FULL WAVEFORM INVERSION WITHOUT LOW FREQUENCY USING WAVEFIELD PHASE CORRELATION SHIFTING METHOD

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ABSTRACT

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Full Waveform Inversion (FWI) of seismic data is a high resolution subsurface imaging tool and there is a lot of effort to fully industrialize it. The method, which uses a gradient based data fitting approach to minimize the misfit between observed and simulated waveforms, strongly requires either a good initial model or low frequency data for the convergence to the global solution. In this paper, we address the cycle skipping phenomena when none of the two above mentioned requirements are met. Then we propose the wavefield phase correlation shifting (WPCS) method to help reduce the influence of local minimum. Compared to the conventional FWI methods, a better inversion result could be achieved by applying the WPCS method without the low frequency components. We test the WPCS method on the Marmousi model and the result is improved compared with conventional method.

KEY WORDS: full waveform inversion, frequency domain, wavefield phase correlation shifting.

INTRODUCTION

FWI is an effective tool to obtain accurate velocity models with complex geological structures. Compared with conventional velocity model building methods (e.g., tomography, migration velocity analysis), the FWI could get a higher resolution result by taking full advantage of kinematics and kinetics information of the prestack seismic records.

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Successful application of FWI requires a good starting model to ensure that the modeled waveforms are kinematically less than half a cycle away from the recorded data. If we could record low frequencies in the field data, initializing FWI iterations from sufficiently low frequencies through a multiscale scheme would provide low wavenumber component of the model in the early stages of FWI and we could obtain the solution without trapping in a local minimum of the misfit function. However, because of the band-limited nature of the acquired seismic data and the limitations in the recording technologies, the low frequency components are practically dropped out and it becomes unrealistic to start FWI from frequencies lower than recording limits ($<$ 5 Hz).

The cycle skipping is a common problem for the FWI method. If one fails to provide the low wavenumber waveforms in the either ways mentioned above, the gradient updates may fall down to the wrong direction and the cycle skipping could happen (Bunks et al., 1995). Figs. 1 and 2 are the schematic diagrams of the cycle skipping problem. Figs. 1a and 1c are the calculated wavefields, and Fig. 1b is the observed wavefield. Comparing Figs. 1b and 1c we could find that the time difference between the corresponding k-th wave crest is less than half a cycle. In this case the cycle skipping will not happen. The time difference between corresponding k-th wave crest in figure 1a and 1b is more than half a cycle. This means cycle skipping will happen and inversion will be trapped in local minimum. Fig. 2 compares the situation of the cycle skipping problem with different frequencies. We can see from Fig. 2 that for the same time difference, low frequencies are less sensitive to the cycle skipping problem.

Many efforts have already been made to minimize the effect of the cycle skipping problem (Son, 2010; Biondi and Almomin, 2012; Warner et al., 2013; Hu, 2014). The most straightforward strategy is to use the travel time inversion result model as the starting model for the FWI (Operto et al., 2004; Brenders and Pratt, 2007). Another strategy is the combination of wave equation tomography and full waveform inversion (Symes, 2008; Biondi and Almomin, 2012). The huge computational cost is the drawback of this method. Shin and Cha (2008) proposed the Laplace domain FWI algorithms, which can recover the large scale structures without low frequency data, but this method is sensitive to signal-to-noise ratio and the accuracy of the source estimation (Ha and Shin, 2012). Choi and Alkhalifah (2011) proposed a frequency domain waveform inversion method with the phase unwrapping procedure to eliminate the phase ambiguity within the seismic data. Warner et al. (2013) presented a scheme that used a non-linear extrapolation to add missing low frequencies into the field data to recover the global minimum model even when the original unextrapolated field dataset is significantly cycle skipped. Bi and Lin (2014) developed an effective cycle skipping reduction strategy through adaptive data selection for full waveform inversion, which assures all input data for FWI is within a half-cycle difference compared with the predicted data by discarding bad traces and muting bad data. Hu (2014) proposed a "beat tone" FWI method, which can extract very low wavenumber components from high frequency seismic data by utilizing two recorded seismic waves with slightly different frequencies.

In this paper, we propose a new WPCS method to overcome the cycle skipping problem in the full waveform inversion. According to the Digital Image Correlation (DIC) method (Hild and Roux, 2012), the seismic wavefield perturbation caused by inaccurate velocity model can be regarded as a "deformation" deviating from the observed wavefield. The aim of FWI is to eliminate this deformation by updating the velocity model to make the simulated wavefield match the observed wavefield gradually. However, the inversion will fall into the local minimum when the deformation is too large or low-frequency components are absent. To avoid this problem, we can estimate the position of this deformation by the correlation algorithm and shift the simulated data to reduce the deformation before inversion. To demonstrate the effectiveness of this algorithm, we perform the conventional FWI method and WPCS FWI method on the Marmousi model, respectively. Numerical tests show that the WPCS FWI can obtain a more accurate result without low-frequencies in seismic data, while conventional FWI is trapped in local minimum due to cycle skipping problem.

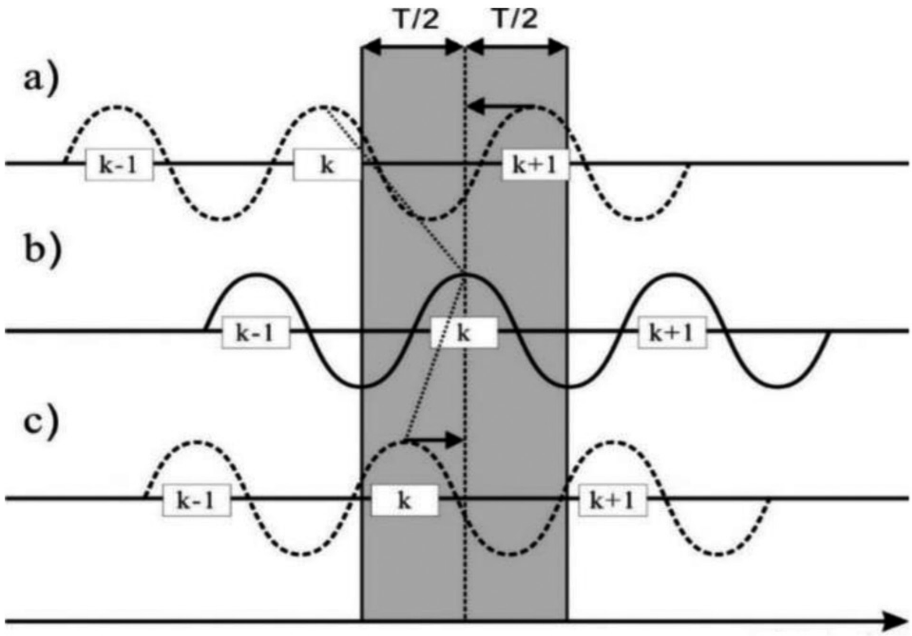


Fig. 1. Schematic diagram of cycle skipping issue in FWI.

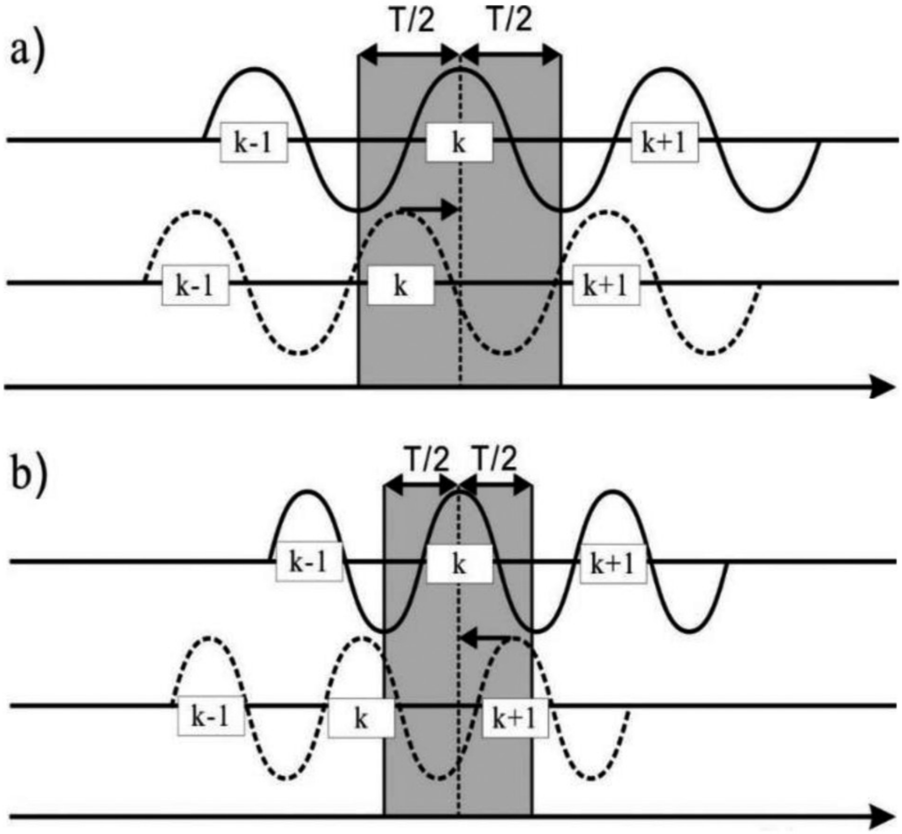


Fig. 2. Comparison of cycle skipping issue with a) low frequency; b) high frequency.

FULL WAVEFORM INVERSION THEORY

FWI algorithm is an iterative process, and every iteration should carry out at least two times forward modeling (one for the incident wavefield and another for the back-propagated residual wavefield). And the cost of the inversion is highly depended on the forward modeling. In this paper, we choose the finite difference method for solving two dimensional frequency-domain acoustic wave equation (Operto and Virieux, 2007):

$$[\omega^2/\kappa(x,z)]p(x,z,\omega) + (\partial/\partial x)\{[1/\rho(x,z)][\partial p(x,z,\omega)/\partial x]\}$$

$$+ (\partial/\partial z)\{[1/\rho(x,z)][\partial p(x,z,\omega)/\partial z]\} = -s(x,z,\omega) \quad , \tag{1}$$

where $p(x,z,\omega)$, $\kappa(x,z)$, and $s(x,z,\omega)$ denote the wavefield, bulk modulus, and source term, respectively, ω is the angle frequency and $\rho(x,z)$ denotes the density.

In the frequency domain, the wave equation reduces to a system of linear equations. Therefore, eq. (1) can be expressed as a linear equation in the following matrix form (Pratt, 1999):

$$AP = S \ , \tag{2}$$

where A indicates the complex impedance matrix, P and S denote the modeled wavefield and source term, respectively.

In the inversion problem, the objective function is expressed as the difference between the real seismic data d^{obs} and the simulated seismic data d^{cal} (Brossier et al., 2010):

$$C(m) = ½[d^{obs} - d^{cal}]^T [d^{obs} - d^{cal}] = ½ \| d^{obs} - d^{cal} \|_2^2 \ , \tag{3}$$

where superscript T denotes conjugate transpose; $\| \cdot \|_2^2$ denotes l_2 norm, and m is the geological model, such as subsurface velocity model.

The process of local optimization is to find the minimum value of the objective function around the starting model m_0. In the framework of the Born approximation, we assume that the updated model m can be expressed as the sum of starting model m_0 and the model update Δm:

$$m = m_0 + \Delta m \ . \tag{4}$$

Conducting Taylor-Lagrange expansion for eq. (3) at m_0, and calculating derivative with respect to m at both ends, we obtain a new equation expressed in matrix form:

$$\partial C(m)/\partial m = [\partial C(m_0)/\partial m] + [\partial^2 C(m_0)/\partial m^2]\Delta m \ . \tag{5}$$

Eq. (3) has the minimum value when $\partial C(m)/\partial m = 0$. Then the model update equals:

$$\Delta m = -[\partial^2 C(m_0)/\partial m^2]^{-1} [\partial C(m_0)/\partial m] \ , \tag{6}$$

where $\partial^2 C(m_0)/\partial m^2$ is the Hessian matrix, and $\partial C(m_0)/\partial m$ is the derivative of the misfit function.

In the optimization process, the most difficult part is to calculate the second order derivative of the objective function with respect to model parameters, i.e., the Hessian matrix. Here we use the L-BFGS algorithm to calculate the Hessian matrix which does not need to calculate it directly. It uses some couples of gradient vector to update the Hessian matrix, thus reduces the storage capacity and improves the computational efficiency.

WPCS FULL WAVEFORM INVERSION METHOD

The process of local optimization requires that the starting model is close to the true model and the seismic data has efficient low-frequency components. Otherwise the FWI algorithms will be easily trapped in the local minimum and yield a bad result. However, the low-frequency components and good starting model are not always available in the field data application and this is one of the main drawbacks of the FWI method.

Digital image correlation and tracking is an optical method that employs tracking and image registration techniques for accurate 2D and 3D measurements of changes in images. It was proposed at the beginning of the 1980's (Sutton, et al., 1983) when applied in solid mechanics. DIC works by comparing digital photographs of a component or test piece at different stages of deformation. By tracking blocks of pixels, the system can measure surface displacement and build up the full field 2D and 3D deformation vector fields and strain maps.

In this paper we introduce the WPCS method by combining DIC method and FWI method to mitigate the effect of cycle skipping problem. Here we assume that the model is elastic, no attenuation and the phase do not change along wave propagation. The whole procedure is as follows:

1. Compute the cross-correlation coefficients of the phase between calculated data and observed data trace by trace before inversion:

$$Cr_i(\tau) = (1/N) \sum_{t=0}^{N-1} \varphi_i^{obs}(x,t)\varphi_i^{cal}(x,t+\tau), i = 1,2,\ldots,N \quad , \tag{7}$$

where Cr is the cross-correlation coefficients, φ denotes phase, i denotes trace number, and N denotes the total trace number.

2. Select the value of τ when $Cr_i(\tau)$ is maximal. And shift the modeled data as follow:

$$d_i^{\prime cal}(x,t) = d_i^{cal}(x,t+\tau) \quad , \tag{8}$$

where $d_i^{\prime cal}$ is the shifted calculated data. In the frequency domain, eq. (8) can be written as:

$$D_i^{\prime cal} = D_i^{cal} \cdot e^{i\omega\tau} \quad , \tag{9}$$

where D_i^{cal} is the frequency domain calculated data, and $D_i^{\prime cal}$ is the shifted frequency domain calculated data.

Thus, the deformation between calculated data and observed data caused by model perturbation is reduced after shifting, which mitigate the probability of cycle skipping.

NUMERICAL TEST

We test the WPCS method on a modified 2D Marmousi P-wave velocity model. We add a water layer above and do not update the water layer during the nonlinear procedure. Here we assume the model is elastic and with no attenuation. Fig. 3(a) and 3(b) are the true model and the starting model, respectively. The starting velocity model is a smoothed version of the true model. We use a Ricker wavelet with a peak frequency of 12 Hz. The source is located on the surface with an interval of 24 m. The grid size is 24 m and the

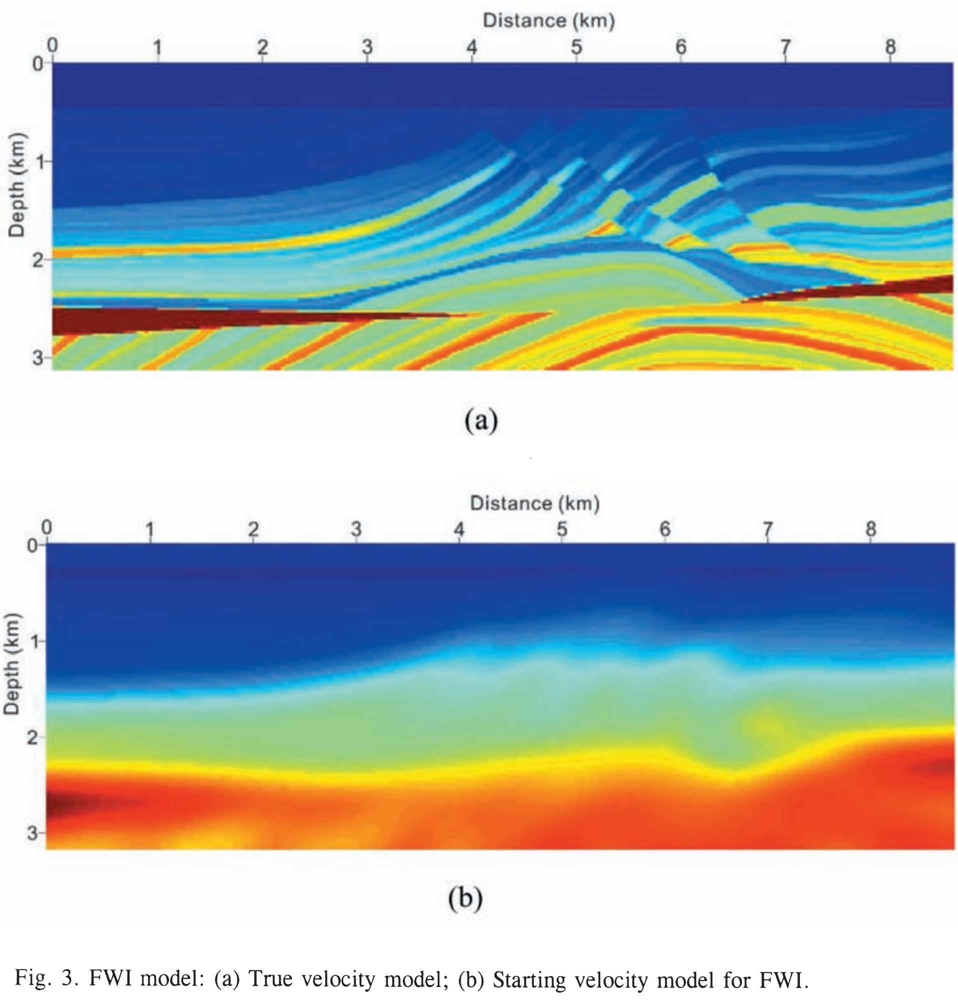
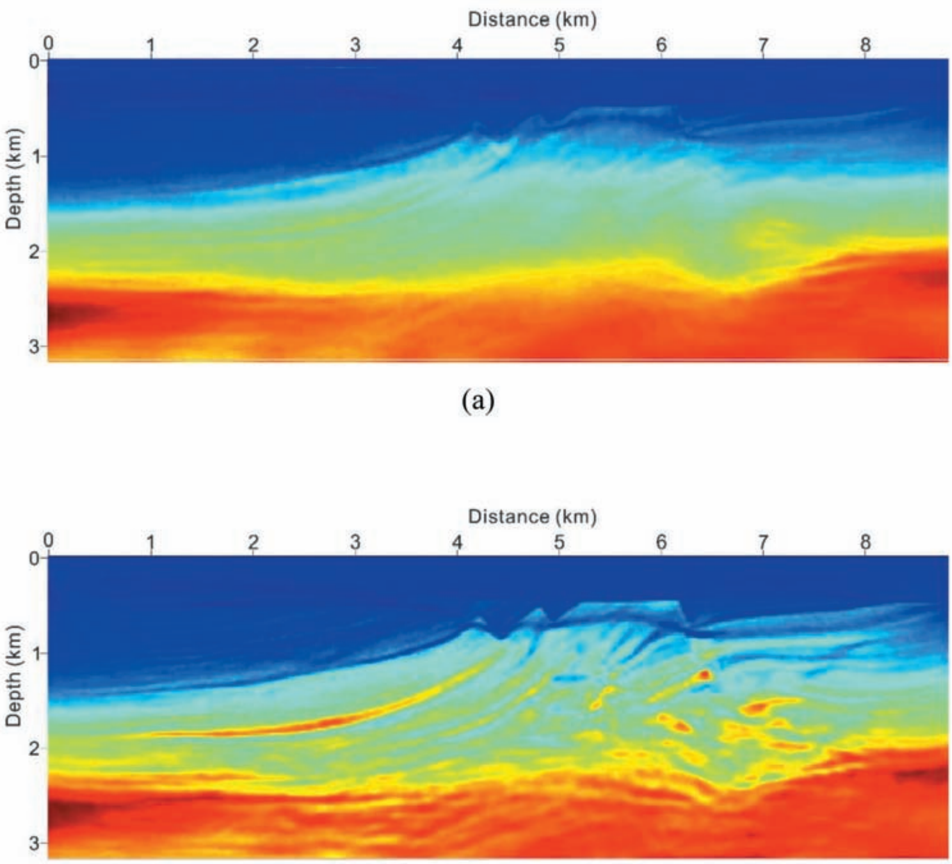


Fig. 3. FWI model: (a) True velocity model; (b) Starting velocity model for FWI.

sampling rate is 4 ms. All the grid points on the surface acts as receivers. The inversion frequency band is 7-30 Hz. In addition, to reduce the computational cost, we also apply the blended seismic source technology (Berkhout et al., 2009; Ben-Hadj-Ali and Operto, 2011; Han et al., 2013).

The inversion results are shown in Fig. 4. Fig. 4(a) is the result of conventional FWI method, and Fig. 4(b) is the result of WPCS FWI method. As we start the inversion at a relatively high frequency, conventional FWI method fails to obtain an accurate result due to cycle skipping, while our new



(a)

(b)

migration results using the velocity results obtained by two FWI methods respectively. From the two migrated figures, we can see that more detail information of subsurface structure is recovered in the WPCS FWI result than that in the conventional FWI result, as marked in the square.

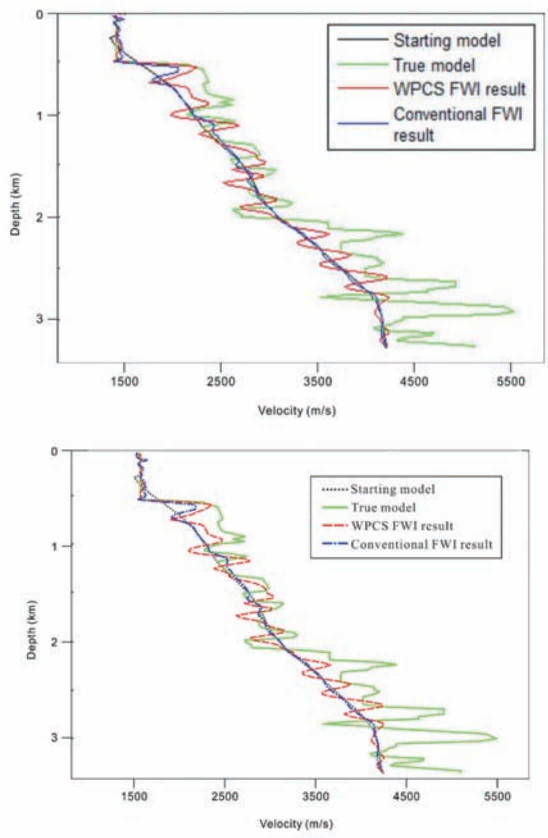
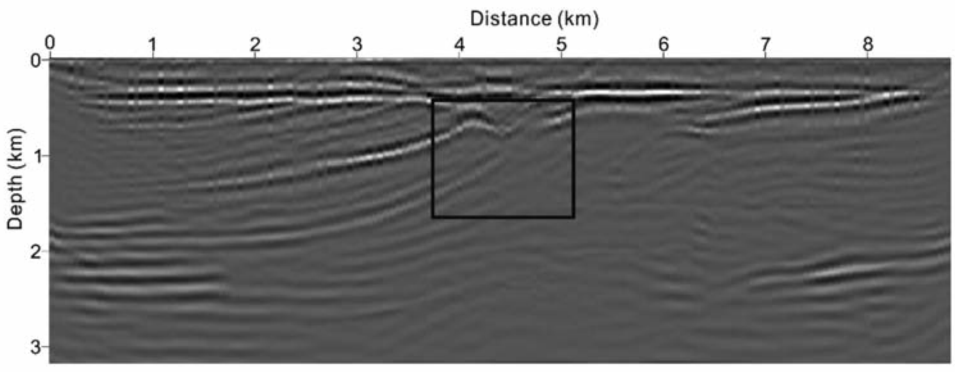


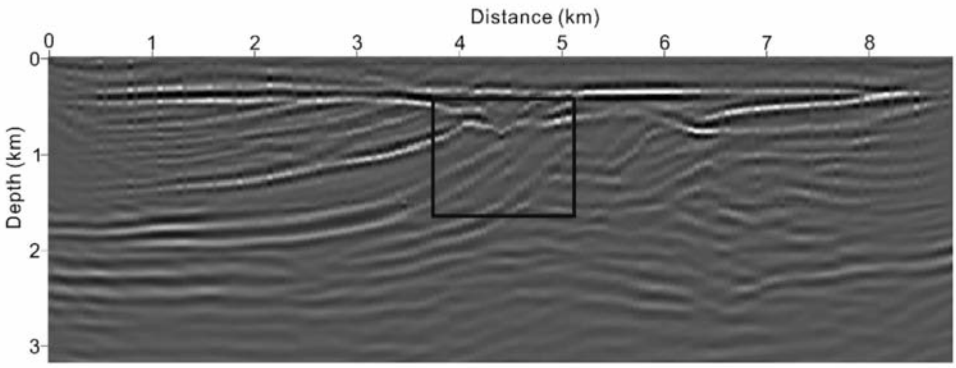
Fig. 5. 1D vertical velocity curves extracted from inversion models (location at receiver No: 241) by different methods (Black line represents the starting model, green line represents the true model, and red line and blue line represent the WPCS FWI result and the conventional FWI result, respectively).

CONCLUSION

FWI is a promising tool for deriving accurate velocity models in areas with complex geological conditions. The advantage of FWI is the high accuracy of the inversion result by using a large quantity of waveform information. But many weaknesses are still existing in FWI, such as easy to get stuck in local minima and cycle-skipping if the starting model is too far from the actual model or seismic data lacks of low frequencies.



(a)



(b)

Fig. 6. Migration results using the velocity models of (a) Conventional FWI; (b) WPCS FWI method.

In this paper we propose the wavefield phase correlation shifting FWI method to mitigate the effect of the cycle skipping problem. Seismic wavefield perturbation caused by the inaccurate velocity model can be regarded as a "deformation" deviating from the observed wavefield. We could estimate the position of this deformation by correlation algorithm and shift the simulated data to reduce the deformation before inversion. Numerical examples on the Marmousi model show that the new method can mitigate the effect of cycle skipping problem and obtain a better result with the absence of low frequencies than the conventional FWI method.

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