A NOVEL METHOD FOR SIMULTANEOUS SEISMIC DATA INTERPOLATION AND NOISE REMOVAL BASED ON THE L_0 CONSTRAINT

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ABSTRACT

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The Projection Onto Convex Sets (POCS) method is an efficient iterative method for seismic data interpolation. In each iteration, observed seismic data is inserted into the updated solution. If observed seismic data contains some random noise, the noisy data would be inserted into the final solution and it reduces the Signal to Noise Ratio (SNR) of the interpolated seismic data. Weighted POCS method can weaken the noise effects because it uses a weight factor to scale the observed seismic data, then fewer noisy data is inserted into the updated solution, but it still inserts some random noise and the final performance is unsatisfactory. In this paper, a novel method is proposed by combining the advantages of the weighted POCS method and the Iterative Hard Threshold (IHT) method: the weighted POCS method used for interpolation and the IHT method used for random noise elimination. The novel method can be used for simultaneous seismic data interpolation and random noise removal, and its superior performances are demonstrated on synthetic and real datasets.

KEY WORDS: Projection Onto Convex Sets (POCS), Iterative Hard Threshold (IHT), interpolation, noise removal.

INTRODUCTION

Spatial irregularity and random noise in the observed seismic data can affect the performances of Surface-Related Multiple Elimination (SRME), wave-equation based migration and inversion. Therefore, interpolation and random noise elimination is a pre-requisite for multi-channel processing techniques.

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Interpolation methods can be divided into four categories: signal analysis and mathematical transform based methods, prediction filter based methods, wave-equation based methods and rank-reduction based methods (Gao et al., 2013; Wang et al., 2014). Among these interpolation methods, mathematical transform based ones are easy to implement and have drawn much attention. In this paper, the POCS method is employed and extended to achieve seismic data interpolation and noise removal, simultaneously.

The POCS method is an efficient iterative algorithm for seismic data interpolation, belonging to transform based methods, but it cannot handle noisy data interpolation properly, thus a novel method is proposed based on the IHT method to overcome that defect. The POCS method (Bregman, 1965) was first used in image reconstruction (Stark and Oskoui, 1989; Wang et al., 2015) and the applications in irregular seismic data interpolation was started by Abma et.al. (Abma and Kabir, 2006). Many effective strategies were developed based on its original idea. Gao et al. (Gao et al., 2010) achieved irregular seismic data interpolation using Fourier transform based POCS method with the exponential threshold model and the performances of different threshold models were compared to further improve the convergence rate (Gao et al., 2012). Curvelet transform (Candès et al., 2006), a sparse transform, better characterization for curved seismic events compared with Fourier-based methods, was also used for seismic data interpolation with the POCS method (Yang et al., 2012; Zhang and Chen, 2013). Dreamlet transform in which the basic atom is a physical wavelet, is used for seismic data interpolation with the POCS method (Wang et al., 2014). While the POCS method cannot eliminate the random noise properly because it inserts the noisy observed data in each iteration, then a weighted strategy is adopted to weaken random noise effects (Gao et al., 2012), but it still inserts some random noise into the reconstructed data, affecting its final performance. In order to overcome this defect, a novel method is proposed based on the IHT method, taking advantage of the threshold method to eliminate random noise (Daubechies et al., 2004; Herrmann et al., 2007).

In this paper, defects of the POCS method and the weighted POCS method are analyzed: the POCS method cannot handle noisy seismic data interpolation properly; the weighted POCS method can weaken the random noise effects but it still inserts some random noise which affects the final performance. Thus a novel method is proposed by combining the advantages of the IHT method and the weighted POCS method, then simultaneous seismic data interpolation and noise removal can be achieved. During which, improved jittered under-sampling strategy (Wang et al., 2014) is adopted to obtain the irregular seismic data and the curvelet transform is used to decompose seismic data. Numerical examples on synthetic and field data verified the validity of the proposed method.

THEORY

The POCS and weighted POCS theory and their limitations

Irregular sampled noisy seismic data d_{obs} can be obtained through eq. (1),

$$d_{obs} = Rd_0 + n \ , \tag{1}$$

where d_0 represents the noise-free seismic data, R denotes the sampling matrix and n indicates the random noise. Because of random noise and limited bandwidth of observed seismic data, it is always ill-posed to solve eq. (1). Due to the fact that seismic data can be sparsely represented in the curvelet domain, the non-constraint objective functional can be constructed based on sparse constraint,

$$\Phi(x) = \| d_{obs} - RC^T x \|_2^2 + \lambda P(x) \ , \tag{2}$$

where x represents the curvelet coefficient vector, C^T denotes the inverse curvelet transform (C denotes the curvelet transform) and P(x) indicates a sparse constraint. Eq. (2) with $P(x) = \| x \|_0$ can be solved by the POCS method with a hard threshold function (Gao et al., 2010; Gao et al., 2012; Wang et al., 2014; Yang et al., 2012; 2013) and the exact formula is shown below,

$$\hat{d}_{k+1} = d_{obs} + (I-R)C^T T_{\lambda_k}(C\hat{d}_k) \ , \tag{3}$$

where $\hat{d}_k$ is the k-th iterative solution and T_{λ_k}, subject to eq. (4), denotes the hard threshold operator performed element-wise.

$$T_\lambda(x^i) = \begin{cases} x^i \ , & |x^i| \geq \tau \\ 0 \ , & |x^i| < \tau \end{cases} \ , \tag{4}$$

where x^i is the i-th element of vector $\mathbf{x}$, $\tau = \sqrt{\lambda}$ is the threshold determined by a threshold model, like the exponential threshold model shown in eq. (5),

$$\tau^k = \tau_{max} e^{c(k-1)/(N-1)} \ , \quad c = \ln(\tau_{min}/\tau_{max}) \ , \quad k = 1,2,\ldots,N \ , \tag{5}$$

where τ_{min}, τ_{max} are the minimum and maximum thresholds, determined by the observed seismic data. The POCS procedures can be summarized as follows (Gao et al., 2010; Wang et al., 2014):

Step 1: Set the maximum iteration number N and choose the thresholds τ according to eq. (5). Input the sampling matrix $\mathbf{R}$, the observed data d_{obs} and set the initial solution to $k = 1$, $\hat{d}_k = d_{obs}$.

Step 2: Take the curvelet transform towards the k-th solution and update the curvelet coefficients by applying threshold operator $\alpha_k = T_{\lambda_k}(C\hat{d}_k)$.

Step 3: Perform the inverse curvelet transform towards the updated curvelet coefficients and get the solution in the data domain $d_k = C^T\alpha_k$.

Step 4: Project the solution d_k onto the observation plane $\{\hat{d}_k \mid R\hat{d}_k = d_{obs}\}$, which means inserting the observed seismic data, and the updated solution can be obtained by $\hat{d}_{k+1} = d_{obs} + (I-R)d_k$.

Step 5: $k = k + 1$. If $k < N$ return to step 2; if $k = N$, output the $\hat{d}_k$ as the final solution.

From the procedures, it can be noted that the POCS method assumes that the observed seismic data should have a high SNR because it inserts the observed data in each iteration. But the observed seismic data always contain random noise, hence some authors take advantage of the weighted strategy in noisy situations to weaken the random noise effects (Gao et al., 2012; Oropeza and Sacchi, 2011; Stanton and Sacchi, 2013; Yang et al., 2013). Thus eq. (3) can be modified into eq. (6) with the weighted strategy,

$$\hat{d}_{k+1} = \alpha d_{obs} + (I-\alpha R)C^T T_{\lambda_k}(C\hat{d}_k) \quad , \tag{6}$$

where $\alpha \in (0,1]$ indicates a weight factor determined by random noise levels. Though the weighted POCS method can weaken random noise effects, it still inserts some random noise into the updated solution, affecting the final performance. When the first few solutions are far from the true solution and the value of α is small, the interpolation performance is unsatisfactory. For example, when α tends to 0 ($\alpha \to 0$), eq. (6) degenerates into $\hat{d}_{k+1} = C^T T_{\lambda_k}(C\hat{d}_k)$, then the final performance is unsatisfactory if the first few solutions are far from the real solution because there is no data residual constraints in the updating procedures. Thus, a novel method is proposed to eliminate the effects of random noise as well as guarantee the interpolation performance.

A novel method

From the analysis in the above section, performances of the POCS and weighted POCS methods may be unsatisfactory when random noise exists. In order to overcome the defects, a novel method is proposed, based on the IHT method and the weighted POCS method. Next, more detailed explanations are given.

Eq. (2) with the L_0 norm constraint can be solved by the IHT method (Blumensath and Davies, 2008; 2009; Loris et al., 2010), and the iterative solution is obtained as follows:

$$x_{k+1} = T_{\lambda_k}[x_k + (RC^T)^T(d_{obs} - RC^Tx_k)] \quad , \tag{7}$$

where x_k is the k-th iterative solution in the curvelet domain, T_{λ_k}, subject to eq. (4), is the hard threshold operator performed element-wise. The procedures can be summarized as follows:

Step 1: Set the maximum iteration number N and choose the thresholds τ according to eq. (5). Input the sampling matrix R, the observed data d_{obs} and set the initial solution to $k = 1$, $x_k = Cd_{obs}$.

Step 2: Apply the inverse curvelet transform to the k-th solution in the curvelet domain and obtain the data residual $\delta d = d_{obs} - RC^Tx_k$.

Step 3: With the sampling matrix R and the curvelet transform, we can get the curvelet coefficient increment $\delta x = (RC^T)^T\delta d = CR^T\delta d = CR\delta d$.

Step 4: With the threshold operator, the updated coefficient in the curvelet domain can be obtained $x_{k+1} = T_{\lambda_k}(x_k + \delta x)$.

Step 5: $k = k + 1$. If $k < N$ return to step 2; if $k = N$, output the $d_k = C^Tx_k$ as the as the final solution.

The aim of seismic data interpolation is to obtain the complete seismic data in the data domain, therefore project the updated solution x_{k+1} onto the observation plane $\{\hat{d}_k | R\hat{d}_k = d_{obs}\}$, which can improve convergence rate. Because observed data always contains random noise, only parts of observed seismic data αd_{obs} are inserted into the updated solution (7) with the weighted strategy. Then the new weighted POCS formula is derived as follows:

$$\begin{aligned}
\hat{d}_{k+1} &= \alpha d_{obs} + (I-\alpha R)C^Tx_{k+1} \\
&= \alpha d_{obs} + (I-\alpha R)C^TT_{\lambda_k}[x_k + (RC^T)^T(d_{obs} - RC^Tx_k)] \\
&= \alpha d_{obs} + (I-\alpha R)C^TT_{\lambda_k}[Cd_k + C(R^Td_{obs} - R^TRC^Tx_k)] \quad , \\
&= \alpha d_{obs} + (I-\alpha R)C^TT_{\lambda_k}[Cd_k + C(d_{obs} - Rd_k)] \\
&= \alpha d_{obs} + (I-\alpha R)C^TT_{\lambda_k}\{C[\alpha d_{obs} + (I-\alpha R)d_k + (1-\alpha)(d_{obs} - Rd_k)]\}
\end{aligned} \tag{8}$$

where $d_k = C^Tx_k$ is the solution in data space, and $\hat{d}_{k+1}$ is the solution after inserting αd_{obs}. Denoting $\hat{d}_k = \alpha d_{obs} + (I-\alpha R)d_k$, then a new weighted POCS formula can be achieved,

$$\hat{d}_{k+1} = \alpha d_{obs} + (I-\alpha R)C^TT_{\lambda_k}\{C[\hat{d}_k + (1-\alpha)(d_{obs} - Rd_k)]\} \quad . \tag{9}$$

The new weighted POCS method, different from the original one [eq. (6)], is controlled by the weight factor α as well as the data residual term $d_{obs} - Rd_k$. Even if the α value is small, it can still speed up convergence rate efficiently because of the constraints of data residual. From the IHT procedures, it notes that the IHT method can eliminate random noise because it has a threshold operator to filter out random noise before the updating procedure. Therefore, in order to attenuate the random noise effects, the order of the IHT operator and the projection operator is exchanged and the novel method is obtained as follows:

$$\begin{aligned} \hat{d}_k &= \alpha d_{obs} + (I-\alpha R)d_{k-1} \\ x_k &= T_{\lambda_k}\{C[\hat{d}_k + (1-\alpha)(d_{obs} - Rd_{k-1})]\} \quad . \\ d_k &= C^T x_k \end{aligned} \tag{10}$$

This novel method can handle the spatial irregularity and random noise effects properly, thus it can achieve seismic data interpolation and noise removal, simultaneously. When $\alpha = 0$, the novel method degenerates to the IHT method in data space domain; when $\alpha = 1$, it degenerates to the POCS method with random noise elimination by the threshold strategy. Therefore, the novel method has more flexibility and wider applications. Equation (10) can be integrated into one tight equation shown below,

$$d_k = C^T T_{\lambda_k}\{C[\alpha d_{obs} + (I-\alpha R)d_{k-1} + (1-\alpha)(d_{obs} - Rd_{k-1})]\} \quad . \tag{11}$$

The interpolated data is controlled by α as well as the data residual term $d_{obs} - Rd_{k-1}$, guaranteeing the solution's convergence. Compared with the weighted POCS method, the novel method implements random noise elimination procedure by the threshold operator after each interpolation procedure, thus the interpolated seismic data with higher SNR can be obtained. Simultaneous seismic data interpolation and noise removal can be achieved via the proposed method.

NUMERICAL EXAMPLES

Firstly, synthetic seismic data is used to prove the validity of the proposed method. The results show that the proposed method is superior to the POCS method, the weighted POCS method and the popular spgl1 method when random noise exists. Secondly, numerical examples of field data further demonstrate the performance of the proposed method. In the synthetic and real data examples, 50% traces are missing based on improved jittered under-sampling strategy (Wang et al., 2014).

Synthetic example

The synthetic data is shown in Fig. 1(a) including 201 traces with 1001 samples per trace. The trace interval and time sampling interval are 12.5 m and 2 ms respectively. The incomplete noisy seismic data with 50% traces missing is shown in Fig. 1(b) and the colorbar for all the synthetic tests is shown in Fig. 2(a). The maximum iteration number is set to 50, and the interpolated results and the corresponding residuals are shown in Fig. 3. Here the residual is defined as the difference between the interpolated seismic data and the noise free original data, that is the difference between left column of Fig. 3 and Fig. 1(a). The smaller residual means the better method. The recovered SNRs for the POCS method, the weighted POCS method ($\alpha = 0.6$) and the proposed novel method ($\alpha = 0.6$) are 8.1, 11.6, 16.6 dB, respectively. The definition of SNR is shown below,

$$\mathrm{SNR} = 20 \log_{10} \| d_0 \|_2 / \| d_{rec} - d_0 \|_2 \ , \tag{12}$$

where d_0 is the noise free original data and d_{rec} is the reconstructed seismic data.

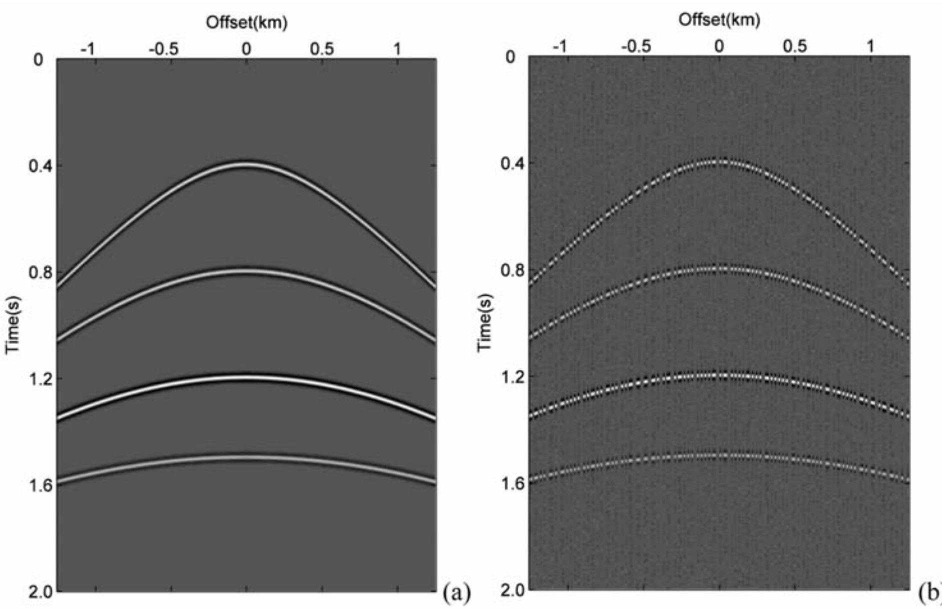

Fig. 1. (a) Complete synthetic data; (b) Noisy data with 50% traces missing.

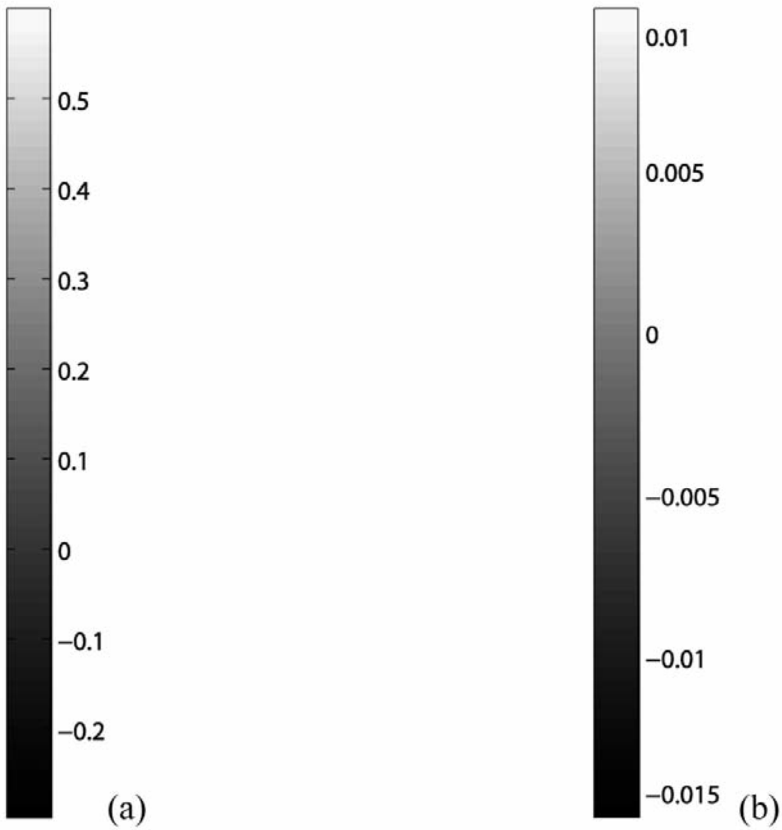

Fig. 2. Colorbar of synthetic data tests (a); field data tests (b).

In order to demonstrate the superiority of the proposed method, a comparison has been made between the popular spgl1 method (Van Den Berg and Friedlander, 2008) and the proposed method. The reconstructed result of the spgl1 method is shown in Fig. 3(g) and Fig. 3(h) denotes the reconstruction error. The recovered SNR is 13.48 dB lower than that of the proposed method. We amplified the magnitude of each error section three times to observe the details of reconstruction errors, which are shown in Fig. 4(a)-(d), indicating that the proposed method is the best method among the four methods.

Figs. 3-4 indicate that the POCS method and the weighted POCS method insert noisy observed data into reconstructed result which reduces the recovered SNRs; the proposed method is superior to the spgl1 method for signal preservation. Tests in this simple synthetic data demonstrate the validity of the proposed method, then the application of real marine data is given to further prove its validity.

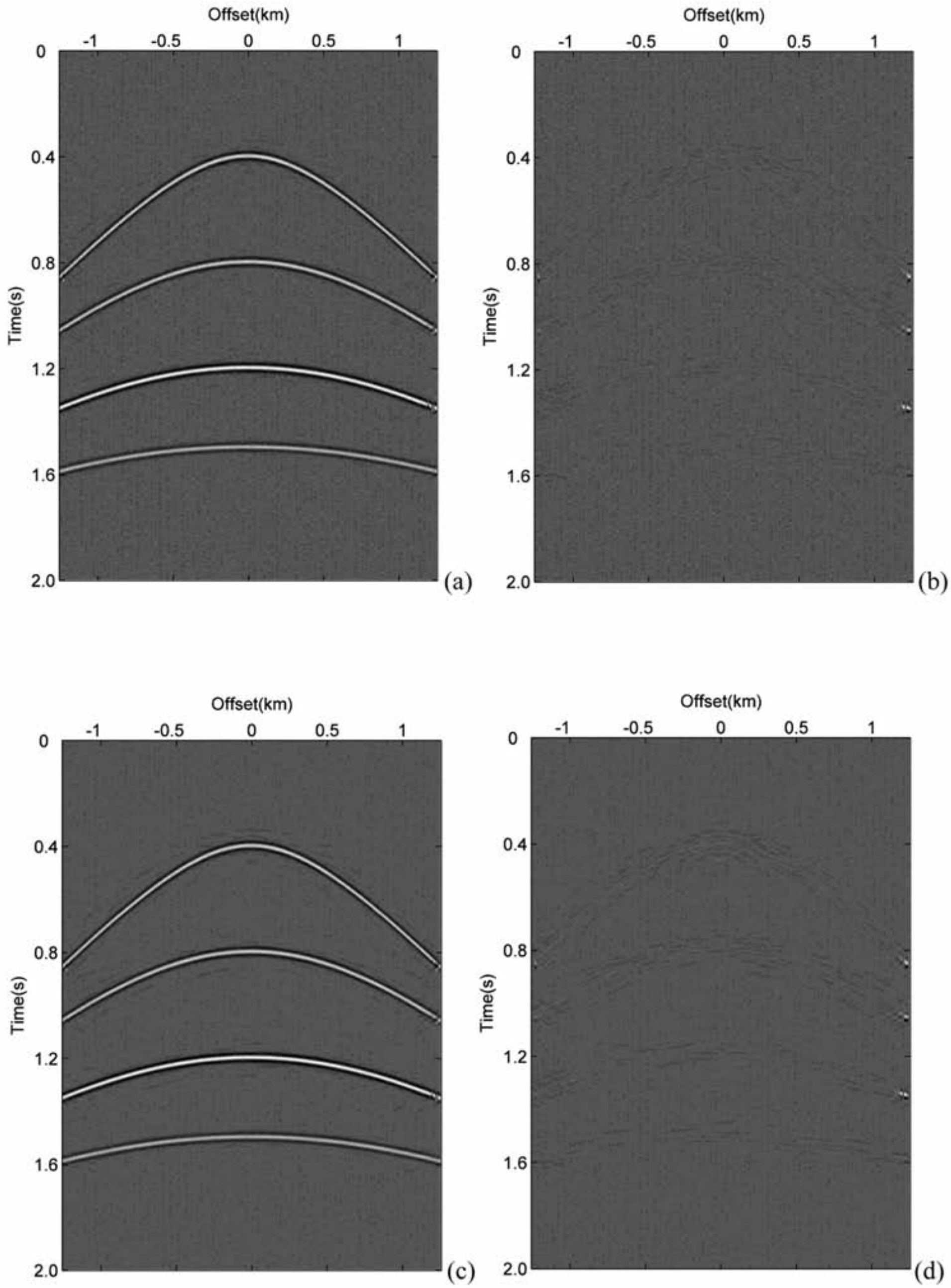

Fig. 3. Interpolated results (left column) and corresponding residuals (right column). (a)-(b) obtained by the POCS method ($\alpha = 0.6$); (c)-(d) obtained by the weighted POCS method ($\alpha = 0.6$).

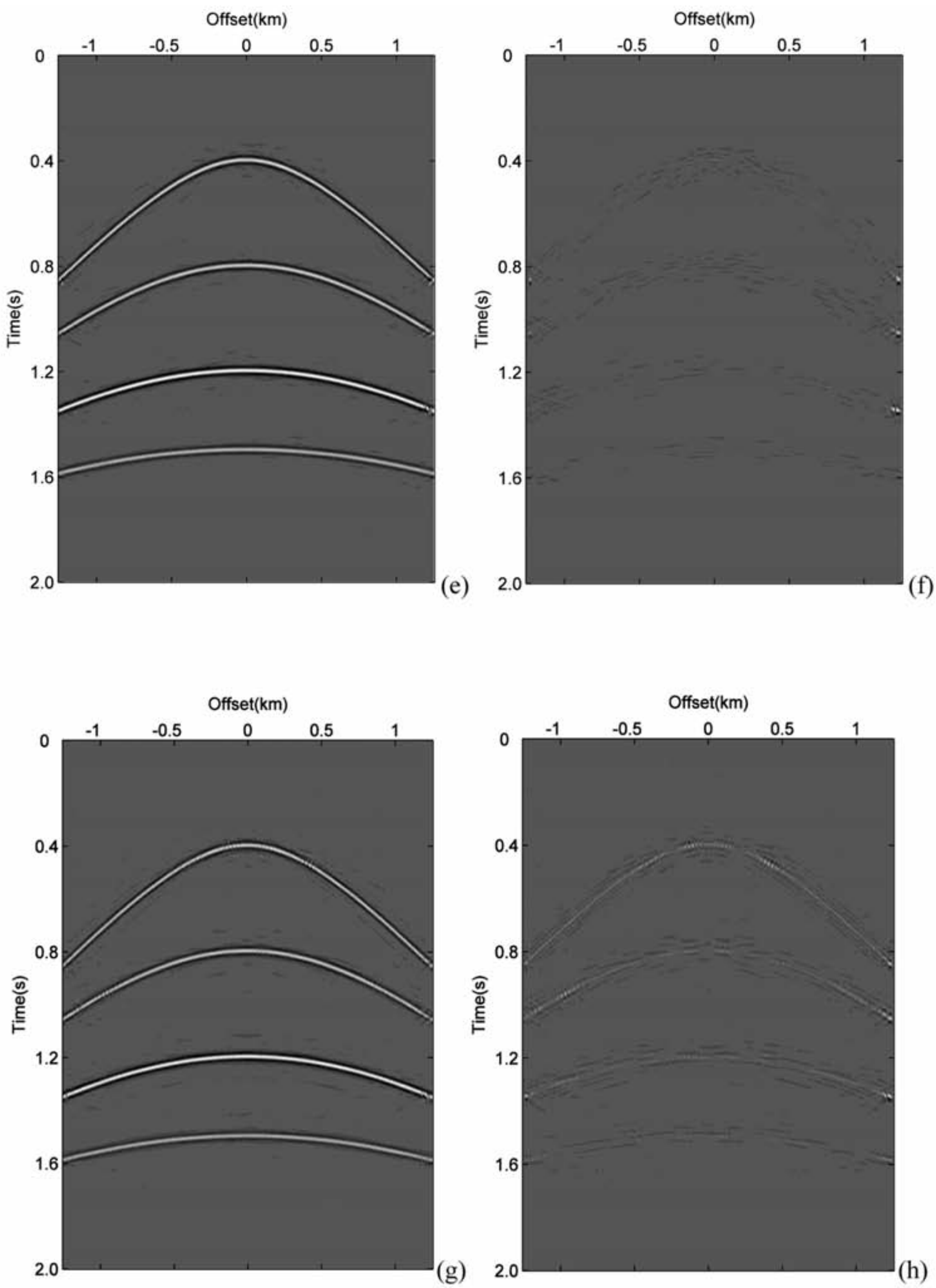

Fig. 3. Interpolated results (left column) and corresponding residuals (right column). (e)-(f) obtained by the proposed novel method($\alpha = 0.6$); (g)-(h) obtained by the spgl1 method.

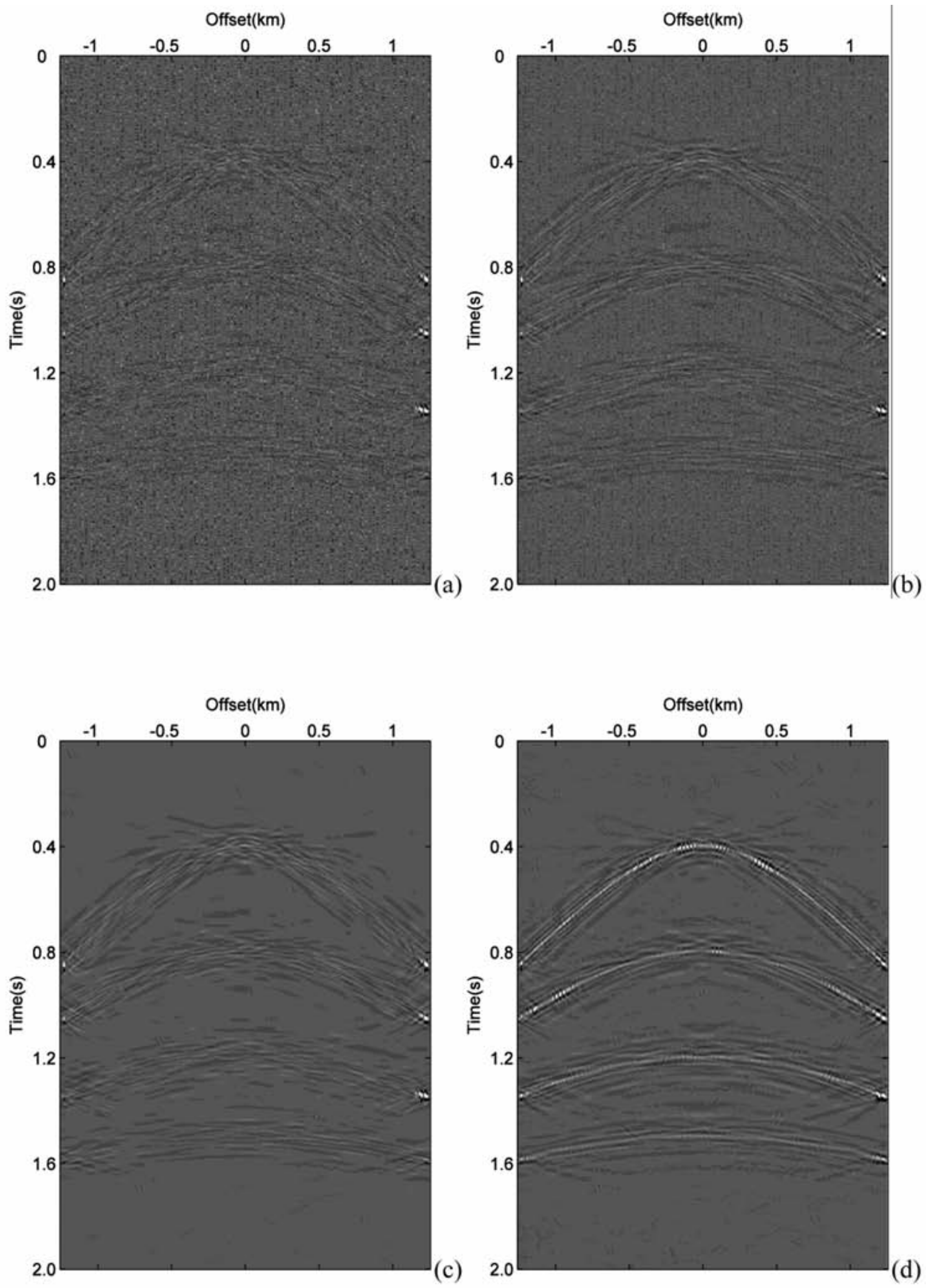

Fig. 4. Reconstruction Error (amplified three times) obtained by the POCS method (a); the weighted POCS method ($\alpha = 0.6$) (b); the proposed novel method (c); the spgl1 method (d).

Real data application

Real marine data is shown in Fig. 5(a), containing 150 traces and 1500 samples in each trace. The trace interval is 12.5 m and the time sampling interval is 4 ms. In order to demonstrate the superiority of the novel method, random noise is added into the original data, and the noisy data with 50% traces missing is shown in Fig. 5(b). The colorbar of all the field data tests is shown in Fig. 2(b).

For convenience of comparisons, the POCS method, the weighted POCS method ($\alpha = 0.6$) and the novel method ($\alpha = 0.6$) are tested with the same thresholds and the maximum iteration is set to 50. The interpolated results and the corresponding residuals are plotted respectively in Fig. 6, and the residual is still defined as the difference between reconstructed data (left column of Fig. 6) and the original noise free data (Fig. 5(a)). The reconstructed result by the spgl1 method is shown in Fig. 6(g) and Fig. 6(h) shows the construction error. In order to show the details of error sections, similar to synthetic data, we amplified the magnitude of each error section three times shown in Fig. 7(a)-(d).

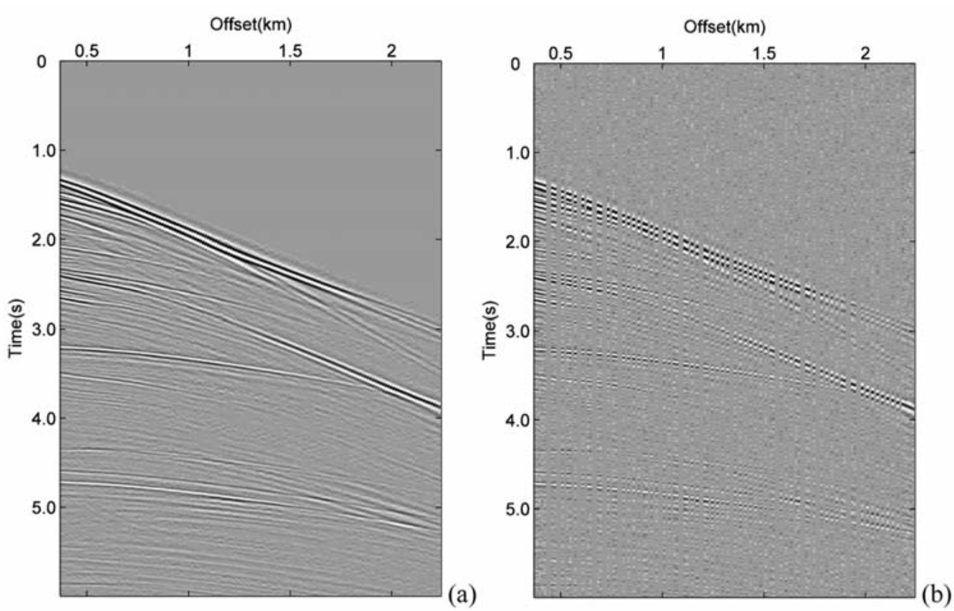

Fig. 5. Complete real marine data (a); noisy data with 50% traces missing (b).

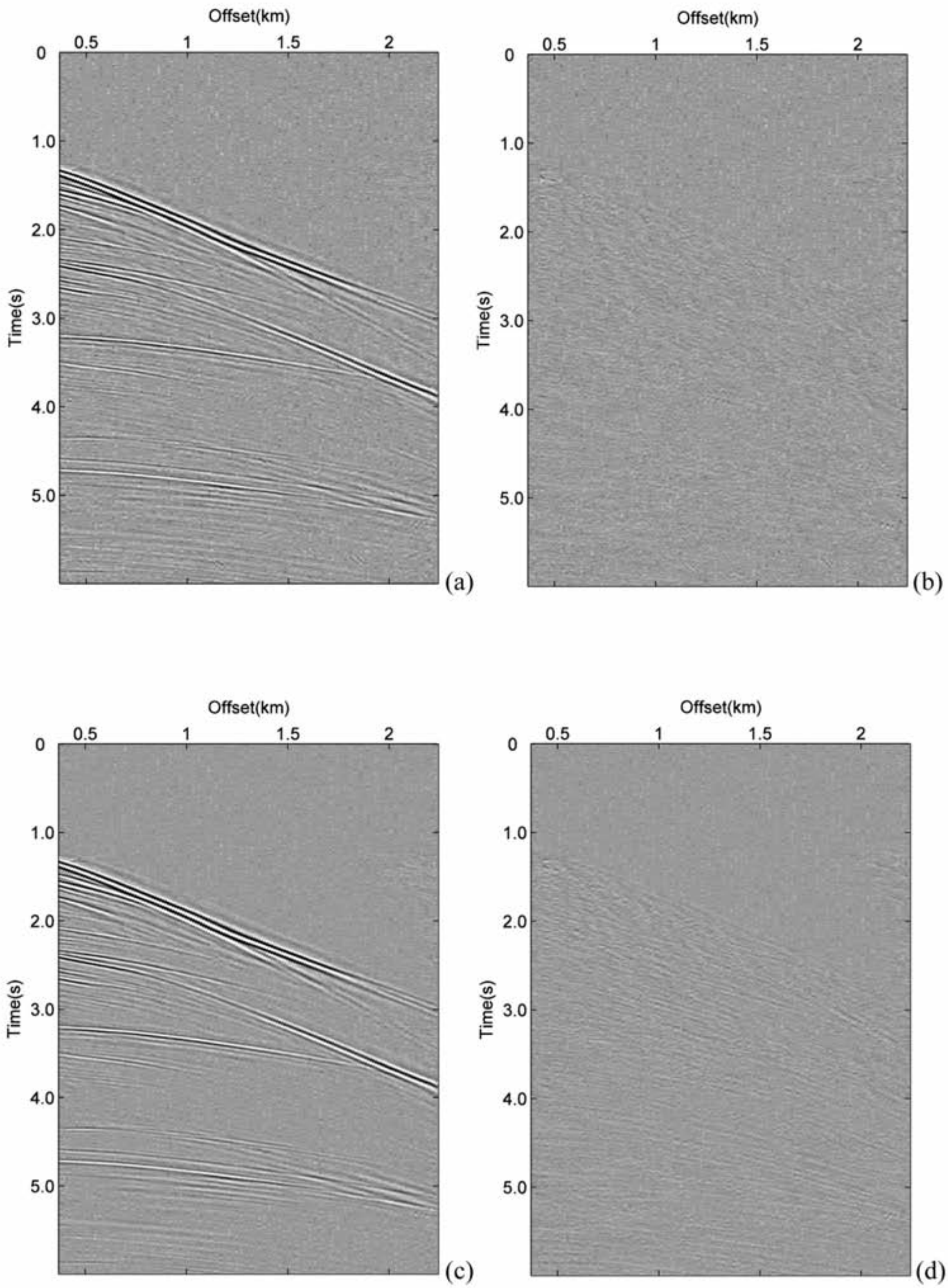

Fig. 6. Interpolated results (left column) and corresponding residuals (right column). (a)-(b) obtained by the POCS method; (c)-(d) obtained by the weighted POCS method ($\alpha = 0.6$).

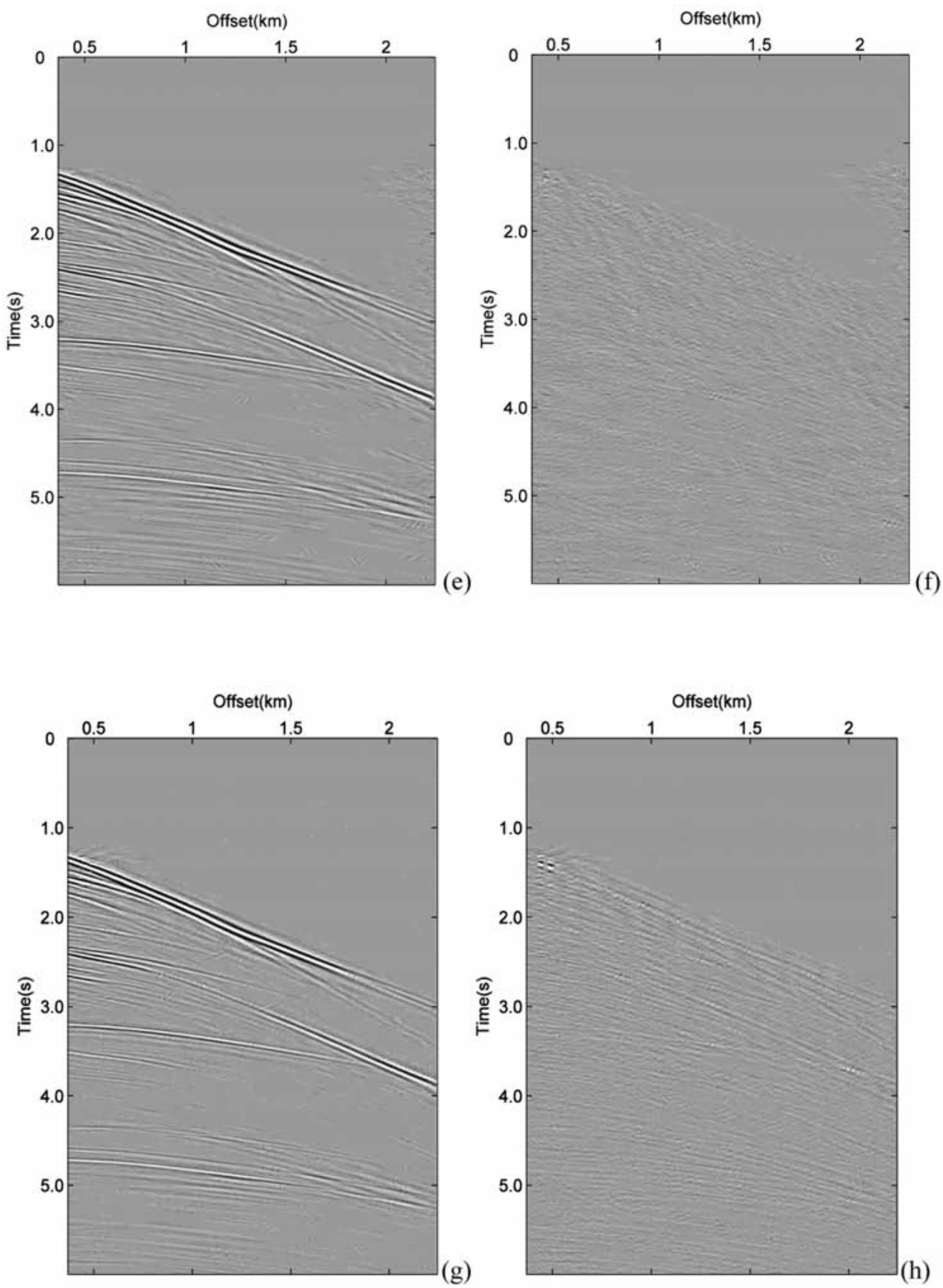

Fig. 6. Interpolated results (left column) and corresponding residuals (right column). (e)-(f) obtained by the proposed novel method($\alpha = 0.6$); (g)-(h) obtained by the spgl1 method.

From the reconstructed data [Fig. 6 (left column)], it can be concluded that the proposed method is the most effective method among the four methods. The construction error section [Fig. 6 (right column)], especially the magnitude amplification section shown in Fig. 7 indicates that the POCS and weighted POCS methods insert random noise into the interpolated data decreasing the recovered SNRs; the spgl1 method can eliminate random noise while damaging the signal to some extent; our proposed method can protect the signal as well as eliminate random noise by the threshold strategy. Fig. 6-7 demonstrate that the novel method can obtain the best interpolated result which is consistent with original data [Fig. 5(a)]; the weighted POCS method can attenuate random noise compared with the POCS method, but its performance is still unsatisfactory; the spgl1 method can eliminate random noise while damaging the signal inevitably, which decreases the recovered SNR. The final recovered SNRs are 6.5, 9.5, 12.9 and 11.07 dB for the POCS method, the weighted POCS method (α = 0.6), the proposed novel method (α = 0.6) and the spgl1 method, respectively. It should be noted that, the proposed method can eliminate the random noise using the threshold strategy, but there is minor signal leak into the error section which is the minor disadvantage of our proposed method.

Figs. 4 and 7 demonstrate that the reconstruction error of the proposed method is the minimum among the four methods and can be used effectively in seismic data interpolation and noise removal simultaneously, compared with the spgl1 method and the weighted POCS method. The recovered SNRs in synthetic and field data applications demonstrate the validity of the proposed method.

CONCLUSION

Simultaneous seismic data interpolation and random noise elimination have been achieved by the proposed novel method with the L_0 norm constraint. Defects of the POCS and weighted POCS methods have been analyzed for noisy data interpolation: the POCS method assumes the observed seismic data with a high SNR and cannot handle noisy data interpolation properly; the weighted POCS method can weaken the random noise effects, but it still inserts some random noise and its performance is unsatisfactory when the value of α is small. Thus, based on the IHT method, a novel method is proposed for interpolation and noise removal, simultaneously. Numerical examples of synthetic and real data confirm that the performances of the novel method are better than those of the POCS method, the weighted POCS method and the spgl1 method in terms of recovered SNRs. The proposed novel method can be used to obtain interpolated data in the situation of existing random noise, hence it is more appropriate for real data applications.

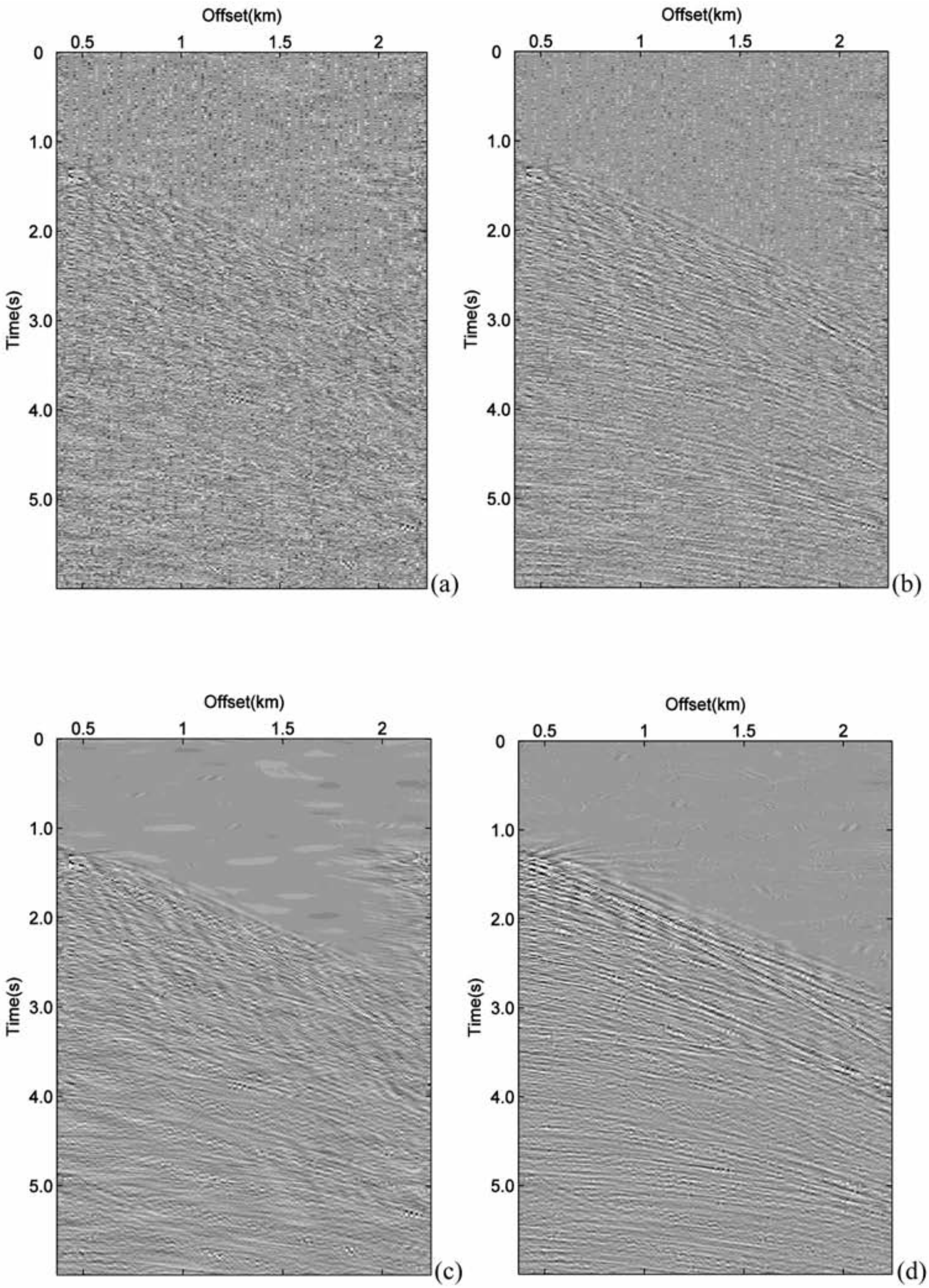

Fig. 7. Reconstruction Error (amplified three times) by the POCS method (a); the weighted POCS method (α = 0.6) (b); the proposed novel method (c); the spgl1 method (d).

The interpolated results in this paper are based on the improved jittered under-sampling strategy and cannot handle regular sampling data. Anti-aliasing strategy (Gao et al., 2012; Naghizadeh, 2012; Naghizadeh and Sacchi, 2010) can be incorporated in the future. Weak signal preserved random noise elimination method is another research topic because there is minor signal leakage by the threshold strategy to eliminate random noise.

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