

A LINEARIZED GROUP VELOCITY APPROACH FOR TWO-POINT qP RAY TRACING IN A LAYERED ORTHORHOMBIC MEDIUM

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ABSTRACT

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Using a linearized approximation for the quasi-compressional phase velocity, v_{qP} in an orthorhombic anisotropic medium, which is a subset of the related quasi-compressional (qP) wave propagation in a general 21 parameter anisotropic medium, a linearized compressional group velocity may be derived as a function of group angles only. In addition, linearized analytic expressions for the components of the slowness vector in terms of group velocities and angles are also obtained. These expressions are used to define two nonlinear equations which are a generalization of Snell's Law. The solutions of these are used to determine the propagation directions of the reflected and transmitted rays due to an incident ray at an interface between two orthorhombic media. The axes of anisotropy, in both media are, in general, not aligned with the interface separating them. Computer code has been written to consider ray tracing in media defined by a type of large scale 3D finite element blocking (blocky structures). However, a plane parallel layered medium will be used in preliminary investigations. Additionally, in each of these elements (layers) the anisotropic parameters are initially assumed to be constant, although provisions for at least minimal spatial variations of the anisotropic parameters have been considered.

KEY WORDS: orthorhombic media, two-point ray tracing, group velocity, layered anisotropic media.

INTRODUCTION

In the geophysical literature related to wave propagation in anisotropic media, specifically quasi-compressional qP waves in a medium displaying orthorhombic symmetry, a linearized approach to derive an approximate phase velocity expression for quasi-compressional qP wave propagation has been presented in Backus (1965) and more recently by Pšenčík and Farra (2005).

These and other methods, such a perturbation theory, have been employed to formulate expressions for quantities related to the propagation of qP waves as well as for the two shear wave modes, qS_1 and qS_2 , in a general 21 parameter anisotropic medium (Every, 1980; Every and Sachse, 1992; Jech and Pšenčík, 1989; Pšenčík and Gajewski, 1998; Song and Every, 2000; Song et al., 2001; Pšenčík and Farra, 2008). Once linearized phase velocity approximations have been obtained, eikonal equations with comparable accuracy may be written. From these, using the method of characteristics, (Courant and Hilbert, 1962; Červený, 2002) the formulae for the vector components of group velocity may be obtained.

Employing an extension of this method of approximation, Daley and Krebes (2006) obtained expressions for the qP group velocities in an orthorhombic anisotropic medium, expressed in terms of group angles. It was further shown that for a weakly anisotropic medium the average deviation of this approximation from the exact expression for the qP group velocity, in an orthorhombic medium, over a range of polar angles for a number of azimuths, Φ , was of the order of a few percent, for media that could be classified as *weakly* anisotropic. The expression for the qP group velocity was also compared with the linearized qP group velocity expressions obtained using the First Order Ray Tracing (FORT) method presented in Pšenčík and Farra (2005), with similar accuracy, for an orthorhombic medium. For this reason it was chosen for use in preliminary ray tracing procedures where speed, together with reasonable accuracy, was required.

It will be convenient to first consider the case where the anisotropic parameters and orientation of anisotropy are assumed constant within any finite element. What is required to be determined is an analogue of Snell's Law at the plane interface between two adjacent elements. Within the context of "two-point" ray tracing this topic will be treated in what follows for an orthorhombic anisotropic medium.

THEORETICAL BACKGROUND

The linearized quasi-compressional (qP) phase (wavefront normal) velocity, $v_{qP}(n_k)$, in an orthorhombic medium may be written in Voigt notation, where the density (ρ) normalized stiffness coefficients, $A_{ij} = C_{ij}/\rho$, ($i, j = 1, 2, \dots, 6$) have the dimensions of velocity squared, as

$$v_{qP}^2(n_k) = A_{11}n_1^2 + A_{22}n_2^2 + A_{33}n_3^2 + E_{12}n_1^2n_2^2 + E_{13}n_1^2n_3^2 + E_{23}n_2^2n_3^2 \quad , \quad (1)$$

$$\mathbf{n} = (n_1, n_2, n_3) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \quad , \quad (2)$$

(Backus, 1965; Daley and Krebes, 2006) where (θ, ϕ) are the polar and azimuthal angles associated with the phase or wavefront normal velocity, v_{qP} . The E_{ij} are the anellipsoidal terms, specifying the deviation of the slowness or ray surface from the ellipsoidal in the "i j" symmetry plane, (i j = 12,13,23). Their definitions are as follows

$$E_{12} = 2(A_{12} + 2A_{66}) - (A_{11} + A_{22}) \quad , \quad (3)$$

$$E_{13} = 2(A_{13} + 2A_{55}) - (A_{11} + A_{33}) \quad , \quad (4)$$

$$E_{23} = 2(A_{23} + 2A_{44}) - (A_{22} + A_{33}) \quad . \quad (5)$$

These expressions may be compared to those given for an arbitrary anisotropic orthorhombic medium and presented in Gassmann (1964) or Schoenberg and Helbig (1996) as an indication of the how linearization simplifies the phase velocity expression.

The 3D phase velocity propagation direction vector, \mathbf{n} , is defined by eq.(2). The related slowness vector in terms of these quantities is of the form

$$\mathbf{p} = (p_1, p_2, p_3) = \{[n_1/v_{qP}(n_k)], [n_2/v_{qP}(n_k)], [n_3/v_{qP}(n_k)]\} \quad . \quad (6)$$

It was shown in Daley and Krebes (2006) that the slowness vector \mathbf{p} , for the qP case may be further approximated for the linearized case and written in terms of group velocity quantities, those being the azimuth and polar ray angles Φ and Θ , and the group/ray velocity, $V_{qP}(\Theta, \Phi)$, as

$$\mathbf{p} = (p_1, p_2, p_3) = \{[N_1 V_{qP}(N_k)/A_{11}], [N_2 V_{qP}(N_k)/A_{22}], [N_3 V_{qP}(N_k)/A_{33}]\} \quad . \quad (7)$$

In the above \mathbf{N} is the orthonormal group (ray) vector defined by

$$\mathbf{N} = (N_1, N_2, N_3) = (\sin\Theta\cos\Phi, \sin\Theta\sin\Phi, \cos\Theta) \quad . \quad (8)$$

It is along the ray that energy propagates from one point in a medium to another. The linearized group velocity expression for a qP ray in an orthorhombic medium is given by the formula

$$[1/V_{qP}^2(N_k)] = (N_1^2/A_{11}) + (N_2^2/A_{22}) + (N_3^2/A_{33}) \\ - (E_{12}N_2^2N_3^2/A_{11}A_{22}) - (E_{13}N_1^2N_3^2/A_{11}A_{33}) - (E_{23}N_1^2N_3^2/A_{22}A_{33}) \quad , \quad (9)$$

with the E_{ij} being previously defined in eqs. (3)-(5). Eq. (9) may be expressed in terms of (Θ, Φ) using eq. (8).

As the slowness vector $\mathbf{p} = (p_1, p_2, p_3)$ is assumed to be known, the sines of the group angles ($\xi_1 = \sin\Theta$, $\xi_2 = \sin\Phi$) will be used as an alternate parameterization of group velocity and related quantities. In the ellipsoidal case ($E_{12} = E_{13} = E_{23} = 0$) ξ_1 and ξ_2 may be obtained analytically as

$$\xi_1 = (p_r/p_3)[(\cos^2\Phi/A_{11}^2) + (\sin^2\Phi/A_{22}^2)]^{-1/2}/A_{33}^2 \\ \times \{1 + (p_r/p_3)^2[(\cos^2\Phi/A_{11}^2) + (\sin^2\Phi/A_{22}^2)]^{-1}/A_{33}^2\}^{-1} , \quad (10)$$

and

$$\xi_2 = (A_{22}p_2/A_{11}p_1)/[1 + (A_{22}p_2/A_{11}p_1)^2]^{1/2} . \quad (11)$$

The radial component of the slowness vector, p_r , is defined as

$$p_r = (p_1^2 + p_2^2)^{1/2} . \quad (12)$$

The relationship between the phase angles (θ, ϕ) and the group angles (Θ, Φ) in the ellipsoidal case are calculated by equating individual components in eqs. (6) and (8) which are used to derive eqs. (10)-(12).

SNELL'S LAW ANALOGUE

Given a three dimensional slowness surface that may be rotated at any orientation with respect to the model axis system, it would be expected that Euler Angles be introduced. However, as one of the motivations for this work is to introduce 3D ray tracing in an orthorhombic anisotropic medium in the simplest possible manner, this will not be done. When more complex ray tracing methods (comparatively) are addressed, such as FORT, this will be necessary. As the slowness vector in terms of phase angles and velocities is not used here, one level of theoretical and numerical complexity has been removed, allowing for the subsequent formulation.

Two orthorhombic media in welded contact are separated by a plane boundary with the axes of anisotropy in both the upper (1) and lower (2) medium being aligned with some Cartesian model coordinates system within which the plane boundary is parallel to the horizontal (x_1, x_2) model coordinate plane. As previously mentioned, this is a very simplistic problem type, but once solved may be extended to more complex geometries with minimal effort.

Given that the incident ray angles (Θ_i, Φ_i) are known it is required to determine the ray angle and magnitude of the ray velocity for either the reflected ray in the upper medium or the transmitted ray in the lower medium. Specifically, the two horizontal components of the slowness vector, $\mathbf{p}_h = (p_1, p_2)$

at the point of incidence at an interface, measured with respect to the model coordinates, are required to be determined. Once this is done, the third component of the slowness vector, p_3 , follows. It has been initially assumed that the plane interface is aligned with the model coordinate axes as are the qP slowness surfaces in both media. As a consequence of these assumptions, the projections of the slowness surface onto the p_1p_3 and p_2p_3 planes may be realized. From eq. (7), the continuity of the horizontal components of the slowness vector may be stated in terms of (ξ_1, ξ_2) by the two equations

$$F_1(\xi_1, \xi_2) = p_1 A_{11} - \xi_1(1 - \xi_2^2)^{1/2} V_{qP}(\xi_1, \xi_2) = 0 \quad , \quad (13)$$

$$F_2(\xi_1, \xi_2) = p_2 A_{22} - \xi_1 \xi_2 V_{qP}(\xi_1, \xi_2) = 0 \quad . \quad (14)$$

These coupled nonlinear equations are required to be solved numerically. For coupled equations of this type Newton's Method, or some variant, would be a reasonable choice as a solution method, the standard formulation of which is given here for completeness,

$$\xi_{k+1} = \xi_k - \mathbf{F}_k(\xi_1, \xi_2) [\mathfrak{S}_k(\xi_1, \xi_2)]^{-1} \quad , \quad (15)$$

where $\xi_k = (\xi_1, \xi_2)_k^T$, $\mathbf{F}_k(\xi_1, \xi_2) = [F_1(\xi_1, \xi_2), F_2(\xi_1, \xi_2)]_k^T$ and the Jacobian $\mathfrak{S}_k(\xi_1, \xi_2)$, is defined as

$$\mathfrak{S}_k(\xi_1, \xi_2) = \begin{bmatrix} \partial F_1 / \partial \xi_1 & \partial F_1 / \partial \xi_2 \\ \partial F_2 / \partial \xi_1 & \partial F_2 / \partial \xi_2 \end{bmatrix}_k \quad . \quad (16)$$

Its inverse is given by

$$[\mathfrak{S}_k(\xi_1, \xi_2)]^{-1} = (1/D_k) \begin{bmatrix} \partial F_2 / \partial \xi_2 & -\partial F_1 / \partial \xi_2 \\ -\partial F_2 / \partial \xi_1 & \partial F_1 / \partial \xi_1 \end{bmatrix}_k \quad . \quad (17)$$

$$D_k = [(\partial F_1 / \partial \xi_1)(\partial F_2 / \partial \xi_2) - (\partial F_2 / \partial \xi_1)(\partial F_1 / \partial \xi_2)]_k \quad . \quad (18)$$

The superscripts "T" and "-1" indicate transpose and inverse, respectively.

The partial derivatives of $F_j(g_1, g_2)$, ($j = 1, 2$) with respect to ξ_1 and ξ_2 analytic expressions for the partial derivatives of $V_{qP}(\xi_1, \xi_2)$ with respect to ξ_1 and ξ_2 are given in the Appendix. The initial "guesses" for the two components of the quantity ξ_0 may be found using the ellipsoidal (degenerate) eqs. (10) and (11). Once $\xi = (\xi_1, \xi_2)$ has been determined to within a specified tolerance, the group velocity and as a result, the three components of the slowness vector in the medium of reflection or transmission, may be determined using eqs. (7).

SNELL'S LAW ANALOGUE AT A PLANE INTERFACE WHERE THE AXES OF ANISOTROPY ARE NOT ALIGNED WITH THE INTERFACE

For this problem, it is assumed that the slowness vector \mathbf{p} , relative to the model coordinates is known. To solve the problem, that is to determine the polar ray angles (Θ, Φ) relative to the model coordinates, the angles in the rotated system $(\Theta' = \Theta + \gamma, \Phi' = \Phi + \zeta)$ must be determined, under the assumption that the polar and azimuthal rotation angles (γ, ζ) are known. The primed angles define the slowness vector, \mathbf{p}' , in the rotated coordinate system and the angle pair (γ, ζ) specifies the relationship between (Θ, Φ) and (Θ', Φ') .

With $\mathbf{p}'_h = (p'_1, p'_2)$ known, a system of nonlinear equations in terms of ξ'_1 and ξ'_2 is required to be determined where

$$\xi'_1 = \sin(\Theta + \gamma) = \sin\Theta' \quad , \quad (19)$$

and

$$\xi'_2 = \sin(\Phi + \zeta) = \sin\Phi' \quad . \quad (20)$$

The following nonlinear system is, apart from the "primed" notation, the same as that for the unrotated (unprimed) case and may be solved in a similar manner.

$$F_1(\xi'_1, \xi'_2) = p'_1 A_{11} - V_{qp}(\xi'_1, \xi'_2) \xi'_1 (1 - \xi'^2_2)^{1/2} = 0 \quad , \quad (21)$$

$$F_2(\xi'_1, \xi'_2) = p'_2 A_{22} - V_{qp}(\xi'_1, \xi'_2) \xi'_1 \xi'_2 = 0 \quad . \quad (22)$$

What is required to solve this system using numerical methods is initial estimates for ξ'_1 and ξ'_2 . This, as previously done, may be accomplished by assuming the ellipsoidal case and solving for $(\xi'_1)_{\text{ellip.}}$ and $(\xi'_2)_{\text{ellip.}}$ using the sequence of eqs. (10)-(12) in the "primed" system. As the system of nonlinear equations above is essentially the same as the unrotated problem, the solution for this case follows.

Once (ξ'_1, ξ'_2) has been determined, the three components of the slowness vector in the primed system may be obtained. Given that the vector \mathbf{p}' is known, \mathbf{p} may be obtained using the inverse of the rotational transforms given by eqs. (19) and (20).

It should be noted that the orientation of the local vertical axis in both slowness space and ray space must be taken in the proper sense as a reflected and transmitted ray and slowness vertical component of each individual vector have different signs. It is often better to do away with the signs and measure the angles as acute with respect to either the positive or negative direction of the vertical axis and one of the horizontal axes in model coordinates.

RAY TRACING METHOD

Assume a medium composed of N plane parallel homogeneous anisotropic layers where both the source and receiver are located at the surface. The source is situated at the origin of a Cartesian system aligned with the plane layered medium, with the vertical Cartesian axis, x_3 , chosen to be positive downwards. A description of unconverted qP ray propagation is given by the two coupled nonlinear equations in terms of ξ_1 and ξ_2 as

$$\begin{aligned} R_1(\xi_1, \xi_2) &= r[1 - (\xi_2^2)]^{1/2} - \sum_{j=1}^M \{n_j^\uparrow h_j(\xi_1)_j^\uparrow [1 - (\xi_2^2)_j^\uparrow]^{1/2} / [1 - (\xi_1^2)_j^\uparrow]^{1/2}\} \\ &\quad - \sum_{j=1}^M \{n_j^\downarrow h_j(\xi_1)_j^\downarrow [1 - (\xi_2^2)_j^\downarrow]^{1/2} / [1 - (\xi_1^2)_j^\downarrow]^{1/2}\} = 0 \quad , \end{aligned} \quad (23)$$

$$\begin{aligned} R_2(\xi_1, \xi_2) &= r(\xi_2^2)_{\text{surf}}^\uparrow - \sum_{j=1}^M \{n_j^\uparrow h_j(\xi_1)_j^\uparrow (\xi_2)_j^\uparrow / [1 - (\xi_1^2)_j^\uparrow]^{1/2}\} \\ &\quad - \sum_{j=1}^M \{n_j^\downarrow h_j(\xi_1)_j^\downarrow (\xi_2)_j^\downarrow / [1 - (\xi_1^2)_j^\downarrow]^{1/2}\} = 0 \quad , \end{aligned} \quad (24)$$

where M ($M \leq N$) is the deepest layer traversed by the ray. The integers n_j are the number of P-ray segments in the j -th layer, with the superscripts " \uparrow " and " \downarrow " indicating whether the ray segment(s) are upward or downward propagating. As upward and downward propagating ray segments within a layer do not generally make the same angle with vertical or horizontal axes they are considered separately. The thickness of the j -th layer is taken to be h_j . What should be noted in eqs. (23)-(24) is that there are no "primes" on the ξ_i ($i = 1, 2$). However, they are implied by the use of the " \uparrow " and " \downarrow " notation. The angle $\Phi_{\text{surf}}[(\xi_2)_{\text{surf}}^\uparrow, (1 - (\xi_2^2)_{\text{surf}}^\uparrow)^{1/2}]$ is the angle measured in a positive (clockwise) direction relative to the x_1 axis from the source to a receiver on the surface at $\mathbf{r} = (x_1^R, x_2^R)$, with $r = |\mathbf{r}|$.

Embedded in these two equations are the two coupled nonlinear equations for the analogue of Snell's Law discussed in an earlier section. Thus, this "two-point" ray tracing scheme is quite computationally intensive. The method or methods used in the solution of these numerical problems include those of Newton among others. The technique chosen is usually based on previous experience. It should however be capable of dealing with more complex situations, a detail to be kept in mind when devising the ray tracing code. In the next section examples will be presented.

Finally, the traveltime along the ray may also be written in terms of the sines $[(\xi_i)_j^\uparrow]$ ($i = 1, 2$, $j=1, M$) of the angles $(\Theta_j^\uparrow, \Phi_j^\uparrow, j=1, M)$ as

$$\begin{aligned} \tau[(\xi_1)_j^{\uparrow\downarrow}, (\xi_2)_j^{\uparrow\downarrow}] &= \sum_{j=1}^M n_j^{\uparrow} h_j / \{1 - [(\xi_1)_j^{(qP^{\uparrow})}]^2\}^{1/2} V[(\xi_1)_j^{\uparrow}, (\xi_2)_j^{\uparrow}] \\ &+ \sum_{j=1}^M n_j^{\downarrow} h_j / \{1 - [(\xi_1)_j^{(qP^{\downarrow})}]^2\}^{1/2} V[(\xi_1)_j^{\downarrow}, (\xi_2)_j^{\downarrow}] \quad . \quad (25) \end{aligned}$$

COMMENTS ON THE SOLUTION AND NUMERICAL RESULTS

The problem discussed here may be approached in a number of ways, depending on what the ray tracing code is being used for. It will be assumed that ray tracing for the purposes of synthetic seismogram production, Born-Kirchhoff migration or some related task is being considered.

The 3D ray tracing algorithm for qP rays in an orthorhombic anisotropic medium is reasonably complex. However, for it to be used in a productive manner, a relatively sophisticated model building program is an essential addition. In 2D, a grid with all of the anisotropic parameters, together with other required quantities, could be specified at each node of this grid. At present, a realistic 3D grid is not an option unless top level computer hardware is available. Few persons would have access to this current generation of "super computers". A reasonably fast laptop with 2 - 4 gigabytes of memory should be adequate if the model is specified as a "blocky type structure", composed of slabs or wedges or other similar types of constituents, separated by plane interfaces. For some applications these interfaces may be smoothed, but the ray tracing program should be written to handle jump discontinuities in the anisotropic parameters defining each of the blocks in the model. Although these types of model building programs exist, obtaining a *freeware* type is not as easy as might be expected. Those which are in the public domain usually require more than a minimum of effort to be of use for an existing ray tracing program. It is highly advisable to choose and become acquainted with one of these and then proceed to write the ray tracing code. It should not have to be mentioned that the possibility of errors in these types of codes is high. Additionally, the known 3D model building software requires at least one, usually two, licensed numerical software packages, which introduces more inconveniences. If alternatives in the public domain are used to replace the equivalent of the licensed packages, integrating these into the model building code can produce another source of error as well as an increased development time. This leaves the preferable option of spending the time to write one's own (not an insignificant undertaking).

The ray tracing described here is computationally fast. Most of the time required is iterating to a solution, which increases if bad initial guesses are used.

Consequently, shooting the initial ray of a linear sequence at near vertical incidence is done followed by considering a line of receivers, increasing in distance at polar angle increments from the origin, at a constant azimuth. This is applicable to both offset shooting and shooting to some reference depth.

In the first example considered here a plane layered three layer model is used. The surface layer is chosen to be isotropic. Layer two has an orthorhombic type structure similar to olivine, with the third layer being an orthorhombic sandstone type medium. In layer 2 the symmetry axes are not aligned with "model coordinates" but rather rotated about the vertical axis by an angle " $\Phi_{\text{rot}} = 30^\circ$ ", measured from the positive x_1 -axis. In the third layer, a rotation of the ray (slowness) surface(s) of " $\Theta_{\text{rot}} = 15^\circ$ " about the x_2 -axis is introduced.

Two-point ray tracing which varies somewhat from that described in the text is first considered. Rather than solve for Θ_j^{\dagger} and Φ_j^{\dagger} , ($j = 1,2,3$), the parameters used are Θ_j^{\dagger} ($j = 1,2,3$) and offset. This was done so that the Φ_j^{\dagger} ($j = 1,2,3$) could "float" to provide an indication of how the azimuthal rotation in the second layer affects the arrival at the surface receivers, which are a fixed distance apart. Put another way, what is done here is to keep the azimuthal phase angle constant for a given ray, maintain a constant receiver spacing and solve for the polar ray angle.

Three views of this is given in Fig. (1). The scale varies along all axes in the figure to produce plots that are easily viewed. It should be noted that proper ray 3D plotting software could significantly enhance the presentation of results. A similar plot for a number of azimuths is given in Fig. (2). Again, the ray azimuthal angles, in each layer, along each ray, are not used as solution parameters. The anisotropic parameters used are given in Tables 1 and 2. Next, a ray tracing problem is considered in which the polar and azimuthal ray angles in each layer and along each ray, together with the distance between the surface located source and receivers are required to satisfy certain conditions. This is the type of ray tracing which was discussed in this paper. Upon viewing Fig.3, where results are presented for this case, it may be seen that the progression of the receiver location describes a straight line from the source to the furthest offset.

An alternate example uses the same model as in the previous case and the "two-point" ray tracing described in the text is used. Here a number of azimuths are chosen and the rays are computed from a source located at the surface down to some reference depth (Fig. 4). The motivation for this is that there are occasions in seismic processing when a "time to depth" map is useful. If the coverage is sufficient the computed data may be used in migration programs. Usually, the coverage must be enhanced which may be done using any number of methods or strategies. The pursuit of these is beyond the scope of this work.

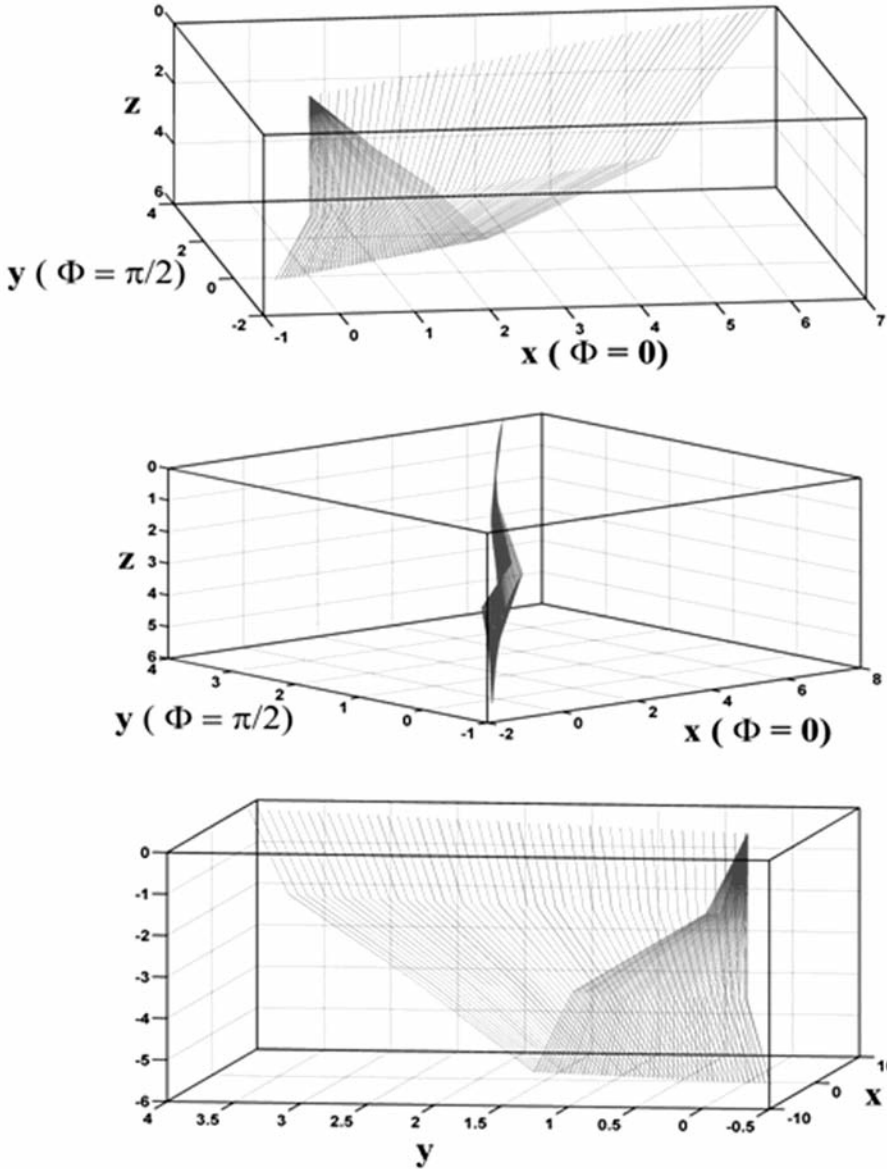


Fig. 1. Three layer model. The first layer is isotropic, the second layer is orthorhombic with a rotation in the $x - y$ plane about the z -axis and the third layer is also orthorhombic with a rotation in the $x - z$ plane. The line shot is very close to two-point ray tracing, requiring some minor tweaking. The analogue of Snell's Law at an interface is a system of two coupled nonlinear equation in two unknown angles (sines of the transmitted or reflected angles). The x - and y -axes in the three panels have different scaling to enhance the effects of the rotations of the orthorhombic ray surfaces. The line is shot at an angle of 70 degrees with respect to the x -axis. The modified two-point ray tracing algorithm consists of two coupled nonlinear equations in two unknown angles per layer in which is embedded the other nonlinear equation set for Snell's Law. Different scaling parameters are used on three panels to improve visibility.

Table 1. The anisotropic parameters (density normalized stiffnesses) in the three layers in Voigt notation. The A_{ij} have the dimensions of velocity squared (km^2/s^2).

A_{11}	A_{22}	A_{33}	A_{44}	A_{55}	A_{66}	A_{12}	A_{13}	A_{23}
7.00	7.00	7.00	2.50	2.50	2.50	2.00	2.00	2.00
9.900	6.023	7.093	1.964	2.448	2.438	1.926	2.074	2.225
10.00	9.84	5.94	2.00	1.60	2.18	3.60	2.25	2.40

Table 2. The anellipsoidal parameters in the three layers. The E_{ij} have the dimensions of velocity squared (km^2/s^2) and are defined in terms of the A_{ij} using eqs. (3)-(5) in the text.

E_{12}	E_{13}	E_{23}
0.0	0.0	0.0
-2.319	-3.053	-0.81
-3.92	-5.04	-2.98

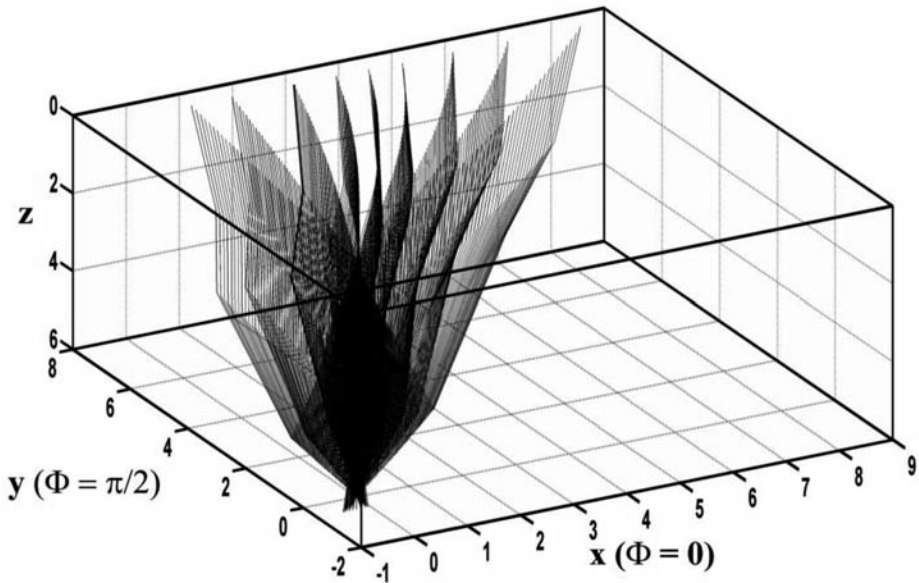


Fig. 2. Three layer model. The model used is the same as that in Fig. 1, except that a number of quasi-azimuthal lines are displayed. The modified ray tracing procedure using the polar angle Θ and a fixed distance between adjacent receivers are again used as the dependent variables.

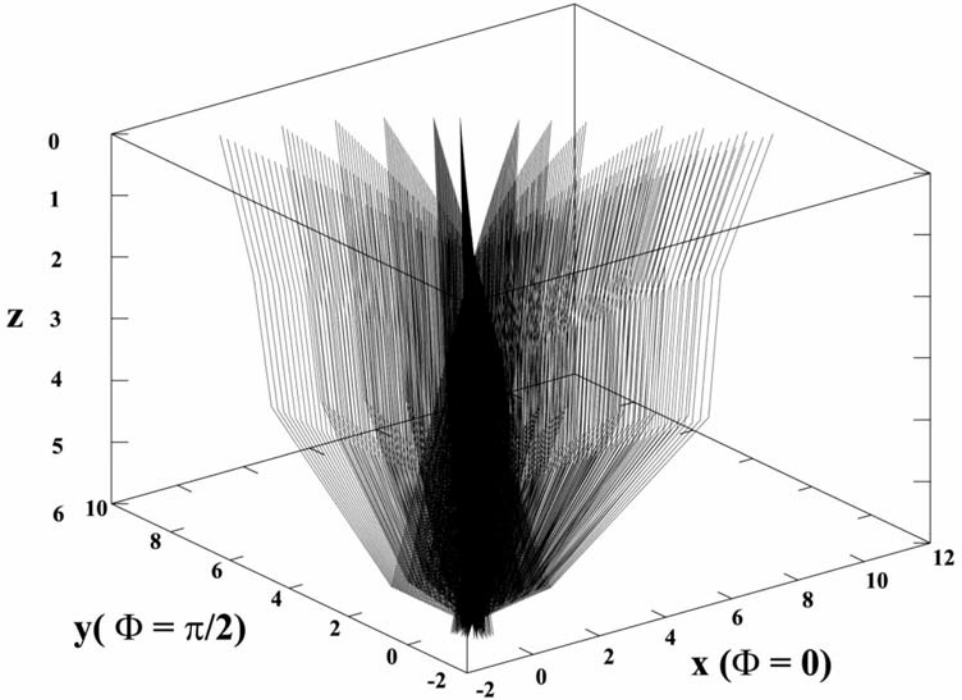


Fig. 3. Three layer model. The model used is the same as that in Fig. 2, except that the polar angle Θ and azimuthal angle Φ are used as the dependent variables. The surface receivers for some azimuthal line are now in a straight line at the surface.

CONCLUSIONS

The basic formulae and solution method for tracing quasi-compressional (qP) rays in a plane layered orthorhombic anisotropic medium where the axes of anisotropy (polar and azimuthal) in any of the layers may be rotated with respect to the general Cartesian coordinate system have been presented. A linearized form of the qP ray velocity is used for this. Proceeding in this manner removes one "level" of nonlinear equations. Included in this discussion is a presentation of an analogue of Snell's Law for reflected and transmitted rays at an interface separating two layers. In general, the "two-point" ray tracing problem consists of two nonlinear equations (Snell's Law) embedded in two nonlinear (qP) ray tracing equations. Writing the shear ray velocities as is done for the qP group velocity does not present itself in a manner as that for the qP case. Obtaining accurate group velocity expressions for the two possible shear wave modes is also a fairly complex undertaking.

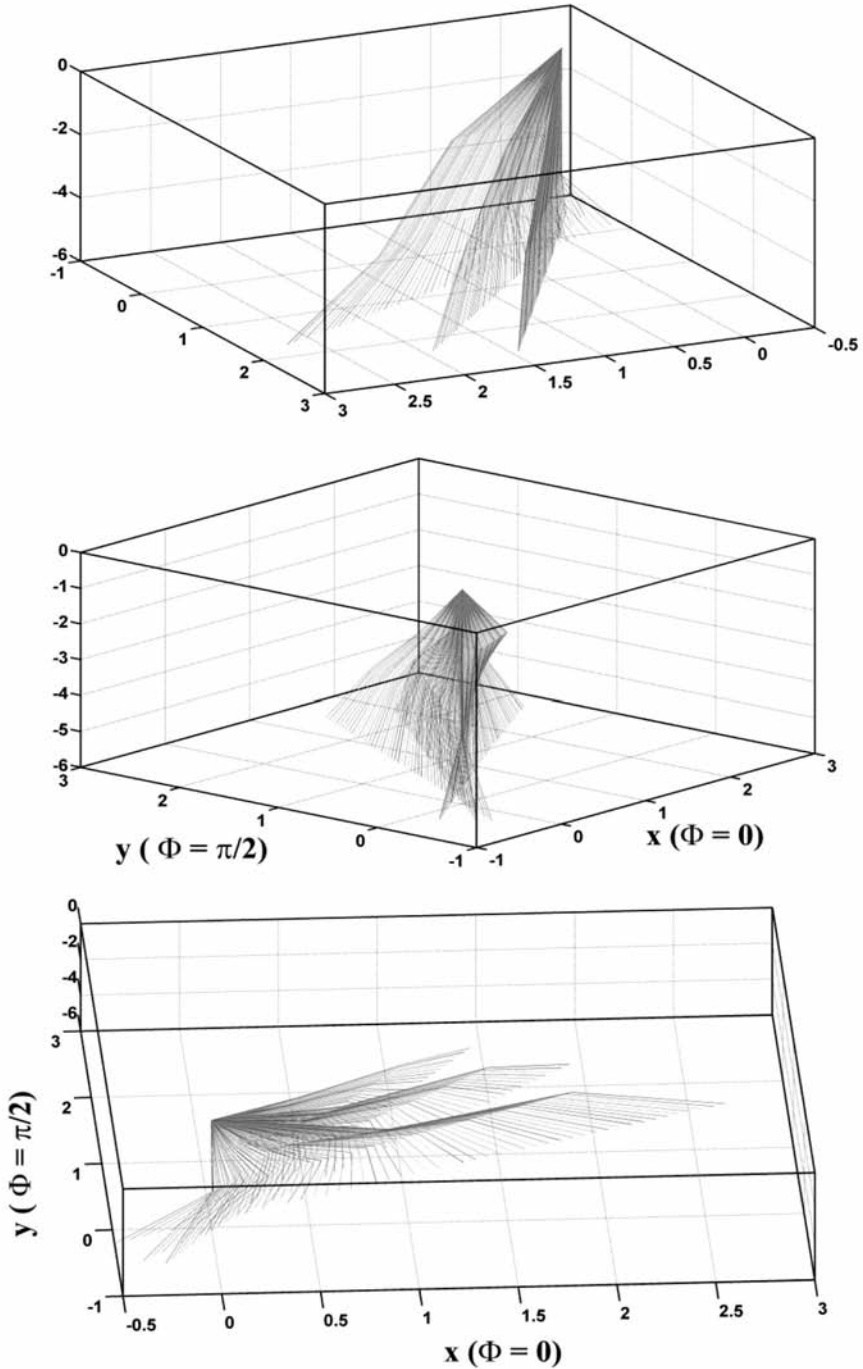


Fig. 4. Three layer model. In this case what is wanted is time to some reference depth. A sampling of three azimuths are shown. The anisotropic parameters describing the layers are given in Tables 1 and 2.

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APPENDIX

PARTIAL DERIVATIVES OF THE GROUP VELOCITY

The partial derivatives of the functions $F_1(\xi_1, \xi_2) =$ and $F_2(\xi_1, \xi_2)$ with respect to $\sin\Theta = \xi_1$ and $\sin\Phi = \xi_2$ are given by

$$\begin{aligned} [\partial F_1(\xi_1, \xi_2)/\partial \xi_1] &= -\{(1 - \xi_2^2)^{1/2} V_{qp}(\xi_1, \xi_2) \\ &+ \xi_1(1 - \xi_2^2)^{1/2} [\partial V_{qp}(\xi_1, \xi_2)/\partial \xi_1]\} , \end{aligned} \quad (A-1)$$

$$\begin{aligned} [\partial F_1(\xi_1, \xi_2)/\partial \xi_2] &= -\{[\xi_1 \xi_2 / (1 - \xi_2^2)^{1/2}] V_{qp}(\xi_1, \xi_2) \\ &\leq \xi_1(1 - \xi_2^2)^{1/2} [\partial V_{qp}(\xi_1, \xi_2)/\partial \xi_2]\} , \end{aligned} \quad (A-2)$$

and

$$[\partial F_2(\xi_1, \xi_2)/\partial \xi_1] = -\{\xi_2 V_{qp}(\xi_1, \xi_2) + \xi_1 \xi_2 [\partial V_{qp}(\xi_1, \xi_2)/\partial \xi_1]\} , \quad (A-3)$$

$$[\partial F_2(\xi_1, \xi_2)/\partial \xi_2] = -\{\xi_1 V_{qp}(\xi_1, \xi_2) + \xi_1 \xi_2 [\partial V_{qp}(\xi_1, \xi_2)/\partial \xi_2]\} . \quad (A-4)$$

The partial derivatives of the group velocity, $V_{qp}(\xi_1, \xi_2)$, with respect to ξ_1 and ξ_2 are obtained using the following sequence of steps:

$$\begin{aligned} (\partial/\partial \xi_1)[1/V_{qp}^2(\xi_1, \xi_2)] &= -[2V_{qp}(\xi_1, \xi_2) \partial V_{qp}(\xi_1, \xi_2)/\partial \xi_1] / V_{qp}^4(\xi_1, \xi_2) \\ &= -[2\partial V_{qp}(\xi_1, \xi_2)/\partial \xi_1] / V_{qp}^3(\xi_1, \xi_2) , \end{aligned} \quad (A-5)$$

$$[\partial V_{qp}(\xi_1, \xi_2)/\partial \xi_i] = -[V_{qp}^3(\xi_1, \xi_2)/2](\partial/\partial \xi_i)[1/V_{qp}^2(\xi_1, \xi_2)] , \quad (A-6)$$

$$\begin{aligned} (\partial/\partial \xi_i)[1/V_{qp}^2(\xi_1, \xi_2)] &= 2\xi_1 \{(1 - \xi_2^2)/A_{11} + \xi_2^2/A_{22} - 1/A_{33} \\ &- 2E_{12} \xi_1^2 \xi_2^2 (1 - \xi_2^2)/A_{11} A_{22} \\ &- E_{13} (1 - \xi_2^2) (1 - 2\xi_1^2)/A_{22} A_{33} \\ &- E_{23} \xi_2^2 (1 - 2\xi_1^2)/A_{22} A_{33}\} , \end{aligned} \quad (A-7)$$

$$\begin{aligned} (\partial/\partial \xi_2)[1/V_{qp}^2(\xi_1, \xi_2)] &= 2\xi_2 \{-\xi_1^2/A_{11} + \xi_1^2/A_{22} - E_{12} \xi_1^4 (1 - \xi_2^2)/A_{11} A_{22} \\ &+ E_{12} \xi_1^4 \xi_2^2 / A_{11} A_{22} + E_{13} \xi_1^2 (1 - \xi_1^2)/A_{11} A_{33} \\ &- E_{23} \xi_1^2 (1 - \xi_1^2)/A_{22} A_{33}\} . \end{aligned} \quad (A-8)$$