JOURNAL OF SEISMIC EXPLORATION **25**, 285-298 (2016)

THEORY FOR A LOW FREQUENCY MARINE DIPOLE SEISMIC SOURCE

[redacted]

[redacted]

[redacted]

[redacted]

far-field amplitude. The other is a source ghost which cancels out radiated wavefields at low frequencies. A dipole source operates differently, exciting acoustic waves by oscillatory translation rather than by volume modulation, and its ghost reinforces radiated wavefields at low frequencies rather than canceling (Duren and Meier, 2008; Hixson, 2009; Meier and Duren, 2014; Meier et al., 2015a,b). These features benefit the dipole source for low frequency marine seismic acquisition.

SPHERICAL SOURCES

We examine spherical sources which may be considered idealized representations of marine seismic sources. The wavefield created by a spherical source is readily solved by matching the general solution of the wave equation to boundary conditions describing the source dynamics (Blackstock, 2000). The boundary condition for a spherical monopole is given by

$$\mathbf{u}_m(r = a) = u_o e^{-j\omega t}\hat{r} \quad , \tag{1}$$

describing a pulsating sphere with a boundary at radius a that is oscillating radially with peak velocity of u_o. The monopole describes a source acting on the surrounding fluid by modulating volume, a manner consistent with modern commercial marine sources. The wave equation solution in a homogeneous whole space for a monopole source with the described boundary condition is given by

$$\mathbf{u}_m = u_o[1/(1 - jka)][(a^2/r^2) - jka(a/r)]e^{j[k(r-a)-\omega t]}\hat{r} \quad , \tag{2}$$

where $k = \omega/c_o$, and c_o is the speed of wave propagation in the homogeneous medium. The monopole flow field has a term that scales with $1/r^2$, often referred to as the near-field component of the wavefield, and a term that scales with $1/r$, often referred to as the far-field component. Both near and far-field flows are radial and uniform in all directions, as shown in Fig. 1. They are 90 degrees out of phase with each other, and their phase relative to the monopole boundary motion given in eq. (1) depends on the magnitude of ka, or correspondingly, frequency. The near-field component dominates at distances $r < \lambda/2\pi$, where $\lambda = 2\pi/k$ is the wavelength of propagating waves. This region is often referred to as the near-field. The far-field component dominates at distances $r > \lambda/2\pi$, often referred to as the far-field region.

The boundary condition for a spherical dipole is given by

$$\mathbf{u}_d(r = a) = u_o e^{-j\omega t}\hat{z} \quad , \tag{3}$$

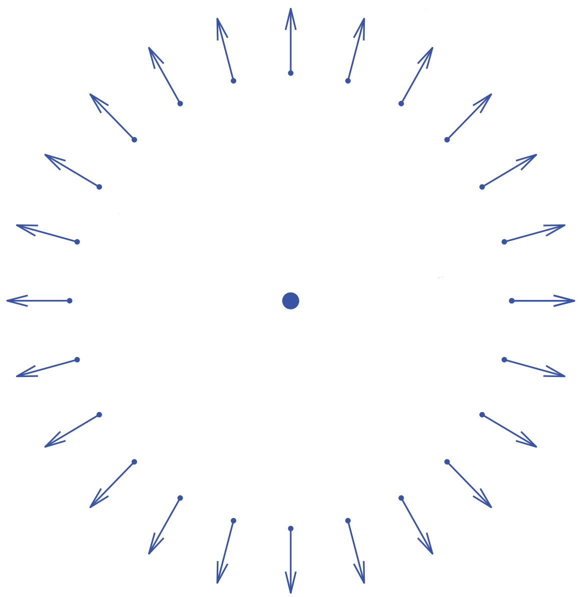

Fig. 1. Arrows indicate direction and relative magnitude of flow every 15 degrees around a circle centered on a monopole. The near and far-field patterns are the same, though the phases differ.

describing a sphere with radius a oscillating along the direction $\hat{z}$ with a peak velocity of u_o. The volume does not modulate. Rather, the dipole source is a rigid volume which acts on the surrounding fluid by translating. The wave equation solution in a homogeneous whole space for a dipole source with the described boundary condition is given by

$$\mathbf{u}_d = u_o\{1/[2 - 2jka - (ka)^2]\} \times$$

$$\{[(a^3/r^3) - jka(a^2/r^2)](3\cos\theta\hat{r} - \hat{z}) - (ka)^2(a/r)\cos\theta\hat{r}\}e^{j[k(r-a)-\omega t]} \quad , \qquad (4)$$

where θ is the angle from the axis of translation. The dipole flow field has two

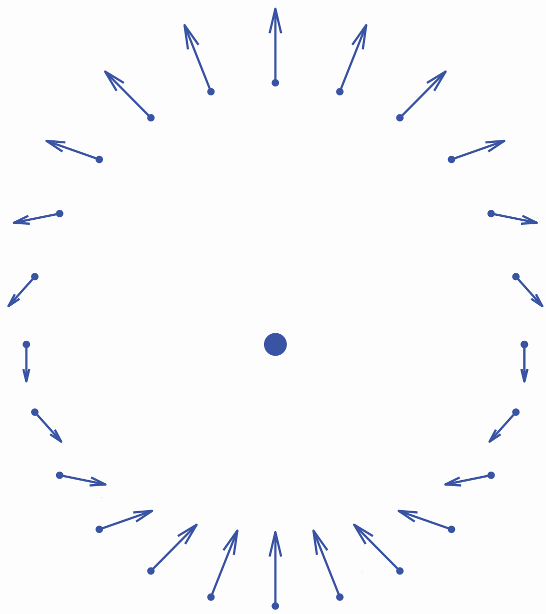

Fig. 2. Arrows indicate direction and relative magnitude of near-field flow every 15 degrees around a circle centered on a dipole. Dipole translation is in the vertical direction.

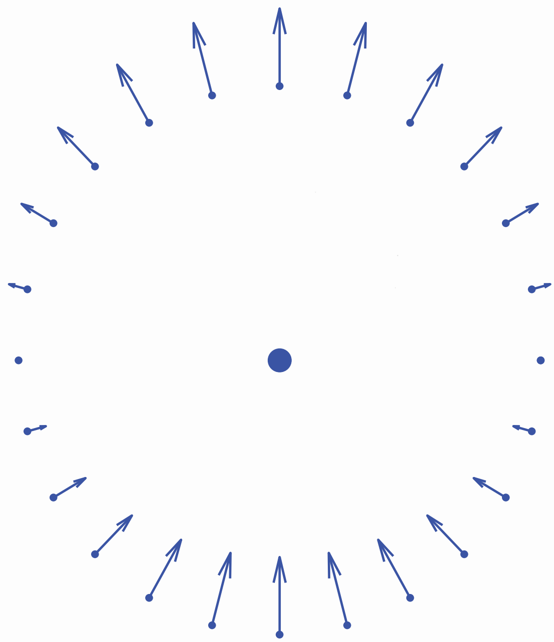

Fig. 3. Arrows indicate direction and relative magnitude of far-field flow every 15 degrees around a circle centered on a dipole. Dipole translation is in the vertical direction.

theory, the drag force on a uniformly moving body in such a fluid is zero, an observation often referred to as d'Alembert's paradox after the French mathematician. The reaction force associated with the above $1/r^3$ term can be shown to be zero, consistent with d'Alembert's observation.

Wavefield pressure relates to flow, $\nabla p = -\rho_o \partial \mathbf{u}/\partial t$, where ρ_o is the medium density. The monopole and dipole pressure corresponding to the flow fields in eqs. (2) and (4), respectively, are

$$p_m = \rho_o c_o u_o[-jka/(1 - jka)](a/r)e^{j[k(r-a)-\omega t]} \quad , \qquad (5)$$

and

$$p_d = \rho_o c_o u_o\{-jka/[2 - 2jka - (ka)^2]\}$$
$$\times [(a^2/r^2) - jka(a/r)]\cos\theta e^{j[k(r-a)-\omega t]} \quad . \qquad (6)$$

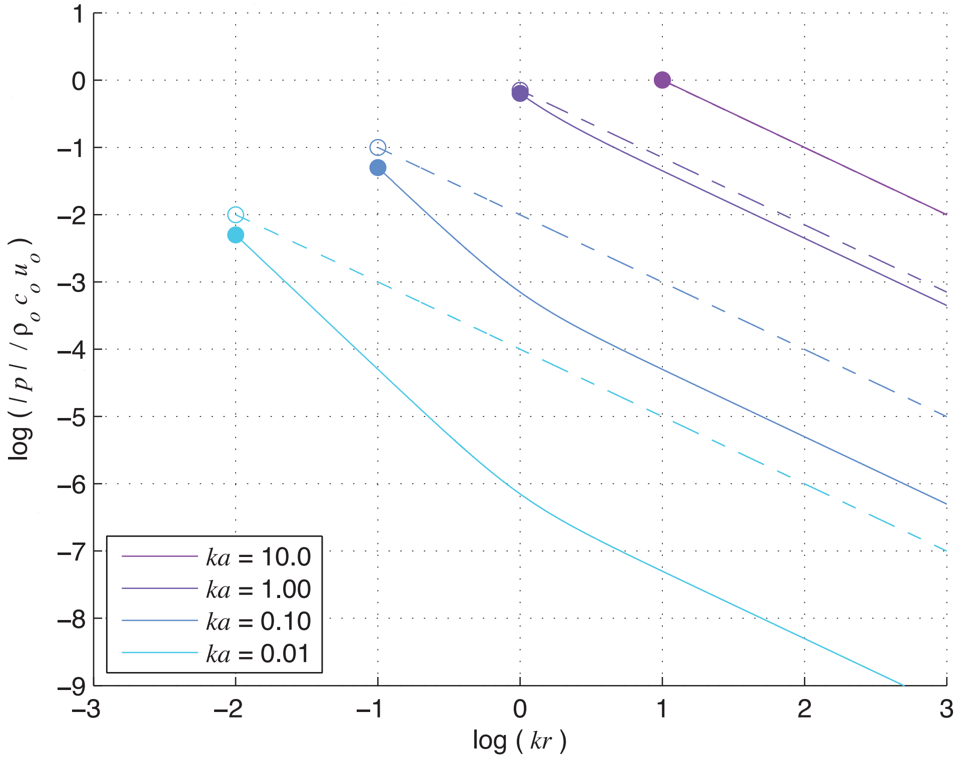

Fig. 4. The wavefield pressure amplitudes for monopole and dipole sources in a whole space, indicated by dashed and solid lines respectively, and normalized by $\rho_o c_o u_o$, are plotted for various values of ka. The magnitude of the boundary condition, u_o, is the same for both sources. The direction is $\theta = 0°$. Boundaries, $r = a$, are indicated by open and filled circles for monopole and dipole fields, respectively. For the case $ka = 10$, the monopole and dipole pressures overplot each other.

The monopole pressure scales with 1/r and has the same frequency dependence as the far-field flow in eq. (2). The dipole pressure has both near-field and far-field components, with distinctive dependencies on ka, similar to dipole flow. Figs. 4 and 5 show the magnitude and phase relative to boundary motion for both monopole and dipole pressure wavefields and several values of ka.

It is the far-field radiation characteristic that is of greatest importance to seismic exploration. Logistically, it is important that source dimensions are allowed to be small relative to radiated wavelengths. If we consider the low frequency condition, ka $<<$ 1, also implying that radiated wavelengths are much greater than the source dimensions, far-field pressures for monopole and dipole sources are approximated from eqs. (5) and (6),

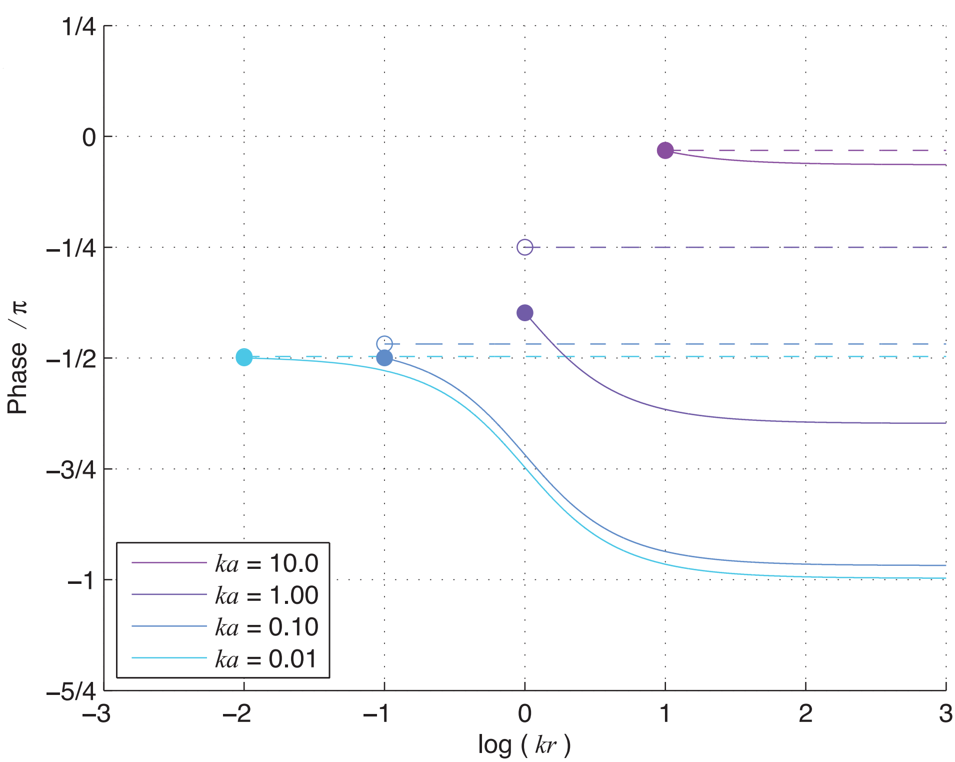

Fig. 5. The pressure wavefield phases for monopole and dipole sources in a whole space, indicated by dashed and solid lines respectively, and referenced to boundary velocity, are plotted for various values of ka. The boundaries, r = a, are indicated by open and filled circles for monopole and dipole fields, respectively. Phase due to propagation is not included.

$$p_{mf} = -\rho_o c_o u_o (jka)(a/r)e^{j[k(r-a)-\omega t]} \quad , \tag{7}$$

$$p_{df} = -\rho_o c_o u_o [(ka)^2/2](a/r)]\cos\theta e^{j[k(r-a)-\omega t]} \quad , \tag{8}$$

where the subscripts mf and df are used to denote the monopole far-field and dipole far-field, respectively. The ratio between the dipole and monopole far-fields is given by

$$p_{df}/p_{mf} = (-jka/2)\cos\theta \quad . \tag{9}$$

Given the condition that $ka << 1$, we conclude that the dipole far-field in a whole space is much smaller than that of the monopole.

Clearly, the homogeneous whole space solution suggests a dipole source has poorer radiation characteristics and no apparent advantages over a monopole source at low frequencies. This conclusion may strongly discourage consideration of dipole sources for marine seismic applications. However, interest in the concept of marine dipole seismic sources is regained, especially for low frequencies, when consideration includes other issues inherent in marine seismic exploration, such as half space effects and forcing requirements.

HALF SPACE

Marine seismic sources are typically towed behind seismic vessels and operate near the sea surface. These sources may be ideally represented as monopole type sources in a half space. The presence of the sea surface substantially impacts the radiation characteristics of a monopole source, especially at wavelengths much larger than the tow depth of the source. In this case, the low frequency radiation characteristic of the monopole source is greatly attenuated and loses much of its advantage over the dipole source. In contrast, the low frequency radiation characteristic of the dipole source is amplified by the sea surface.

A single monopole source positioned in a half space with a pressure-release surface, that is, the pressure is zero on the surface, may be equivalently represented as two opposite polarity monopole sources in a homogeneous whole space, on either side and equidistant from the half space surface. This is known as the image source construction. The radiated field is the linear superposition of the source and image fields. The source pair may be described as a doublet, but should not to be confused with a dipole. The pressure far-field solution for a monopole in the half space may be obtained from

$$p^H_{mf} = p_{mf}(r - d\cos\theta) - p_{mf}(r + d\cos\theta) \quad , \tag{10}$$

where the superscript H is used to denote a half space solution, d is the distance between the monopole and half space boundary, and r now references the midpoint position between the source and its image. For kd $<<$ 1, implying low frequencies with wavelengths much larger than d, the pressure far-field is approximated by

$$p^H_{mf} = -\rho_o c_o u_o (ka)(2kd)(a/r)\cos\theta \; e^{j[k(r-a)-\omega t]} \quad . \tag{11}$$

For a dipole source in a half space under the same conditions, the pressure far-field is approximated by

$$p^H_{df} = -\rho_o c_o u_o (ka)^2 (a/r)\cos\theta \; e^{j[k(r-a)-\omega t]} \quad . \tag{12}$$

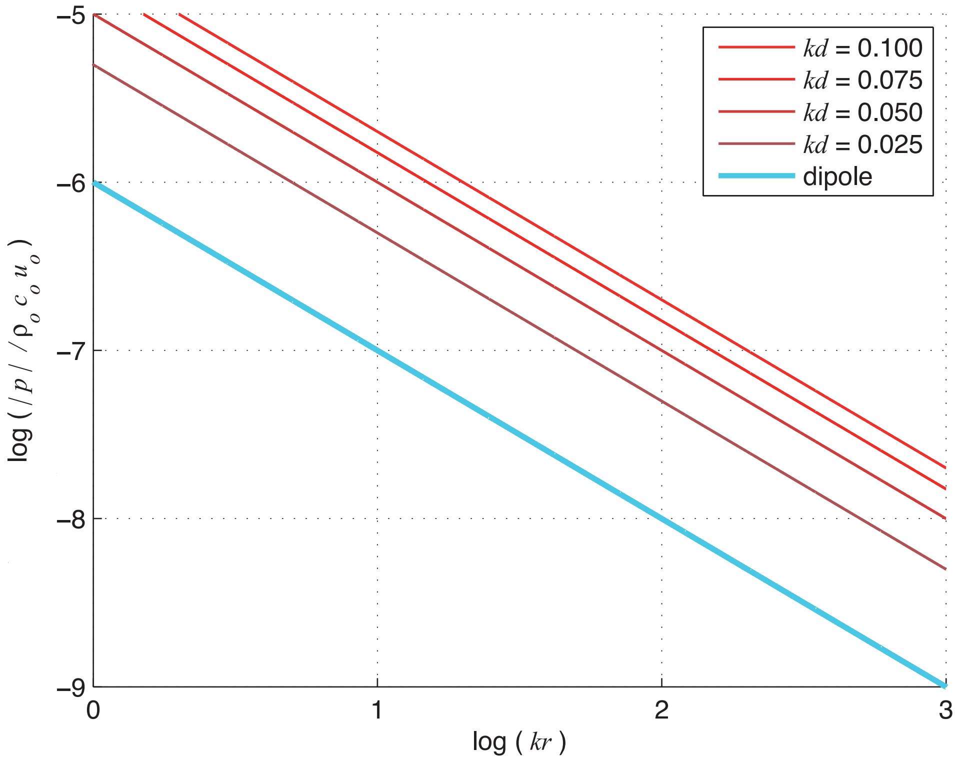

Fig. 6. The far-field pressure amplitudes for ka = 0.01 and direction $\theta = 0°$, normalized by $\rho_o c_o u_o$, of monopole and dipole sources in a half space are plotted. The monopole far-field depends on kd. The magnitude of the boundary condition, u_o, is the same for both sources.

Both dipole and monopole pressure fields have a cosθ radiation pattern. Fig. 6 shows the monopole and dipole far-field pressure amplitudes for the low frequency case, ka = 0.01, and several values of kd ranging from 0.100 to 0.025. For a source depth of six meters and propagation velocity of 1500 meters per second, this range for kd corresponds to a frequency range of 4 to 1 Hz. Recall the dipole far-field pressure is independent of kd at low frequencies.

The ratio between the dipole and monopole far-fields in a half space is given by

$$p^H_{df}/p^H_{mf} = a/2d \quad . \tag{13}$$

The far-field amplitudes of the dipole and monopole are similar in order, which is a very different result from the whole space case. Even so, source dimensions smaller than the tow depths may normally be expected, a < d. In this case, the dipole far-field is still somewhat smaller than the monopole.

FAR-FIELD AMPLITUDE AND FORCING

The far-field ratios given in eqs. (9) and (13) hold for the case when dipole and monopole boundaries modulate with the same peak velocity given by u_o, as described in eqs. (1) and (3). However, from a design point of view, the peak boundary velocity is not necessarily the best reference for comparing far-field amplitudes. A source is designed to deliver a force on the surrounding fluid medium. It takes considerably more force to achieve a given oscillatory peak boundary velocity for a monopole than it does for a dipole. So, a more useful reference for comparison is the force needed to achieve a desired far-field amplitude.

Force delivered on a surrounding fluid medium can be determined from pressure on the source boundary. A force acting on a surface element relates to the pressure on that element, and is expressed in scalar and vector form by

$$d\boldsymbol{F} \cdot \hat{n} = p dS \quad , \tag{14}$$

$$d\boldsymbol{F} = p d\mathbf{S} \quad , \tag{15}$$

where $\hat{n}$ is the surface normal, $d\mathbf{S} = \hat{n} dS$. The total force is obtained from integration over the source surface, recognizing that integrating scalar force is different from integrating vector force and gives different results delineating the basic nature of the source. For example, a net scalar force and no net vector force describes forcing to drive volume modulation, but not translation. Contrastingly, a net vector force and no net scalar force describes forcing to drive translation, but not volume modulation. Surface integrals can be related to volume integrals through integral theorems,

$$\oint_S \mathrm{p}d\mathrm{S} = \int_V \nabla\cdot(d\boldsymbol{F}/d\mathrm{S})d\mathrm{V} \quad , \tag{16}$$

$$\oint_S \mathrm{p}d\mathbf{S} = \int_V \nabla \mathrm{p}d\mathrm{V} \quad . \tag{17}$$

Eq. (16) shows that surface integration over scalar force relates to volume integration of force divergence, and eq. (17) shows that surface integration over vector force relates to volume integration of pressure gradient. Loosely speaking, finite values from (16) and (17) describe a "pressure source" and "pressure gradient source" nature, respectively.

The pressure on the boundaries of monopole and dipole elements are given by evaluation of the corresponding wavefield solutions given in eqs. (5) and (6),

$$\mathrm{p_m}(\mathrm{r} = a) = \rho_o c_o u_o[-\mathrm{j}\mathrm{k}a/(1 - \mathrm{j}\mathrm{k}a)e^{-\mathrm{j}\omega t} \quad , \tag{18}$$

$$\mathrm{p_d}(\mathrm{r} = a) = \rho_o c_o u_o\{[-\mathrm{j}\mathrm{k}a-(\mathrm{k}a)^2]/[2-2\mathrm{j}\mathrm{k}a-(\mathrm{k}a)^2]\}\cos\theta\, e^{-\mathrm{j}\omega t} \quad . \tag{19}$$

Evaluating the surface integrals in eqs. (16) and (17) for the pressure on the surface of the monopole element gives the total forcing required to drive the monopole surface in pulsation with peak velocity of u_o. Surface integration of the scalar quantity in eq. (14) gives a finite result whereas surface integration of the vector quantity in eq. (15) gives zero. This shows the monopole is a type of "pressure source". The monopole forcing for $\mathrm{k}a << 1$ is:

$$F_m = -\mathrm{j}\omega\rho_o u_o 4\pi a^3 e^{-\mathrm{j}\omega t} \quad . \tag{20}$$

Evaluating the surface integrals in eqs. (16) and (17) for the pressure on the surface of the dipole element gives the total forcing required to drive the dipole surface in translational oscillation with peak velocity of u_o. In this case, surface integration of the scalar quantity gives zero whereas surface integration of the vector quantity gives a finite result. This shows the dipole is a type of "pressure gradient source". The dipole forcing for $\mathrm{k}a << 1$ is:

$$\boldsymbol{F}_d = -\mathrm{j}\omega\rho_o u_o(2/3)\pi a^3 e^{-\mathrm{j}\omega t}\hat{z} \quad . \tag{21}$$

The monopole and dipole far-field pressures in a half space can be written in terms of the total respective applied forces;

$$p^H_{mf} = -jkF_m(d/2\pi a)\cos\theta\ [e^{j[k(r-a)-\omega t]}/r] \ , \qquad (22)$$

$$p^H_{df} = -jkF_d(3/2\pi)\cos\theta\ [e^{j[k(r-a)-\omega t]}/r] \ , \qquad (23)$$

where F_m and F_d are force magnitudes defined by the relations $\boldsymbol{F}_m = F_m e^{-j\omega t}$ and $\boldsymbol{F}_d = F_d e^{-j\omega t}\hat{z}$.

Reconsidering the ratio of far-field amplitudes between monopole and dipole for the case of equal force magnitudes, $F_d = F_m$, gives

$$[p^H_{df}/p^H_{mf}]_{\overline{F_d=F_m}} = 3a/d \ . \qquad (24)$$

In the case of equal forcing, the far-field amplitude of a dipole source in a half-space is comparable to that of a monopole source in the same space. Fig.7 shows the far-field pressure amplitudes for the low frequency case $ka = 0.01$ and the same values of kd as in Fig. 6. For sufficiently low frequencies, the dipole far-field pressure is slightly greater than the monopole. The far-field pressures are the same if the source radius is one-third the tow depth, $a = d/3$. With regard to radiation characteristics, the monopole source has no substantial advantage over the dipole source. Each may be comparably considered, and factors other than radiation characteristics may determine the favorability of one over the other.

MARINE DIPOLE DESIGN CRITERIA

The far-field pressure ratio in eq. (24) has dependence on the source dimension, a, and the source depth, d, that comes strictly from monopole far-field pressure given in eq. (22). The dipole far-field pressure, given in eq.(23), has no dependence on the source dimension or depth, provided the requirements ka, $kd << 1$ are satisfied. The dipole far-field pressure depends on the force applied, F_d, but not on the size of the source. This important result shows that a far-field amplitude specification does not directly establish any requirement on the source dimension.

However, a far-field amplitude specification does directly establish a forcing requirement. Force causes acceleration, and source dimension enters into design criteria if there is a maximum acceleration that should not be exceeded. For example, boundary accelerations exceeding acceleration due to gravity may result in undesirable cavitation effects. Dipole forcing given in eq. (21) produces the boundary velocity given in eq. (3). Taking the time derivative to obtain boundary acceleration gives

$$\mathbf{a}_d(r = a) = -j\omega u_o e^{-j\omega t}\hat{z} \ . \qquad (25)$$

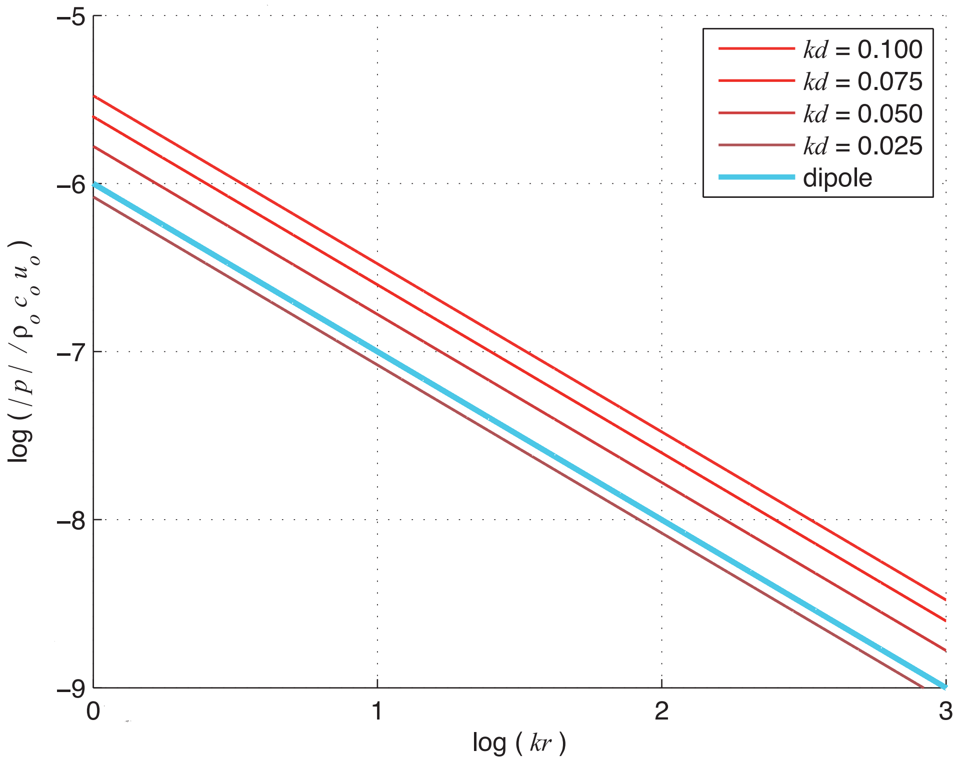

Fig. 7. The far-field pressure amplitudes for ka = 0.01 and direction θ = 0°, normalized by $\rho_o c_o u_o$, of monopole and dipole sources in a half space are plotted. The monopole far-field depends on kd. The forcing is the same for both sources, $F_m = F_d$.

Combining with the forcing equation and using

$$\boldsymbol{F}_d = m_e \mathbf{a}_d(r = a) \quad , \tag{26}$$

to define a quantity referred to as "entrained mass", m_e, gives

$$m_e = (2/3)\pi a^3 \rho_o \quad , \tag{27}$$

which is, equivalently, one half the mass of the displaced fluid. The entrained mass represents the fluid load on the dipole source, and grows with increasing source size. The entrained mass must be sufficiently large so that the source boundary does not exceed g for a desired dipole forcing magnitude,

$$m_e > F_d/g \quad . \tag{28}$$

By way of example, the required radius of the source element may be expressed directly in terms of the far-field pressure,

$$a > (rp^H_{df}/k\rho_0 g)^{1/3} \quad . \tag{29}$$

Table 1 shows example values for a one Hertz dipole source. Note that the source element dimensions are orders of magnitude smaller than the 1,500 meter wavelength of a one Hertz wavefield in water.

Results shown in Table 1 describe time harmonic amplitudes. Commercial application of a marine dipole source will involve finite sweep durations over a desired range of frequencies. The design of marine dipole sweeps to deliver adequate source energy over a frequency band incorporates many of the same design elements used for modern controlled seismic sources. Increased ambient environmental noise levels at low frequencies add to the challenge of delivering sufficient energy. In addition, trace density, determined by source and receiver station intervals, plays a role in discerning the seismic response from ambient environmental noise and unwanted modes. Seismic survey design procedures using a marine dipole source will need to determine forcing, sweep function, and sampling density required to achieve back scatter response amplitudes at desired frequencies that meet particular seismic survey objectives.

Table 1. Producing a one Hertz far-field pressure given in column two at a distance of 1,500 meters along the dipole axis ($\theta = 0°$) requires a dipole force magnitude shown in column three. The radius shown in column four is needed to keep boundary acceleration less than g.

	A One Hertz Dipole Source		
Distance r (m)	Far-field pressure p^H_{df} (Pa)	Dipole forcing F_d (kN)	Radius a (m)
1,500	0.01	7.5	0.71
	0.1	75	1.54
	1.0	750	3.32

CONCLUSIONS

We have shown that a dipole source has radiation characteristics in the low frequency band that may be useful for marine seismic applications. The dipole element dimension does not affect the far-field amplitude, and can be made orders of magnitude smaller than the radiated wavelength. The far-field amplitude does depend on the dipole forcing level, and desired far-field amplitude levels determine dipole forcing specifications. To avoid cavitation, the entrained mass must be sufficiently large to prevent acceleration from becoming too large.

Since dipole radiation is comparable to monopole radiation at low frequencies, factors other than radiation characteristics decide which may be a more effective marine seismic source. Improving low frequency output from existing commercial marine sources has proven very difficult. Their monopole nature requires that the effective boundary peak velocity must increase proportionally with the square of decreasing frequency in order to maintain the same far-field amplitude. This corresponds to radial displacement increasing with the cube of decreasing frequency, requiring very large volume modulation which can be challenging to implement. The marine dipole is a fixed volume, generating acoustic radiation by translation rather than volume modulation. Axial translation of a fixed volume, even over a large displacement, can be a simpler mechanical action to implement. Translation can be driven by an actuator and reaction mass internal to the dipole source element. Engineering and logistical issues may well favor the dipole marine source over monopole for low frequency seismic applications.

ACKNOWLEDGMENTS

The authors gratefully acknowledge Professor Emeritus Elmer Hixson (now deceased) for his interest and early guidance in this work. Support was provided by the ExxonMobil Upstream Research Company.

REFERENCES

Blackstock, D.T., 2000. Fundamentals of Physical Acoustics. John Wiley and Sons, Inc., New York: 335-375.

Duren, R.E. and Meier, M.A., 2008. Marine Seismic Acquisition Method and Apparatus. US Patent 7,377,357.

Hixson, E.L., 2009. A low-frequency underwater sound source for seismic exploration. Proc. 158th Mtg., The Acoust. Soc. Am., San Antonio: 3aEA1.

Meier, M.A. and Duren, R.E., 2014. Two Component Source Seismic Acquisition and Source Deghosting. U.S. Patent 8,833,509.

Meier, M.A., Lewallen, K.T., Otero, J., Heiney, S. and Murray, T., 2015a. Low-frequency seismic acquisition innovations approaching one Hertz. Offshore Technol. Conf., Houston, OTC-25688-MS.

Meier, M.A., Duren, R.E., Lewallen, K.T., Otero, J., Heiney, S. and Murray, T., 2015b. A marine dipole source for low frequency seismic acquisition. Expanded Abstr., 85th Ann. Internat. SEG Mtg., New Orleans: 176-180.