

## THE APPLICATION OF HIGH-ORDER CUMULANTS ZERO SLICE IN WAVELET PHASE CORRECTION

YONGSHOU DAI<sup>1</sup>, YANAN ZHANG<sup>1,2</sup>, MANMAN ZHANG<sup>1</sup>, RONGRONG WANG<sup>1</sup> and PENG ZHANG<sup>1</sup>

<sup>1</sup> College of Information and Control Engineering, China University of Petroleum, Qingdao 266580, P.R. China. ufo995-ufo@163.com

<sup>2</sup> Qing Dao Topscomm Communication Inc., Qingdao 266024, P.R. China.

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### ABSTRACT

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To solve the problem of Gaussian noise sensitivity in the traditional seismic wavelet phase correction criteria, a wavelet phase correction method based on high-order cumulants (HOCs) zero slice was proposed, and its application conditions and scope were researched. The wavelet phase correction results were evaluated based on the criterion of calculating the HOCs zero slice of deconvolution results. Because of HOC's insensitivity to Gaussian noise, the method could effectively achieve the wavelet phase correction under conditions with Gaussian noise pollution. A simulation showed the effectiveness of the method, but the criterion was limited by data length, and the criterion's anti-noise capabilities could be improved with increased data length. The processing of actual seismic data demonstrated the practicability of the method. This method provides a new method of wavelet phase correction, and the criterion based on HOCs zero slice can be used in deconvolution and seismic wavelet estimation.

KEY WORDS: high-order cumulants (HOC), zero slice, wavelet phase correction, evaluation criterion, Gaussian noise.

### INTRODUCTION

Seismic wavelet estimation is an important component of seismic data processing and seismic interpretation, but the wavelet phase is not always estimated accurately in the wavelet estimation process (Yu et al., 2011). Because of inaccurate wavelet phase estimation, deconvolution cannot effectively

improve the resolution of the seismic record. Thus, the wavelet phase correction is needed after deconvolution to improve the signal-to-noise ratio and resolution of seismic sections (Liu et al, 2012; Gao et al, 2009). Traditional wavelet phase correction methods usually use evaluation criteria to restrain the phase correction results and use phase optimization to correct the phase.

Levy introduced the maximum variance norm into the wavelet phase correction and explained its rationale (Levy and Oldenburg, 1987). Zhou used the maximum variance norm to realize the constant phase correction and proved that the variance norm is at its maximum when a discrete sequence is zero phase (Zhou, 1989). To avoid the loss of a weak signal, Cao used Parsimony criterion to realize the deconvolution process, and this criterion was also applied to the wavelet phase correction (Cao et al., 2003; Yu et al., 2012). Yao used the maximum similarity coefficient of a synthetic seismogram based on logging data and phase shift records to correct the wavelet phase (Yao, 1990). In addition, the common criteria in deconvolution can also be applied in wavelet phase correction, such as the norm, variation, absolute abundance, Cauchy criterion and improved Cauchy criterion (Oldenburg et al., 1983; Wang et al., 2011; Brossier et al., 2009; Jeong et al., 2013; Wu et al., 2012; Yuan and Wang, 2010; Yuan and Wang, 2011; Gao and Liu, 2013; Lois et al., 2013; Sacchi, 1997; Li et al., 2013). All of the criteria restrain the optimal value by checking the time domain mathematical characteristics of the test signal. However these methods are sensitive to the noise of changing the signal's time-domain characteristics, this is the inherent defects of these methods. Therefore, these criteria cannot obtain good wavelet phase correction results from signal polluted with noise.

To solve the problem of the wavelet phase correction methods' sensitivity to Gaussian noise, a wavelet phase correction method based on the HOCs zero slice was proposed. This method uses the HOCs zero slice of deconvolution results as the criterion to correct the wavelet phase in conditions containing Gaussian noise. The effectiveness and practicality of this method were checked by simulation and actual seismic data processing.

## DECONVOLUTION MODELING WITH NOISE

By studying the influence of inaccurate wavelet phases on the inversion results of reflection coefficient sequences, the authors have come to the following conclusions: a phase-only filter can be retained in the deconvolution results of inaccurate wavelet phase estimations, the deconvolution results are the convolution of the reflection coefficient sequence and a phase-only filter, and the phase spectrum of a phase-only filter is the phase spectrum difference between the real wavelet and the estimated wavelet (Zhang et al, 2013). These conclusion can be shown as follows:

$$\begin{aligned}\tilde{r}(n) &= F^{-1}[e^{j\varphi_w(\omega)-j\varphi_{\tilde{w}}(\omega)} R(e^{j\omega})] \\ &= F^{-1}[P(e^{j\omega})R(e^{j\omega})] = p(n) * r(n) \quad ,\end{aligned}\quad (1)$$

where  $\tilde{r}(n)$  is the reflection coefficient sequence after deconvolution,  $r(n)$  is the original reflection coefficient sequence,  $F^{-1}$  is the inverse Fourier transform coefficient sequence,  $\varphi(\omega)$  and  $\varphi_{\tilde{w}}(\omega)$  are the phase spectra of the original operator,  $R(e^{j\omega})$  is the frequency-domain representation of original reflection wavelet and estimated wavelet, respectively.  $P(e^{j\omega}) = e^{j\varphi_w(\omega)-j\varphi_{\tilde{w}}(\omega)}$  is the frequency-domain representation of the phase-only filter with unit energy. The term  $p(n)$  is the time-domain sequence of the phase-only filter. This parameter has unit energy in the time domain and an all frequency component, and its phase spectrum is the phase spectrum difference between the real wavelet and the estimated wavelet.

For seismic traces containing additive Gaussian noise, the Robinson convolution model can be written as follows (Robinson, 1967; Chen et al., 2013):

$$x(n) = r(n) * \omega(n) + v(n) \quad , \quad (2)$$

where  $\omega(n)$  is the seismic wavelet,  $v(n)$  is the additive Gaussian noise (Wang and Wang, 2012). These signals shall meet the following assumptions,

1. The reflection coefficient sequence,  $r(n)$ , is a sparse-spike sequence that belongs to an independent identically distributed (IID) sequence, obeys the Bernoulli-Gaussian distribution, has a variance of  $\sigma^2 < \infty$ , and has 4th-order cumulants of  $|\gamma_{4r}| < \infty$ .
2. The environmental noise,  $v(n)$ , is additive colored Gaussian noise with a zero mean and has statistical independence from  $r(n)$  and  $x(n)$ .

When the estimated wavelet  $\tilde{\omega}(n)$  and the original wavelet have the same amplitude spectrum, the estimated wavelet  $\tilde{\omega}(n)$  was used to implement deconvolution. The result is similar to eq. (1):

$$\tilde{r}(n) = p(n) * r(n) + \tilde{b}(n) * v(n) \quad , \quad (3)$$

where  $\tilde{b}(n)$  is the inverse wavelet of the estimated wavelet, i.e.,  $\tilde{\omega}(n) * \tilde{b}(n) = 1$ .

From eq. (3), we can see that in the condition of seismic traces that contain additive Gaussian noise, the deconvolution results are not only the convolution of the phase-only filter and the original reflection coefficient sequence, but also contain the components of noise and the inverse wavelet. Because of the influence of noise, the deconvolution result would deviate from

the original reflection coefficient sequence. If the traditional methods are used to correct this wavelet phase, accurate results would not be achieved.

## WAVELET PHASE CORRECTION CRITERION BASED ON HOC

The HOCs theory is now widely used in production and scientific research, and its characteristic insensitivity to Gaussian noise can solve the problem of noise in seismic trace signal processing (Tang and Yin, 2001; Zhang et al., 2011; Wang et al., 2013). The third-order cumulants of a random sequence with a zero mean and symmetrical distribution is always zero, the higher-than-fourth-order cumulants' calculated amount is too large, and the fourth-order cumulants contain enough phase information (Lazear, 1993). Therefore, the fourth-order cumulants of deconvolution results will be used to evaluate the wavelet phase correction. Eq. (3) can be seen as the sum of the reflection coefficient sequence,  $r(n)$ , passing the phase-only system,  $p(n)$ , and the Gaussian noise,  $v(n)$ , passing the system  $b(n)$ . Then, calculating their fourth-order cumulants (Feng et al., 2011),

$$\begin{aligned}
 C_{4r}(\tau_1, \tau_2, \tau_3) &= \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \sum_{i_3=-\infty}^{\infty} \sum_{i_4=-\infty}^{\infty} p(i_1)p(i_2)p(i_3)p(i_4) \\
 &\quad \times C_{4r}(\tau_1+i_1-i_2, \tau_2+i_1-i_3, \tau_3+i_1-i_4) \\
 &\quad + \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \sum_{i_3=-\infty}^{\infty} \sum_{i_4=-\infty}^{\infty} \tilde{b}(j_1)\tilde{b}(j_2)\tilde{b}(j_3)\tilde{b}(j_4) \\
 &\quad \times C_{4v}(\tau_1+j_1-j_2, \tau_2+j_1-j_3, \tau_3+j_1-j_4) \quad , \quad (4)
 \end{aligned}$$

where  $C_{4r}(\tau_1, \tau_2, \tau_3)$ ,  $C_{4r}(\tau_1+i_1-i_2, \tau_2+i_1-i_3, \tau_3+i_1-i_4)$  and  $C_{4v}(\tau_1+j_1-j_2, \tau_2+j_1-j_3, \tau_3+j_1-j_4)$  are the fourth-order cumulants of the reflection coefficient sequence after deconvolution, the original reflection coefficient sequence and the additive Gaussian noise, respectively.

Because the HOCs of arbitrary Gaussian noise are identical to zero, i.e.,  $C_{4v}(\tau_1+j_1-j_2, \tau_2+j_1-j_3, \tau_3+j_1-j_4) = 0$ , the last equation can be simplified as

$$\begin{aligned}
 C_{4r}(\tau_1, \tau_2, \tau_3) &= \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \sum_{i_3=-\infty}^{\infty} \sum_{i_4=-\infty}^{\infty} p(i_1)p(i_2)p(i_3)p(i_4) \\
 &\quad \times C_{4r}(\tau_1+i_1-i_2, \tau_2+i_1-i_3, \tau_3+i_1-i_4) \quad . \quad (5)
 \end{aligned}$$

Based on the Bartlett-Brillinger-Rosenblatt (BBR) formula of HOC, this equation can be rewritten as

$$C_{4r}(\tau_1, \tau_2, \tau_3) = \gamma_{4r} \sum_{i=-\infty}^{\infty} p(i)p(i+\tau_1)p(i+\tau_2)p(i+\tau_3) , \quad (6)$$

where  $\gamma_{4r}$  is the kurtosis of the original reflection coefficient sequence, .

Because the phase-only filter,  $p(n)$ , has unit energy, if the actual wavelet and the estimated wavelet are exactly the same,  $p(n)$  is a zero-phase filter and an unit pulse in the time domain. If the phase spectra of the real wavelet and the estimated wavelet are different,  $p(n)$  is a nonzero-phase filter and has a divergent energy nature in the time domain. When a discrete series is zero phase, its variance norm is at its maximum (Zhou, 1989). Therefore, the maximum variance norm can effectively identify the condition of a zero-phase  $p(n)$ . The  $L^p$  norm and Variation can also be used in this identification procedure. Similarly, eq. (6) can be used to make  $p(n)$  reach extreme values when  $p(n)$  is zero phase. Thus, the conditions of a zero-phase  $p(n)$  can be identified, and the wavelet phase can be corrected.

When  $\tau_1 = \tau_2 = \tau_3 = 0$ , eq. (6) can be written as

$$\Psi_1 = C_{4r}(0,0,0) = \gamma_{4r} \sum_{i=-\infty}^{\infty} p(i)^4 , \quad (7)$$

The parameter  $p(n)$  is a sequence of unit energy, i.e.,  $\sum_{i=-\infty}^{\infty} p(i)^2 = 1$ , and the maximum variance norm criterion is expressed as follows:

$$\Psi_M(r) = \sum_{i=-\infty}^{\infty} r_i^4 / \left[ \sum_{j=-\infty}^{\infty} r_j^2 \right]^2 , \quad (8)$$

From this, we can conclude that the HOCs of deconvolution results translate into the maximum variance norm criterion when  $\tau_1 = \tau_2 = \tau_3 = 0$ . The zero-phase  $p(n)$  can be identified by the HOCs zero slice of the deconvolution results, and the wavelet phase can then be corrected.

The criterion of wavelet phase correction based on the HOCs zero slice can be obtained from eq. (7). Because of the HOCs' insensitivity to Gaussian

noise, the wavelet phase correction can be determined abstractly in conditions of additive Gaussian noise, and the wavelet phase correction can be achieved by iterative algorithm or nonlinear optimization methods. In addition, this criterion can also be applied to the seismic wavelet extraction and deconvolution processing.

## SIMULATION

To validate the effectiveness of the criterion based on the HOCs zero slice in the wavelet phase correction process, an autoregressive moving average (ARMA) model was used to describe the seismic wavelet and synthesize a seismogram. Seismic wavelets with the same amplitude spectrum and different phase spectra were constructed by symmetrical mapping Pole-Zeros of the ARMA model in the  $z$ -domain, and deconvolution was implemented using those wavelets. The criterion of the wavelet phase correction based on the HOCs zero slice was used to evaluate the deconvolution results to identify the original reflection coefficient sequence and verify the effectiveness of the proposed criterion. The applied range of the criterion was checked by changing the experimental conditions.

An original causal mixed-phase wavelet is used in this experiment, and its difference expression in ARMA model is as follows:

$$\begin{aligned} x(t) - 2.35x(t-1) + 2.12x(t-2) - 0.95x(t-3) + 0.21x(t-4) \\ = r(t) - 0.8r(t-1) + 0.2r(t-2) - 0.82r(t-3) \end{aligned} \quad (9)$$

Its system function in the  $z$ -domain is as follows:

$$\begin{aligned} W(z) = (1 - 0.8z^{-1} + 0.2z^{-2} - 0.82z^{-3}) \\ / (1 - 0.235z^{-1} + 2.12z^{-2} - 0.95z^{-3} + 0.21z^{-4}) \end{aligned} \quad (10)$$

To calculate the poles of the original wavelet in eq. (10), the following expressions are used:  $\alpha_1 = 0.8643 + 0.1666j$ ,  $\alpha_2 = 0.8643 - 0.1666j$ ,  $\alpha_3 = 0.3107 + 0.4177j$ , and  $\alpha_4 = 0.3107 - 0.4177j$ , and the following expressions describe the zeros of the original wavelet:  $\beta_1 = 0.2015$ ,  $\beta_2 = -0.2008 + 0.8013j$ , and  $\beta_3 = -0.2008 - 0.8013j$ . The original seismic wavelet is shown in Fig. 1.

The reflection coefficient sequence is a randomly generated sparse-spike sequence (Zhang et al., 2011) and is used to synthesize a seismogram with the seismic wavelet in Fig. 1. The generated reflection coefficient sequence and synthesized seismogram are shown in Fig. 2.

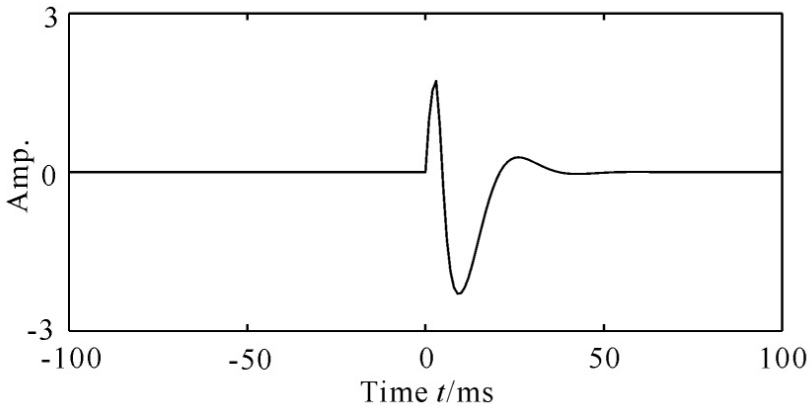


Fig. 1. Original seismic wavelet.

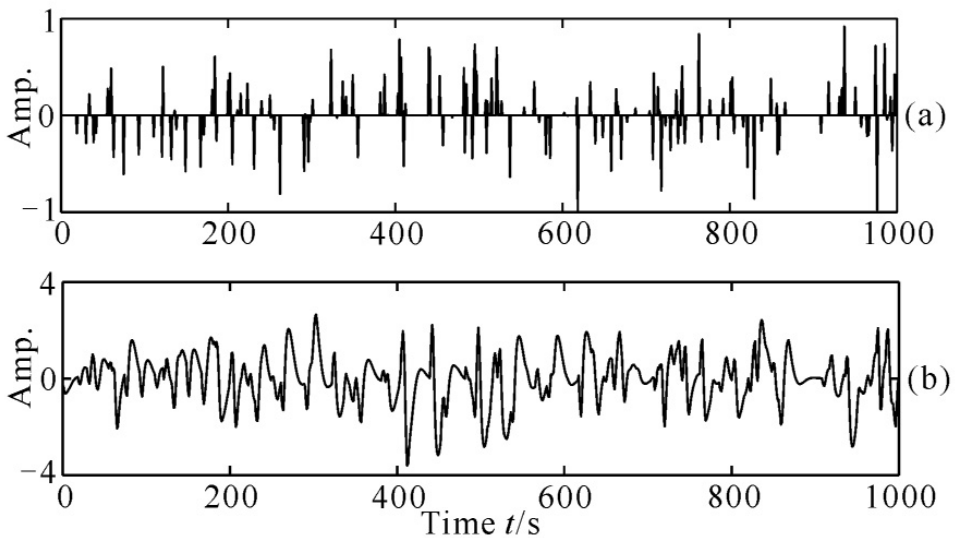


Fig. 2. (a) Reflection coefficient sequence and (b) synthesized seismogram.

The symmetrical mapping of Pole-Zeros in  $z$ -domain of the ARMA model can be used to construct a series of seismic wavelets with the same amplitude spectrum and different phase spectra to simulate the results of wavelet estimation (Zhang et al., 2013). The results of this process are shown in Fig. 3, in which the No. 2 Trace is the original seismic wavelet. The seismic wavelets have the same amplitude spectrum; thus, each trace wavelet should have same energy. Because of the different phase spectra, the wavelets exhibit different properties in the time domain.

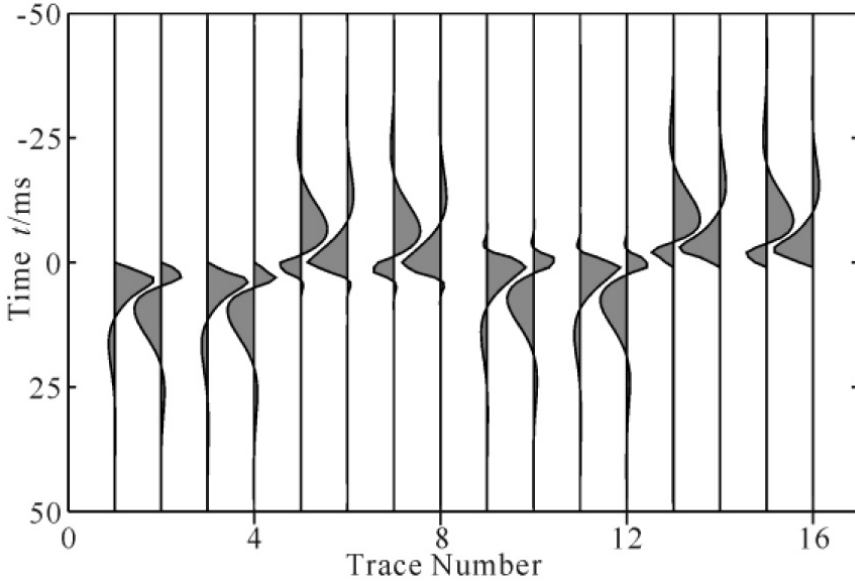


Fig. 3. Constructed seismic wavelets with the same amplitude spectrum and different phase spectra.

Deconvolution was performed on the constructed seismic wavelets in the synthetic seismogram, producing 16 reflection coefficient sequences. The criterion based on the HOCs zero slice was used to evaluate the reflection coefficient sequences after deconvolution. If the No. 2 reflection coefficient sequence can be identified, the criterion in this paper is valid.

The criteria of maximum variance norm and variation, eqs. (8) and (11), respectively, were used as comparisons in the discrimination process.

$$\Psi_V(r) = \sum_{i=0}^{L-1} |r_{i+1} - r_i|, \quad (11)$$

### Criterion validity verification

A reflection coefficient sequence with a length of 1 ms and a sampling interval of 1 ms was generated and used to synthesize a seismogram with the seismic wavelet shown in eq. (9), just like Fig. 2(a). The constructed wavelets have the same amplitude spectrum and different phase spectra and are used to apply deconvolution to the synthetic seismogram. In total, 16 deconvolution results were evaluated by the criterion  $\Psi_1$ , and the criteria of maximum variance norm and variation were used as comparisons. The results are shown in Fig. 4.



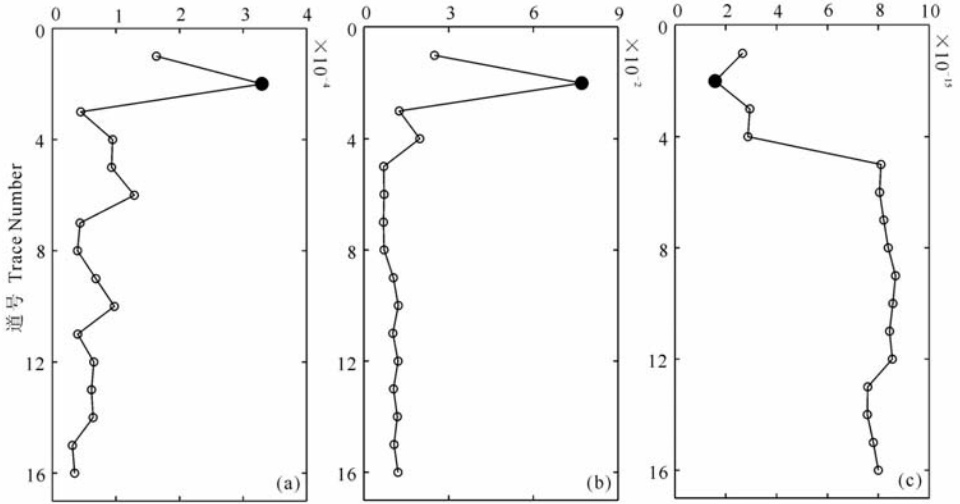


Fig. 4. Evaluation results of the wavelet phase correction criterion using (a)  $\Psi_1$ , (b) Maximum variance norm and (c) Variation.

Fig. 4 shows that the criteria of  $\Psi_1$ , maximum variance norm and Variation can identify the original reflection coefficient sequence when the data length is 10,000 ms. The criterion based on the HOCs zero slice is effective.

**The influence of data length on criterion**

To demonstrate the influence of data length on the criterion, reflection coefficient sequences with lengths of 10,000 ms, 5,000 ms, 2,000 ms, 1,000 ms, 500 ms, 200 ms, 100 ms and 50 ms were generated and used to synthesize seismograms. Deconvolution processing with 16 constructed seismic wavelets was then implemented. The criterion of  $\Psi_1$  was used to identify the 16 deconvolution results. The experiment was repeated 50 times for each time length, and the determination accuracies at the different lengths are shown in Table 1.

Table 1. Determination accuracies of different data lengths.

Data Length (ms)	10000	5000	2000	1000	500	200	100	50
$\Psi_1$ Discriminant Accuracy	100%	100%	100%	100%	100%	100%	92%	36%

From Table 1, we can see that, without the noise, data lengths longer than 200 ms have criterion determination accuracies of 100%. However, at data lengths less than 200 ms, the criterion of  $\Psi_1$  cannot accurately identify the original reflection coefficient sequence. If the data length is too short, the statistical feature of HOCs cannot be expressed well; thus, the application of the criterion is limited by the shortest data length, which means data lengths should be longer than 200 ms.

### The influence of noise on criterion

A 10,000 ms reflection coefficient sequence was generated. A seismogram was synthesized and Gaussian noise with noise-to-signal ratios (NSRs) of 0.1, 0.3 and 0.5[0] was added. Subsequently, 16 constructed seismic wavelets were used to implement deconvolution, and 16 deconvolution results were evaluated by the criterion of  $\Psi_1$ . The criteria of maximum variance norm and Variation were used as comparisons.

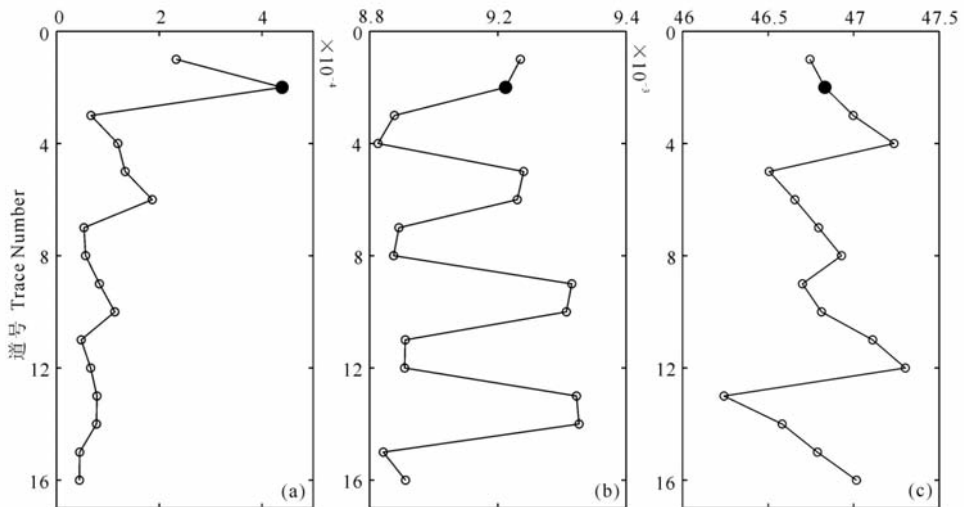


Fig. 5. Evaluation results when  $NSR = 0.1$  with (a)  $\Psi_1$ , (b) Maximum variance norm and (c) Variation.

The evaluation results of the three criteria are shown in Figs. 3 to 5. From Fig. 3, we can see that, when the synthetic seismogram contains 10% Gaussian noise, the criteria of maximum variance norm and Variation cannot identify the original reflection coefficient sequence, but the criterion of  $\Psi_1$  can identify it accurately. With increasing noise components in the synthetic

seismogram (Figs. 5 to 7), the criterion of  $\Psi_1$  based on the HOCs zero slice can still accurately identify the original reflection coefficient sequence, which proves that the criterion based on the HOCs zero slice has good anti-noise capabilities.

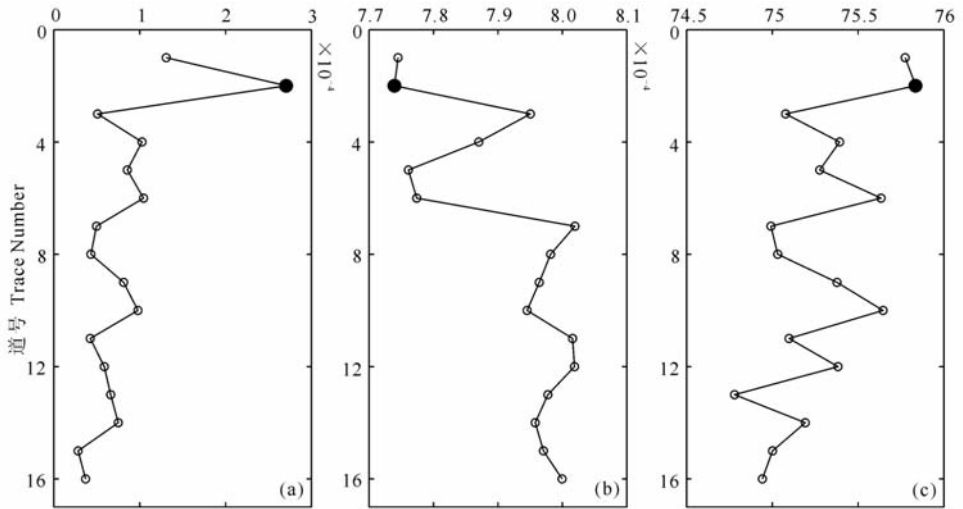


Fig. 6. Evaluation results when NSR = 0.3 with (a)  $\Psi_1$ , (b) Maximum variance norm and (c) Variation.

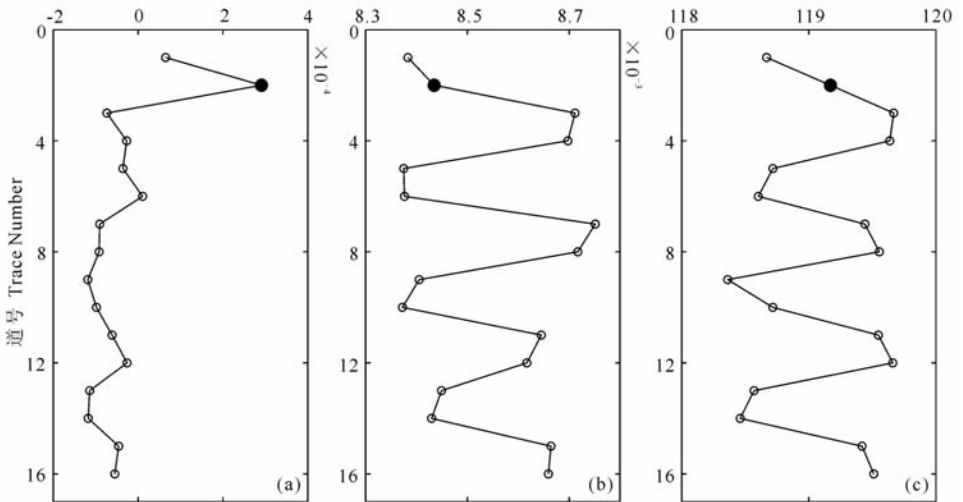


Fig. 7. Evaluation results when NSR = 0.5 with (a)  $\Psi_1$ , (b) Maximum variance norm and (c) Variation.

To validate the applicability of the proposed criterion under the influences of data length and noise, reflection coefficient sequences with lengths of 10,000 ms, 5,000 ms, 2,000 ms, 1,000 ms, 500 ms, 200 ms, 100 ms and 50 ms were generated. Seismograms were synthesized, and Gaussian noise with NSRs of 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45 and 0.5 was added. Deconvolution was implemented with 16 constructed seismic wavelets, and the proposed criterion was used to identify the deconvolution results. Each experiment was repeated 50 times, and the determination accuracy is shown in Table 2.

Table 2. Determination accuracy of  $\Psi_1$ .

	Data Length/ms						
	10000	5000	2000	1000	500	300	200

Noise-to-



Table 2 shows that the criterion of  $\Psi_1$  is accurate when 1) the data length is greater than or equal to 5,000 ms and the NSR is less than or equal to 0.5, 2) the data length is 2,000 ms and the NSR is less than or equal to 0.45, 3) the data length is 1,000 ms and the NSR is less than or equal to 0.2, 4) the data length is 500 ms and the NSR is less than or equal to 0.1, or 5) the data length is 300 ms and the NSR is less than or equal to 0.05. If the data length is less than 300 ms and the NSR is 0.05, the criterion of  $\Psi_1$  cannot identify the original reflection coefficient sequence. Table 2 also shows that 1) when the NSR is 0.05, the shortest accurately discriminated length of  $\Psi_1$  is 300 ms and the best discriminated length can be 400 ms; 2) when the NSR is 0.1, the shortest and best discriminated lengths of  $\Psi_1$  are 500 ms and 600 ms, respectively; 3) when the NSR is 0.15-0.2, the shortest discriminated length of  $\Psi_1$  is 1,000 ms, and the best length is 1,200 ms; 4) when the NSR is 0.25-0.45, the shortest discriminated length of  $\Psi_1$  is 2,000 ms, and the best length is 2,200 ms; and 5) when the NSR is 0.5, the shortest discriminated length of  $\Psi_1$  is 5,000 ms, and, considering the actual length and noise intensity of this post-stack seismic data, the best length under this NSR has little practical significance.

The experimental results show that the proposed wavelet phase correction criterion based on the HOCs zero slice is effective and has good anti-noise capabilities. Because of its statistical method, this method has a minimum data length requirement (i.e., a data sample point should be greater than 200 if it does not contain noise), and its anti-noise capabilities improve with longer data lengths.

## ACTUAL SEISMIC DATA PROCESSING

To validate the practicability of the wavelet phase correction method based on HOCs zero slice, we processed actual seismic data and reflection coefficient sequences from logging. Fig. 8(a) is post-stack seismic data from the Shengli Oil Field. The digitization interval is 2 ms, and the well is near trace No.289. Fig. 8(b) is the reflection coefficient sequence in the time domain, which has been converted from logging data.

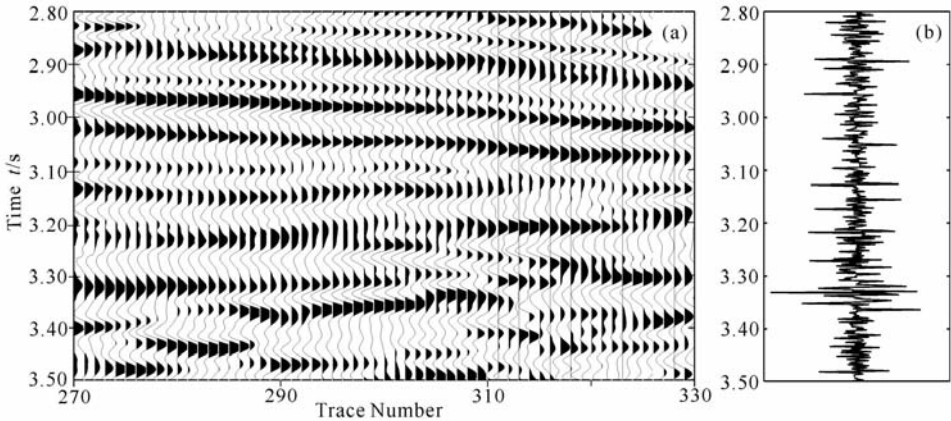


Fig. 8. Actual seismic data: (a) seismogram of the Shengli Oil Field and (b) reflection coefficient sequence of the 289th seismic trace.

The seismic wavelet was estimated from the actual seismic data shown in Fig. 8. The uphole trace wavelet was extracted using the method of fine picking up seismic wavelet at uphole trace (Zhang et al., 2005), and the minimum-phase wavelet was estimated with the traditional statistical seismic wavelet extraction method. The results of the seismic estimation are shown in Fig. 9.

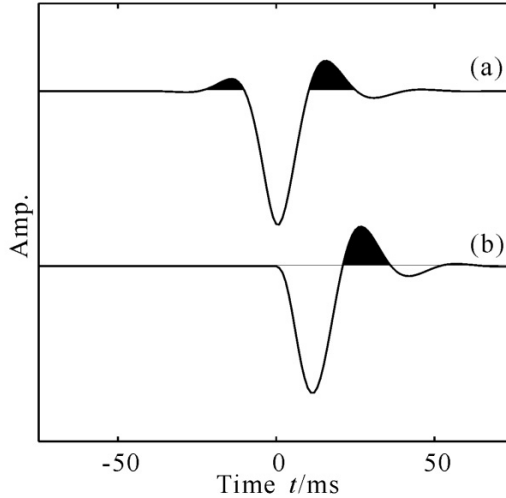


Fig. 9. (a) Seismic wavelet from the uphole trace and (b) minimum phase seismic wavelet based on actual seismic data.

From Fig. 9, the uphole trace seismic wavelet is mix-causal and mix-phase, and the seismic wavelet extracted using the traditional method is causal minimum-phase. Because they have the same amplitude spectrum but different phase spectra, the results of the deconvolution (which was implemented via minimum-phase wavelet for the actual seismic data) must contain the wavelet residual phase. The wavelet phase correction method based on the HOCs zero slice was used to correct the wavelet residual phase. This result was then compared to the reflection coefficient sequences from logging; if the two are similar, the method is demonstrated to be practical.

Under the constraint of the wavelet phase correction criterion based on the HOCs zero slice, improved particle swarm optimization was used to accurately optimize the wavelet residual phase (Dai et al., 2011). The post-stack seismic data suppressed the added Gaussian noise, and a high-order window function was used to filter the HOCs to improve the determination accuracy. Although the available data are short, it can still relatively accurately correct the wavelet residual phase. The logging reflection coefficient sequence and the wavelet phase correction results are shown in Fig. 10.

As shown in Figs. 10(a) and 10(c), the deconvolution results of the minimum phase wavelet differ greatly from the real reflection coefficient sequence. Because of the wavelet residual phase, the deconvolution results do not reflect the stratigraphic information correctly. In contrast with Fig. 10(c), the deconvolution results after wavelet phase correction in Fig. 10(b) are similar to the real reflection coefficient sequence, and the similarity is as high as

73.74%. Due to the seismic trace noise, time-depth conversion error, short data length, etc., the wavelet phase correction and reflection coefficient sequence calculation results contain some error (Cao et al., 2009), but the similarity is within the acceptable range. Thus, the wavelet phase correction method based on the HOCs zero slice has been demonstrated to be practical.

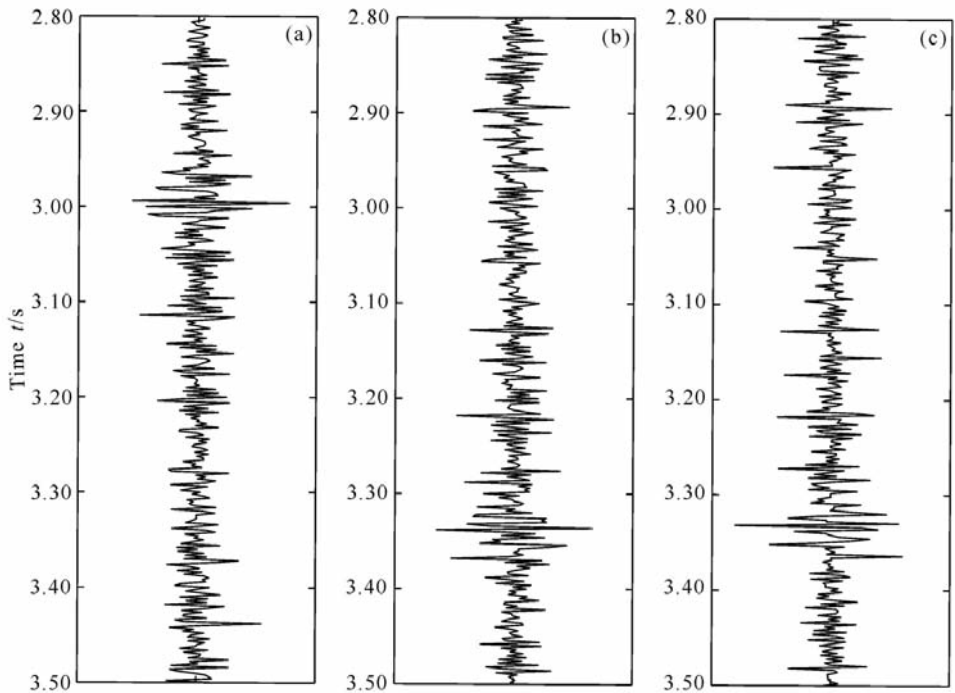


Fig. 10. Reflection coefficient sequence and wavelet phase correction results:  
(a) Minimum phase wavelet deconvolution results, (b) Wavelet phase correction results, and (c) Reflection coefficient sequence of logging.

## CONCLUSIONS

This paper proposes a wavelet phase correction method based on the HOCs zero slice to solve the Gaussian noise sensitivity problem of traditional seismic wavelet phase correction methods. The wavelet phase correction results were identified by the criterion of calculating the HOCs zero slice of deconvolution results. A simulation showed that the criterion is effective conditions involving Gaussian noise[0], but the criterion is limited by data length, and its anti-noise capabilities could be improved by increased data

lengths. To use this criterion, data sample points should be greater than 200 without noise and should be 400 when the data contains 5% noise, and 600 when the data contains 10% noise. The best data sample can be 1,200 when the noise intensity is less than or equal to 20% and 2,200 when the noise intensity is less than or equal to 45%. The processing of actual seismic data showed that the method could effectively process the seismic wavelet phase correction and has a certain practicality. The wavelet phase correction method based on the HOCs zero slice proposed in this paper can provide a new method for seismic wavelet estimation and deconvolution.

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