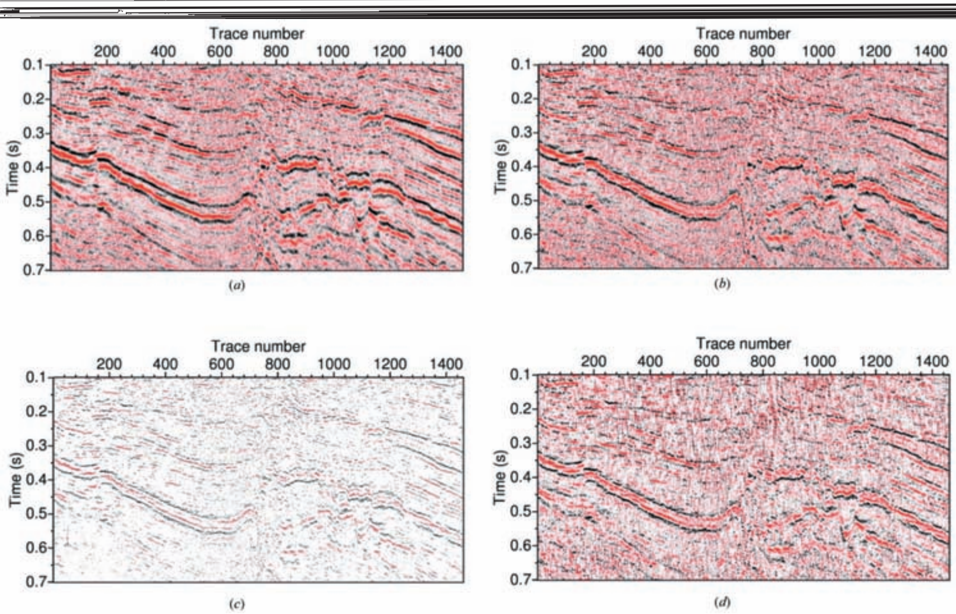


Fig. 5. (a) Small stack. Reflectivity obtained by (b) the least-squares inverse method. (c) L_p-norm deconvolution. (d) Curvelet deconvolution.

Fig. 6 illustrates a field data set and the results obtained by different inversion methods. Fig. 7 further gives a zoomed-in view of the seismic data with the trace number from 400 to 500 and the time from 0.4 to 0.6 second, and also the corresponding inversed reflectivity profiles. It comes to the similar conclusion that the reflectivity inversion based on curvelet transform can suppress the noise in an effective manner, and has a higher resolution than that of the least-squares inverse method. Once again, a better lateral coherency of the structure is preserved when comparing with the reflection profile obtained by L[illegible]-norm deconvolution.



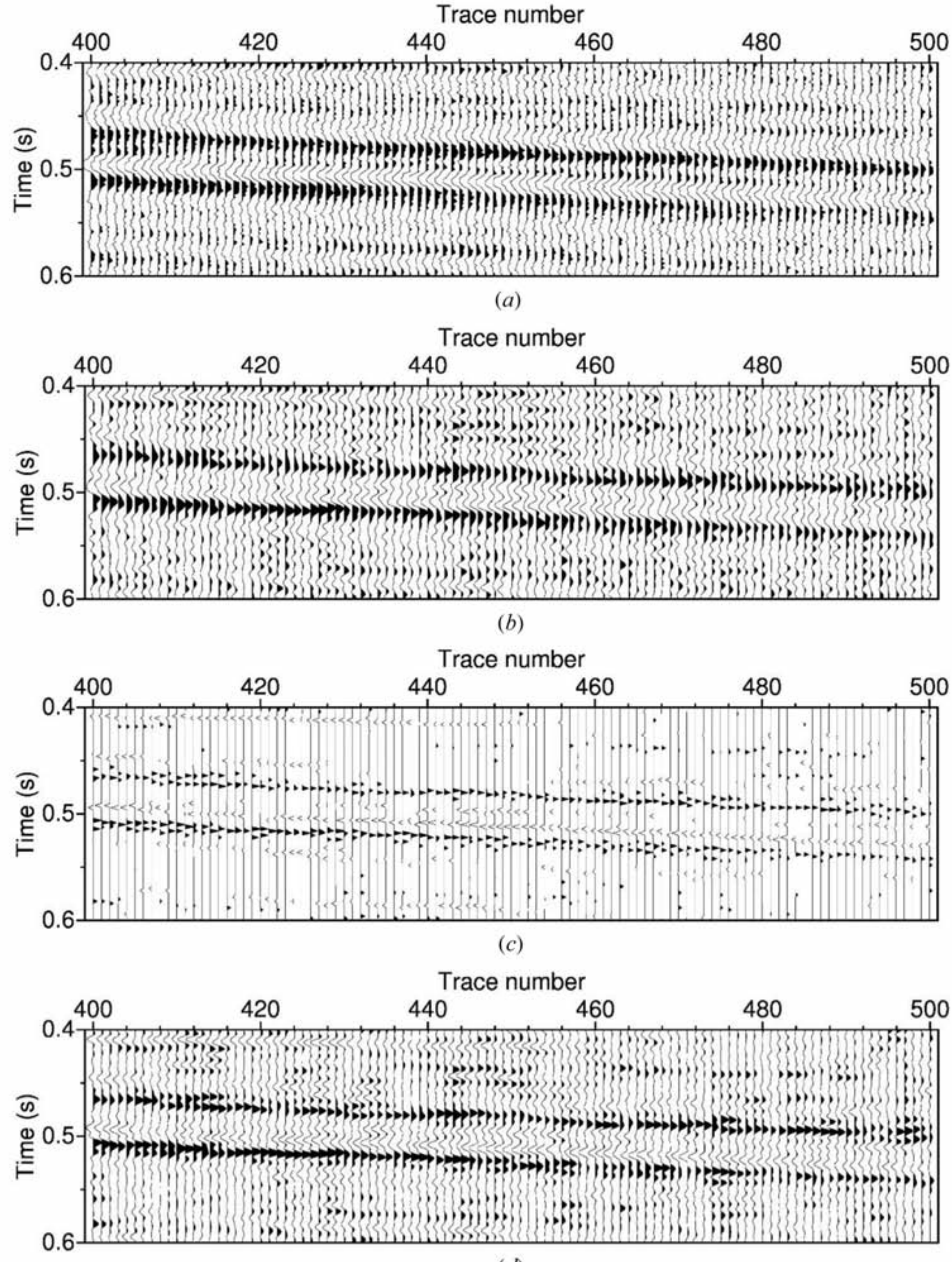


Fig. 7. (a) Zoomed-in view of the field data set. Zoomed-in view of the reflectivity obtained by (b) the least-squares inverse method. (c) L_p-norm deconvolution. (d) Curvelet deconvolution. the

initial model in L_p-norm deconvolution. As observed in the previous section, the curvelet deconvolution has its own advantages when comparing with L_p-norm deconvolution and the least-squares inverse method, we can use the result obtained by the curvelet deconvolution as the initial model to enhance the performance of traditional L_p-norm deconvolution method.

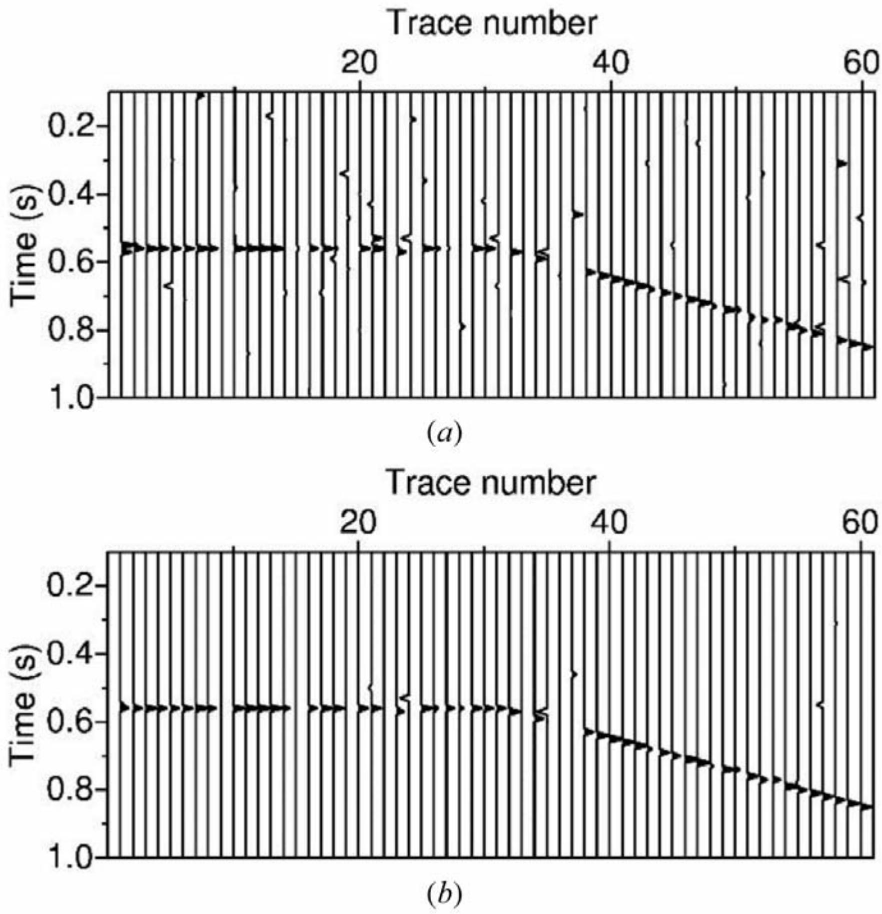


Fig. 8. Reflectivity obtained by (a) L_p-norm deconvolution. (b) Curvelet transform enhanced L_p-norm deconvolution.

As shown in Fig. 8, after using the result from the curvelet deconvolution instead of conventionally used least-squares method as the initial model, the retrieved seismic reflectivity series profile of the noisy synthetic data is cleaner and some missing structures are preserved when comparing with the result by the conventional L_p-norm deconvolution. Furthermore, we compare the conventional and curvelet transform enhanced L_p-norm deconvolution quantitatively, by calculating the residual energy ratio between the retrieved and original data. The equation for calculating the residual energy ratio can be expressed as

$$residual\ energy\ ratio = \frac{|input\ data\ energy - synthetic\ data\ energy|}{input\ data\ energy}. \tag{8}$$

The residual profiles associated with different processes are shown in Fig.9. When using the conventional L_p-norm deconvolution, the residual energy ratio is calculated to be 38.5062%, while the residual energy ratio of the enhance method is only 20.178%. Therefore, the enhanced L_p-norm deconvolution can provide a lower residual energy ratio. From Fig. 9, it can also be seen that when using the curvelet transform enhanced L_p-norm deconvolution method, the noise existed on the reflectivity profile has been largely eliminated, when comparing with the conventional L_p-norm deconvolution.

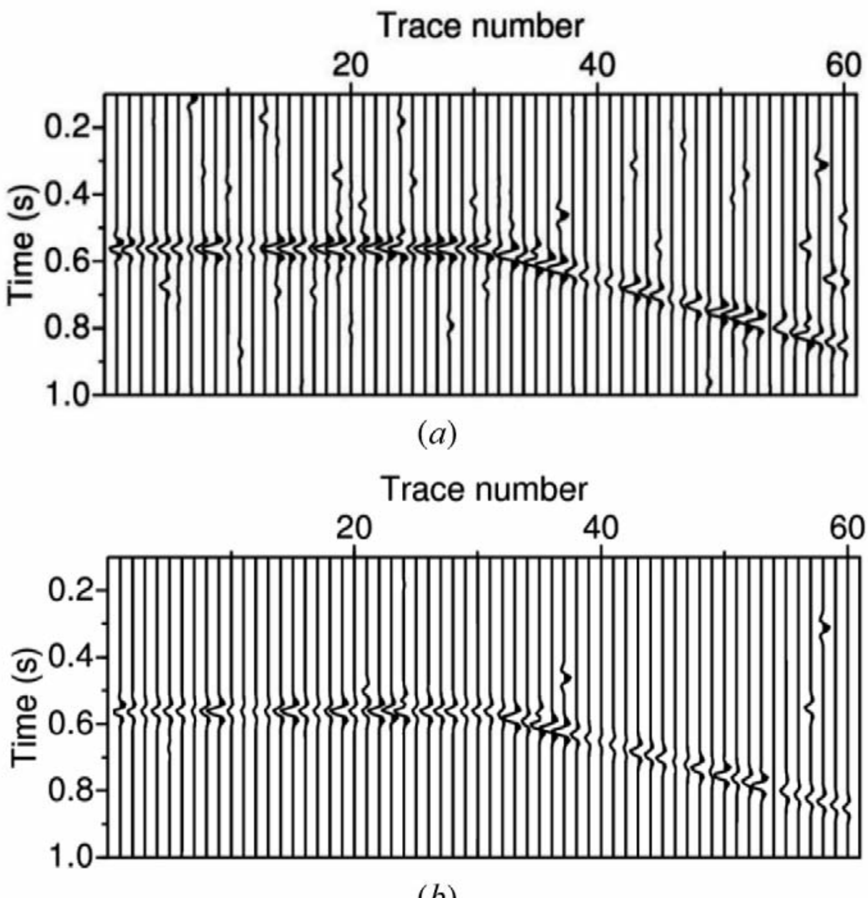


Fig. 9. Residuals associated with the reflectivity profile obtained by (a) L_p-norm deconvolution. (b) Curvelet transform enhanced L_p-norm deconvolution.

The same small stack data in Fig. 5a is used to test the performance of the curvelet transform enhanced L_p-norm deconvolution. As shown in Fig. 10, it is observed that the lateral continuity of the inversed reflectivity is improved using the proposed method, especially within the area denoted by the black circle. By comparing the associated residual profiles, it can be seen that the proposed method can greatly recover the structure, giving a residual profile with small amplitude, as shown in Fig. 11.

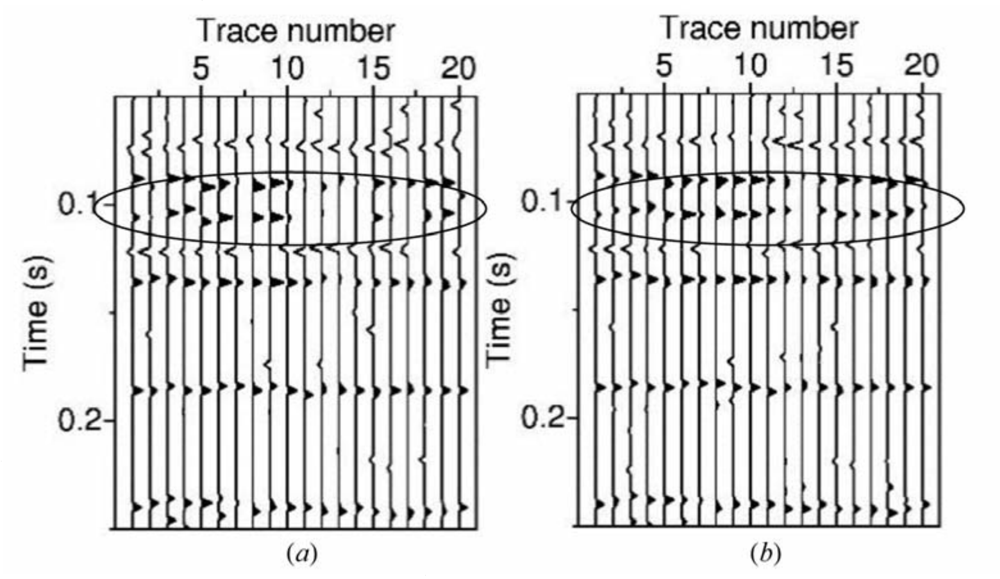


Fig. 10. Reflectivity obtained by (a) L_p-norm deconvolution. (b) Curvelet transform enhanced L_p-norm deconvolution.

Moreover, the same field data set as shown in Fig. 7a is used to evaluate the performance. The inversed reflectivity profiles obtained by the conventional and enhanced L_p-norm deconvolution are illustrated in Fig. 12. Using the proposed method, the structure of the reflectivity profile is more regularized and continuous, which means that our proposed method is superior in terms of the lateral coherency. From the residual profiles shown in Fig. 13, it is seen that the amplitude of residuals associated with the enhanced method is lower than that of the conventional L_p-norm method. Furthermore, the continuous structures shown on the residual profile of the proposed methods are less, which means the lateral continuity of the seismic events is better preserved.

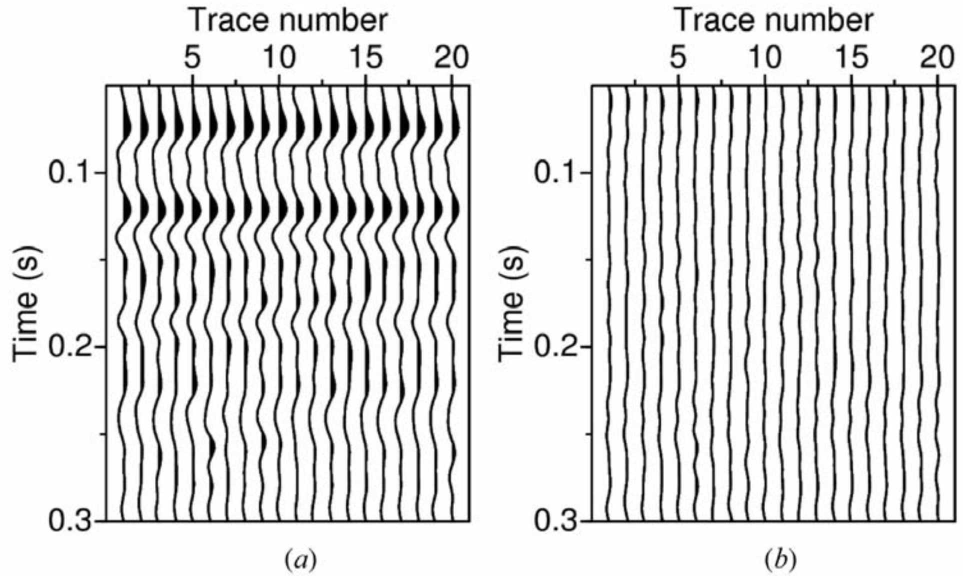


Fig. 11. Residuals associated with the reflectivity profile obtained by (a) L_p-norm deconvolution. (b) Curvelet transform enhanced L_p-norm deconvolution.

CONCLUSIONS

Curvelet transform is able to provide a sparse representation of seismic reflectivity. In this paper, the multichannel curvelet operator was applied during the inversion process to obtain a spiky result with a better lateral continuity. A comparative study was conducted to evaluate the performance of curvelet deconvolution, the least-squares inverse method and L_p-norm deconvolution. It is found that the deconvolution based on curvelet transform can obtain spiky reflectivity profiles which are cleaner and with higher resolution, when compared with those obtained by the least-squares method. On the other hand, it can produce results which are more continuous than those obtained by L_p-norm deconvolution. In order to improve the lateral continuity of spiky seismic reflectivity profiles, an enhanced L_p-norm deconvolution method was proposed by applying the result obtained by curvelet deconvolution, to replace originally used least-squares solution, as the initial model. Numerical results using a noisy synthetic data, a small stack and a large field data all validated the

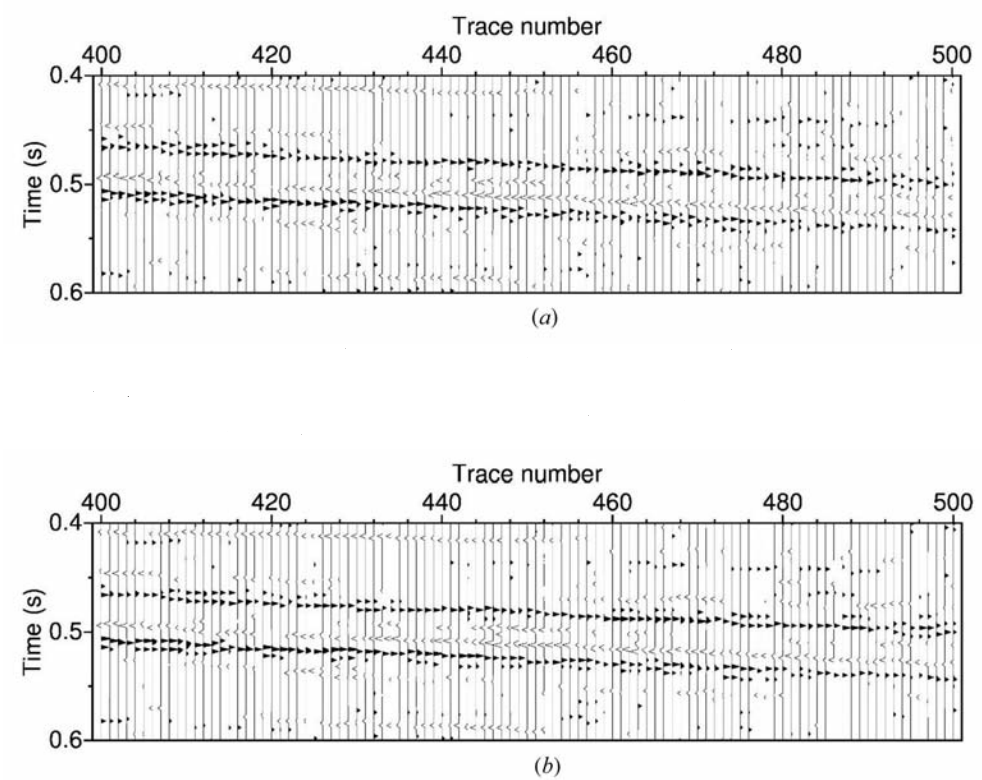


Fig. 12. Zoomed-in view of the reflectivity obtained by (a) L_p-norm deconvolution. (b) Curvelet transform enhanced L_p-norm deconvolution.

proposed method, showing that the structure of the obtained reflection coefficients profile is more regularized and has a better lateral coherency than that of the L_p-norm deconvolution, and has a higher resolution than that of the least-squares method.

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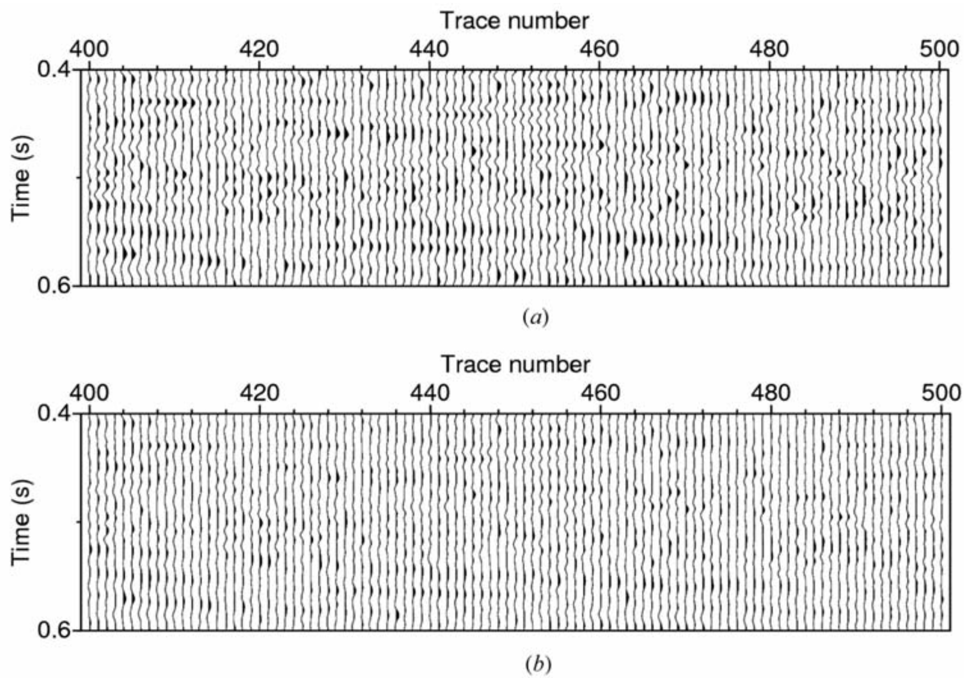

Fig. 13. Residuals associated with the reflectivity profile obtained by (a) L[illegible]-norm deconvolution, (b)

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