VARIABLE STEP SIZE-NORMALIZED SIGN GRADIENT AVO INVERSION ALGORITHM

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ABSTRACT

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Pre-stack seismic inversion faces difficulties when applied to real seismic data because of the existence of many types of noise. As we know, the l_1 norm minimization gives more robust solutions than the l_2 norm does because it is less sensitive to spiky and high-amplitude noise. To take advantage of l_1 norm and constraint on the deviation between two adjacent solutions, a variable step size-normalized sign gradient algorithm (VSS-NSGA) is proposed to obtain a more rational inversion result. By minimizing the l_1 norm of the error vector with a minimum disturbance constraint, the proposed VSS-NSGA not only reduces the computational cost of the large scale seismic inversion problems but also avoids the instability of the l_1 norm solution using the iteratively reweighted least squares (IRLS) algorithm. Furthermore, the variable step size is introduced to overcome the contradiction of the fast convergence rate and small steady-state error brought by fixed step size. Synthetic tests demonstrate that the proposed VSS-NSGA algorithm out-performs the traditional IRLS method in both convergence rate and steady-state error. The real data example shows the validity of the proposed method for AVO inversion.

KEY WORDS: l_1 norm, spiky and high-amplitude noise, minimum disturbance constraint, variable step size-normalized sign gradient algorithm (VSS-NSGA), iteratively reweighted least squares (IRLS).

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INTRODUCTION

AVO inversion has been widely used as an effective approach of carbon-hydrogen detection in the past few years (Riedel et al., 2003; Lavaud et al., 1999; Sun et al., 2004; Zhang et al., 2013). Most of the seismic inversion problems, including AVO inversion, can be formulated as an optimization problem, where the goal is to find a model of subsurface that minimizes the difference between the observed data and the modeled data (Li et al., 2012; Saraswat and Sen, 2012).

The performance of the classic inversion methods, where the traditional l_2 norm of the misfit is minimized based on the Gaussian assumption, tends to deteriorate significantly when the observed data are corrupted by non-Gaussian noise (Liu et al., 2007). As the l_1 norm shows less sensitivity to large measurement errors, it yields far more stable inversion results than the traditional l_2 norm does when handling certain types of errors (e.g., erratic data) and noise distributions (e.g., non-Gaussian) that commonly occur in geophysical applications (Claerbout and Muir, 1973; Chapman and Barrodale, 1983; Guitton and Symes, 2003; Zou et al., 2006). Such insensitivity to large noise has a statistical interpretation: robust l_1 norm is related to a long-tailed density function, whereas l_2 norm is related to the short-tailed Gaussian density function (Tarantola, 1987). However, the l_1 norm is singular where any residual component vanishes, which leads to instability in numerical minimization (Guitton and Symes, 2003). The iterative algorithm called IRLS has been used to solve l_1 norm minimization problems (Gersztenkorn et al., 1986; Scales and Gersztenkorn, 1988; Bube and Langan, 1997; Zhang et al., 2000; Ji, 2006), which provides an easy way to compute than the traditional linear programming techniques (Taylor et al., 1979). However, the IRLS algorithm is cumbersome to use because users must specify numerical parameters with unclear physical meanings (Li et al., 2010) and is time-consuming on updating the re-weighting factors at each iteration in a large scale problem. Furthermore, it lacks of stability because the l_1 norm-based IRLS method gives an infinite weight at the zero point.

To take advantage of l_1 norm and constraint on deviation between two adjacent solutions, we propose a novel objective function by minimizing the l_1 norm of a posterior error vector with a minimum disturbance constraint on the inversion parameters. The new constraint can ensure that the reflection coefficient does not change dramatically and minimize the impact of singularities to obtain a more stable solution. The proposed normalized sign gradient algorithm (NSGA) only needs the sign of the iterative error in the updating process without the need to calculate the weighting matrix as required by the traditional IRLS algorithm; thus, it reduces the computational cost and is easier to implement in large scale problems. However, the NSGA, with fixed step size, leads to a contradiction between the convergence rate and the steady-state error. Therefore, a variable step-size normalized sign gradient algorithm (VSS-NSGA) is proposed to improve the performance of both the convergence rate and steady-state error.

The paper is broadly divided into two major sections. In the first section, we give the detailed derivations of NSGA and VSS-NSGA in the AVO inverse problem and simultaneously analyze the stable condition of the proposed method. In the second section, we test the proposed VSS-NSGA using noisy synthetic seismic data and field-based seismic data.

VSS-NSGA FOR THE AVO INVERSE PROBLEM

Without loss of generality, the forward model of the AVO inverse problems can be described as a large-scale, ill-posed linear equation (Rodi and Mackie, 2001):

$$\mathbf{d} = \mathbf{G}\mathbf{m} + \mathbf{n} \ , \tag{1}$$

where **d** denotes the observed data, **n** denotes the noise vector, **m** represents the model vector to be estimated and **G** is the forward operator including both the convolution and weak contrasts Aki-Richards approximation, which may be ill-conditioned in general. Many different methods are used to find the optimum solution for given data instead of solving the above equation directly. The least-squares algorithm that minimizes the l_2 norm of the misfit is commonly used with the assumption that the noise is consistent with a Gaussian distribution. As the l_1 norm shows less sensitivity to large measurement errors, it yields far more stable estimates of the earth parameters than the traditional l_2 norm does. However, performing numerical minimizations of the l_1 norm is difficult because it is singular where any residual component vanishes. The IRLS algorithm provides an easy way to compute an l_1 solution, but it is cumbersome to use. On the one hand, users must specify numerical parameters with unclear physical meanings. On the other hand, it would result in a high computational cost at each iteration and would require many iterations in large scale problems. Furthermore, because the l_1 norm-based IRLS method gives an infinite weight at any singular point, it lacks stability. In this paper, a novel algorithm named VSS-NSGA is proposed for seismic inversion problems with lower computational costs and greater stability.

NSGA for AVO inverse problem

Mathematically, the l_1 norm of the misfit function can be expressed as:

$$\mathrm{J}(\mathbf{m}) = \| \mathbf{d} - \mathbf{G}\mathbf{m} \|_1 \ . \tag{2}$$

Because the seismic inversion is always ill-posed, we usually add some constraints to stabilize the solution. These constraints should not depend on the data and can improve the stability of the AVO inversion. Due to the singularity of the l_1 norm, we introduce the l_2 norm constraint in the objective function:

$$\min_{\mathbf{m}_{k+1}} = \|\mathbf{d} - \mathbf{Gm}\|_1 \ . \tag{3}$$

$$\text{Subject to } \|\mathbf{m}_{k+1} - \mathbf{m}_k\|_2^2 \leq \delta^2 \ , \tag{4}$$

where $\mathbf{m}_k$ is the estimated value at the k-th iteration, and δ is a parameter that ensures that $\mathbf{m}_k$ does not change dramatically (Vega et al., 2010). We can also view eq. (4) as the minimum disturbance constraint that controls the convergence level of the algorithm and should be as small as possible. Using the Lagrange multipliers method, the unconstrained cost function can be obtained by combining eqs. (3) and (4):

$$J(\mathbf{m}_{k+1}) = \|\mathbf{d} - \mathbf{Gm}\|_1 + \beta(\|\mathbf{m}_{k+1} - \mathbf{m}_k\|_2^2 + \gamma^2 - \delta^2) \ , \tag{5}$$

where β is the Lagrange multiplier, and γ is a slack variable which converts the optimization problems with inequality constraints to optimization problems with equality constraints. The derivative of the cost function (5) with respect to the vector $\mathbf{m}_{k+1}$ is

$$\partial J(\mathbf{m}_{k+1})/\partial \mathbf{m}_{k+1}) = -\mathbf{G}^T\text{sgn}(\mathbf{e}_{k+1}) + 2\beta(\mathbf{m}_{k+1} - \mathbf{m}_k) \ , \tag{6}$$

where $\mathbf{e}_{k+1} = (\mathbf{d} - \mathbf{Gm}_{k+1})$ and sgn(·) denotes the sign function. Setting eq. (6) equal to zero, we obtain:

$$\mathbf{m}_{k+1} - \mathbf{m}_k = (1/2\beta)\mathbf{G}^T\text{sgn}(\mathbf{e}_{k+1}) \ . \tag{7}$$

Substituting eq. (7) into the constraint condition eq. (4) and selecting the equal sign only, we obtain:

$$1/2\beta = \surd(\delta^2 - \gamma^2)/\surd\{\text{sgn}^T(\mathbf{e}_{k+1})\mathbf{GG}^T\text{sgn}(\mathbf{e}_{k+1})\} \ . \tag{8}$$

Substituting eq. (8) into eq. (7), the updated equation for the unknown parameter is then:

$$\mathbf{m}_{k+1} = \mathbf{m}_k + \surd(\delta^2 - \gamma^2)\, \mathbf{G}^T\text{sgn}(\mathbf{e}_{k+1})/\surd\{\text{sgn}^T(\mathbf{e}_{k+1})\mathbf{GG}^T\text{sgn}(\mathbf{e}_{k+1})\} \ . \tag{9}$$

Because the *a posteriori* error vector $\mathbf{e}_{k+1}$ depends on $\mathbf{m}_{k+1}$, which is not accessible before the current update, it is reasonable to approximate this vector $\mathbf{e}_{k+1}$ with the *a priori* error vector $\mathbf{e}_k$. $\surd(\delta^2 - \gamma^2)$ has a similar purpose as the step-size parameter in conventional iterative algorithms; therefore, it can be replaced by a step-size parameter μ_k. To avoid the instability caused by the ill-posed of forward model **G**, we add a small positive regularization parameter ε in the denominator. Then, eq. (9) can be rewritten as:

$$\mathbf{m}_{k+1} = \mathbf{m}_k + \mu_k[\mathbf{G}^T\text{sgn}(\mathbf{e}_{k+1})/\sqrt{\{\text{sgn}^T(\mathbf{e}_k)\mathbf{G}\mathbf{G}^T\text{sgn}(\mathbf{e}_k) + \varepsilon\}} \quad . \tag{10}$$

Because μ_k comes from the minimum disturbance constraint δ, we should choose $0 < \mu_k < 1$ to ensure the stability of the algorithm and to achieve a small steady-state error. Considering only the sign of the iterative error is needed in the updating process and the denominator of eq. (10) can be seen as the normalization of the molecules $\mathbf{G}^T\text{sgn}(\mathbf{e}_k)$, the novel algorithm can be named as a normalized sign gradient algorithm.

As shown in eq. (10), compared with the IRLS method, the proposed NSGA only updates the sign operation of the error vector between the observed and modeled data in each iteration, without calculating the weighting matrix. Determining the weighting matrix is time-consuming and may bring instability by infinite weighting at the zero point. Therefore, it is easier to implement than the traditional IRLS method.

VSS-NSGA for AVO inverse problem

Although the proposed NSGA is more robust and easier to implement than the traditional IRLS algorithm, the fixed step size μ_k will cause a conflict between the convergence rate and steady-state error. A large step size will cause a fast convergence but result in larger steady-state error; the results are reversed when a small step size is used. Therefore, a variable step size algorithm is proposed for the NSGA in this section to speed up the convergence rate and lower the steady-state error simultaneously.

Let the model error vector $\hat{\mathbf{m}}_{k+1} = \mathbf{m}^* - \mathbf{m}_{k+1}$, where $\mathbf{m}^*$ is the optimal solution that we expect to estimate. Then, by eq. (10), we have

$$\hat{\mathbf{m}}_{k+1} = \hat{\mathbf{m}}_k - \mu_k[\mathbf{G}^T\text{sgn}(\mathbf{e}_k)]/\sqrt{\{\text{sgn}^T(\mathbf{e}_k)\mathbf{G}\mathbf{G}^T\text{sgn}(\mathbf{e}_k) + \delta M\}} \quad . \tag{11}$$

By taking the expectations after squaring both sides of (11), the update recursion of mean-square deviation (MSD) is derived as

$$\begin{aligned} E[\|\hat{\mathbf{m}}_{k+1}\|_2^2] &= E[\|\hat{\mathbf{m}}_k\|_2^2] \\ &\quad - 2\mu_k E[\text{sgn}(\mathbf{e}_k^T)\mathbf{G}\hat{\mathbf{m}}_k/\sqrt{\{\text{sgn}^T(\mathbf{e}_k)\mathbf{G}\mathbf{G}^T\text{sgn}(\mathbf{e}_k) + \delta M\}}] + \mu_k^2 \quad . \end{aligned} \tag{12}$$

Using eq. (1), the error of the k-th iteration can be expressed as

$$\mathbf{e}_k = \mathbf{d} - \mathbf{G}\mathbf{m}_k = \mathbf{G}\mathbf{m}^* + \mathbf{n} - \mathbf{G}\mathbf{m}_k$$

$$= \mathbf{G}(\mathbf{m}^* - \mathbf{m}_k) + \mathbf{n} = \mathbf{G}\hat{\mathbf{m}}_k + \mathbf{n} \quad . \tag{13}$$

Then,

$$\mathbf{G}\hat{\mathbf{m}}_k = \mathbf{e}_k - \mathbf{n} \quad . \tag{14}$$

Eq. (12) can be rewritten as

$$E[\|\hat{\mathbf{m}}_{k+1}\|_2^2] = E[\|\hat{\mathbf{m}}_k\|_2^2] - 2\mu_k E[\|\mathbf{e}_k\|_1 \mathrm{sgn}(\mathbf{e}_k^T)\mathbf{n}]/\sqrt{\{\mathrm{sgn}^T(\mathbf{e}_k)\mathbf{G}\mathbf{G}^T\mathrm{sgn}(\mathbf{e}_k) + \delta M\}} + \mu_k^2 \tag{15}$$

Supposing that the iterative error $\mathbf{e}_k$ is uncorrelated with the background noise $\mathbf{n}$, we obtain $E[\mathrm{sgn}(\mathbf{e}_k^T)\mathbf{n}] = 0$. Eq. (15) can be written as:

$$E[\|\hat{\mathbf{m}}_{k+1}\|_2^2] = E[\|\hat{\mathbf{m}}_k\|_2^2] - 2\mu_k E[\|\mathbf{e}_k\|_1/\sqrt{\{\mathrm{sgn}^T(\mathbf{e}_k)\mathbf{G}\mathbf{G}^T\mathrm{sgn}(\mathbf{e}_k) + \delta M\}}] + \mu_k^2 \quad . \tag{16}$$

Taking the derivative of the cost function (16) in respect to the step-size μ_k and equating it to zero, we obtain the optimum solution of μ_k,

$$\mu_k^* = E[\|\mathbf{e}_k\|_1/\sqrt{\{\mathrm{sgn}^T(\mathbf{e}_k)\mathbf{G}\mathbf{G}^T\mathrm{sgn}(\mathbf{e}_k) + \delta M\}}] \quad . \tag{17}$$

To improve the robustness of the algorithm, we propose that μ_k is to be calculated recursively by time-averaging, as follows:

$$\mu_k = \alpha\mu_{k-1} + (1-\alpha)\min([\|\mathbf{e}_k\|_1/\sqrt{\{\mathrm{sgn}^T(\mathbf{e}_k)\mathbf{G}\mathbf{G}^T\mathrm{sgn}(\mathbf{e}_k) + \delta M\}}], \mu_{k-1}) \quad , \tag{18}$$

where $0 \leq \alpha \leq 1$ is the smoothing factor.

Next, the Lyapunov stable condition is employed to guarantee the stability of the proposed VSS-NSGA (Tanaka and Sugeno, 1992). The Lyapunov theorem is defined as an energy-like Lyapunov function, and the theorem states that the system is asymptotically stable when the Lyapunov function is convergent.

Based on eq. (16), the VSS-NSGA will satisfy the Lyapunov stable condition $E[\|\hat{\mathbf{m}}_{k+1}\|_2^2] \leq E[\|\hat{\mathbf{m}}_k\|_2^2]$ when and only when

$$0 \leq \mu_k \leq 2E[\|\mathbf{e}_k\|_1/\sqrt{\{\mathrm{sgn}^T(\mathbf{e}_k)\mathbf{G}\mathbf{G}^T\mathrm{sgn}(\mathbf{e}_k) + \delta M\}}] \quad . \tag{19}$$

Considering eq. (18), it is obvious that

$$0 \leq \mu_k \leq \mu_k^* \leq 2E[\|e_k\|_1/\sqrt{\{sgn^T(e_k)GG^Tsgn(e_k) + \delta M\}}] \ , \ 0 \leq \alpha \leq 1 \quad (20)$$

which satisfies the Lyapunov stable condition.

In conclusion, the VSS-NSGA for AVO inversion can be concluded in Table 1.

Table 1. Pseudo-code for VSS-NSGA.

Input: seismic data $\mathbf{d}$, forward model $\mathbf{G}$

Output: to be estimated earth parameters $\mathbf{m}$

Initialization: the initial point $\mathbf{m}_0$, the initial step-size μ_0, the termination condition η and a small positive regularization parameter ε

while $\|\mathbf{e}_{k+1} - \mathbf{e}_k\|_2^2 \leq \eta$, do

$\mathbf{e}_k = \mathbf{d} - \mathbf{G}\mathbf{m}_k$,

$\mathbf{g}_k = \mathbf{G}^T sgn(\mathbf{e}_k)$,

$\mu_k = \alpha\mu_{k-1} + (1-\alpha)\min\left(\|\mathbf{e}_k\|_1/\sqrt{\{\|\mathbf{g}_k\|_2^2 + \delta M\}}, \mu_{k-1}\right)$,

$\mathbf{m}_{k+1} = \mathbf{m}_k + \mu_k[\mathbf{g}_k/\sqrt{\{\|\mathbf{g}_k\|_2^2 + \delta M\}}]$,

end while

APPLICATION TO AVO INVERSION

In this section, we test our algorithm using the noisy synthetic data and the field data. Our goal is to demonstrate that the proposed VSS-NSGA provides a more robust estimate of the model parameters with lower computational costs when the data are contaminated by impulse noise and has the feasibility of usage in the seismic inversion.

Synthetic data examples

To assess the performance of the proposed algorithm, the time series reflectivity is constructed from the P-wave velocity, S-wave velocity and density shown in Figs. 1(a)-1(c). Then, the synthetic data are generated by convolving the reflectivity series with a 40 Hz Ricker wavelet. The generated synthetic data are shown in Fig. 1(d). The data consists of 60 traces with angles ranging from 0° to 32.56° and depths from approximately 5800 to 6500 m.

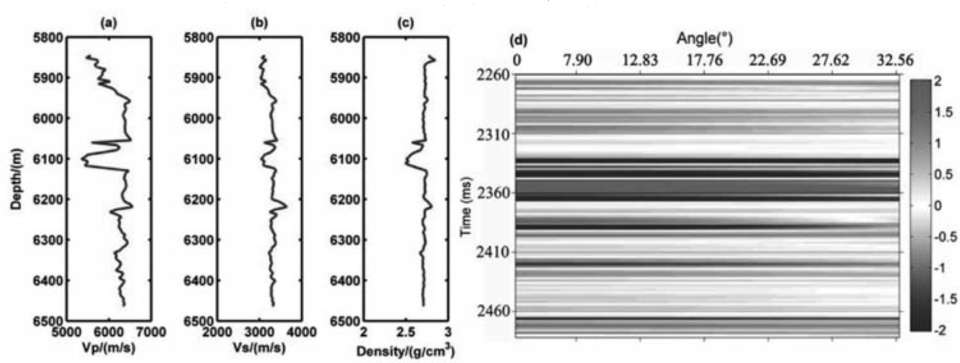

Fig. 1. The real logging data and the corresponding synthetic seismic data.

Fig. 2 shows the "contaminated" CMP gather that is strongly affected by a strong Bernoulli-Gaussian interference signal with a signal to noise ratio (SNR) of five and Pr = 0.05. The Bernoulli-Gaussian (BG) distribution here is used for modeling the interference signal. This signal is generated as the product of a Bernoulli process and a Gaussian process such that n(k) = B(k)G(k), where

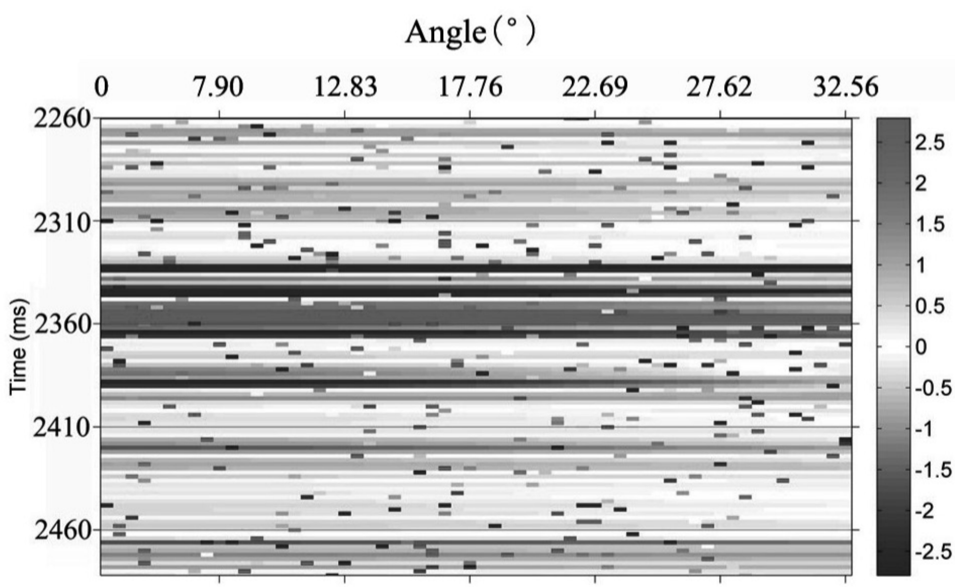

Fig. 2. The "contaminated" synthetic data affected by a strong Bernoulli-Gaussian interference signal with SIR = 5 and Pr = 0.05.

G(k) is a white Gaussian random sequence with a mean of zero and variance δ_n^2, and B(k) is a Bernoulli process with the probability mass function given as P(B) = 1 − Pr for B = 0, and P(B) = Pr for B = 1, where Pr is a probability that satisfies values of 0 < Pr ≤ 1. The average power of a BG process is $Pr \cdot \delta_n^2$. Keeping the average power constant, a BG process is spikier when Pr is smaller. It reduces to a Gaussian process when Pr = 1.

Fig. 3 shows the corresponding inversion results of the IRLS, NSGA with $\mu = 0.1$, NSGA with $\mu = 0.05$ and the proposed VSS-NSGA with initial step-size $\mu_0 = 0.1$. The true models are shown in blue, the initial models that are obtained by a 30 Hz low-pass filter of the true models are shown in green,

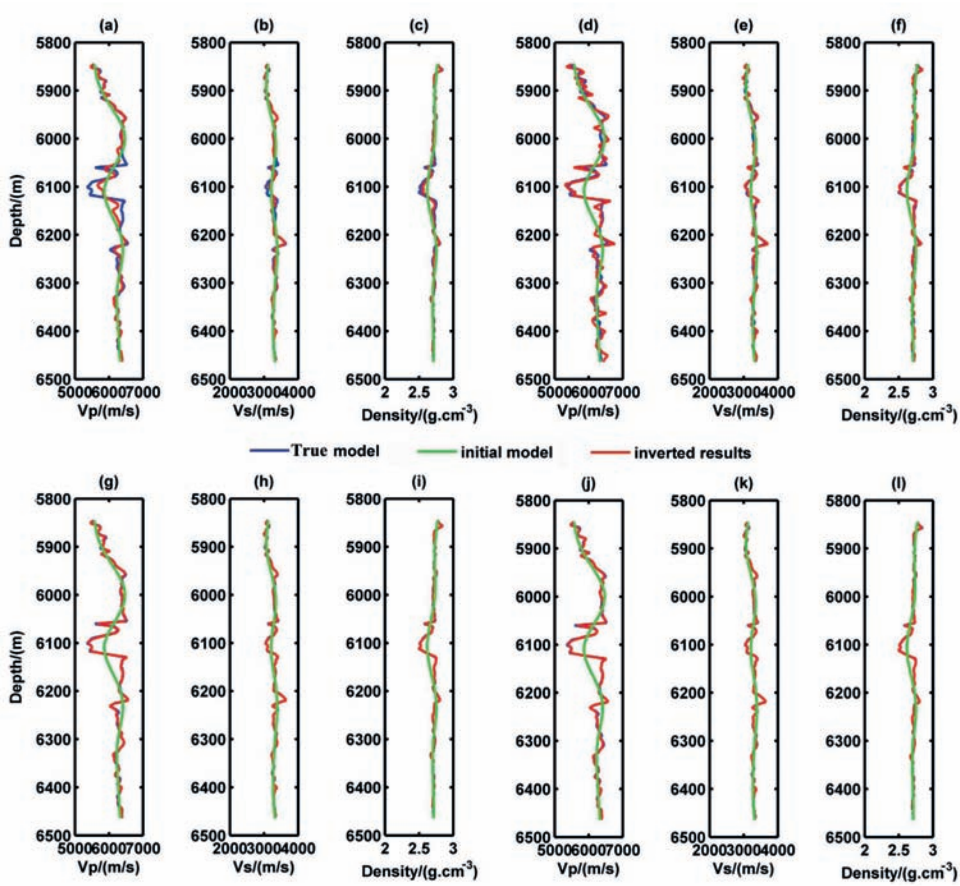

Fig. 3. The inversion results of the noisy synthetic data. (a)-(c) is retrieved by the IRLS algorithm, (d)-(f) is retrieved by the proposed NSGA with $\mu = 0.1$, (g)-(i) is retrieved by the proposed NSGA with $\mu = 0.05$, and (j)-(l) is retrieved by the proposed VSS-NSGA. The red bold line is the inverse result, the green line is the initial model, and the blue bold line is the true model.

and the inverted results are shown in red. We can see that the inversion results of the proposed NSGA and VSS-NSGA are both similar with the truth, whereas the traditional IRLS algorithm can hardly return a reliable inversion result. Fig. 4 shows the difference between the input data and the predicted data that shows the degree of data fitting. Fig. 5 shows the convergence comparison of these methods. Fig. 4 indicates that the VSS-NSGA results in the best degree of the data fitting. The NSGA with $\mu = 0.05$ also reveals a similar degree of data fitting, although it is slower than the VSS-NSGA, as is shown in Fig. 5 (the vertical axis represents the mean square error). The convergence curves also indicate that the proposed VSS-NSGA bears two advantages, faster convergence speed and lower steady error than conventional IRLS algorithm, as well as the fixed step-size NSGA.

To compare the execution efficiency of the IRLS, NSGA and VSS-NSGA, CPU time is used as an index of complexity, although it gives only a rough estimation of complexity. Our simulations are performed in a MATLAB 7.11.0 environment using an AMD Athlon (tm) 64 X2 Dual Core Processor 4400+, 2.3-GHz processor with 2-GB of memory on a Microsoft Windows XP operating system. The final average CPU times (of total ten times, in seconds) are listed in Table 2. μ_0 is the initial value of the step-size of the VSS-NSGA. It can be seen that the proposed NSGA and VSS-NSGA are much faster than the traditional IRLS method.

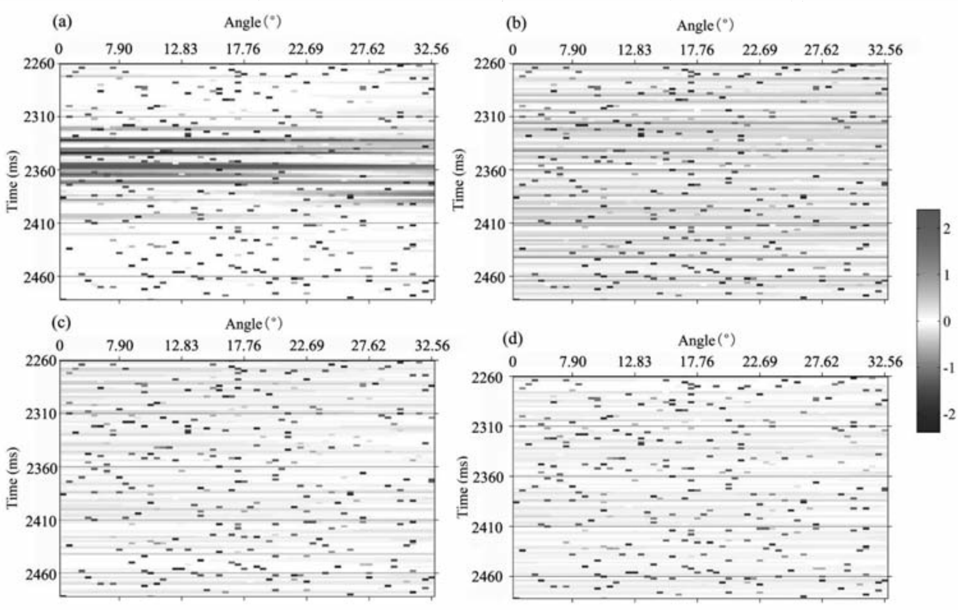

Fig. 4. The difference between the input data (with outliers) and the predicted data using (a) IRLS algorithm, (b) NSGA with $\mu = 0.1$, (c) NSGA with $\mu = 0.05$, and (d) the proposed VSS-NSGA, respectively.

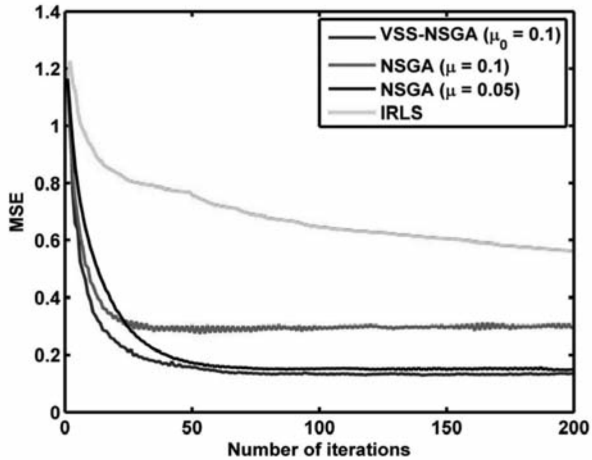

Fig. 5. The convergence performances comparison of the IRLS algorithm, NSGA with $\mu = 0.1$, NSGA with $\mu = 0.05$ and VSS-NSGA in the Bernoulli-Gaussian interference.

Table 2. CPU time comparison of the IRLS, NSGA and VSS-NSGA.

Algorithms	Average CPU times (in seconds)
IRLS	19.49
NSGA ($\mu = 0.1$)	4.90
NSGA ($\mu = 0.05$)	4.89
VSS-NSGA ($\mu_0 = 0.05$)	5.13

Field data example

We now test our algorithm with a field data example. As the reef reservoirs in Western China are usually deeply buried (approximately 6000-7500 m underground), it is difficult to have high amplitude fidelity of seismic signals. Because of the complex acquisition conditions and the disorder reflection characteristics of the reef reservoir, the SNR of the seismic data is very low, especially of the pre-stack seismic data. The low SNR of the seismic data is one of the main factors that hinder reservoir prediction. Fig. 6 is the actual stack section of a reef reservoir in a work area in western China, where there is a drilling well in the 1060-th CMP and a reef reservoir at 2.43 seconds, 6100 m, with daily gas production of 500×10^3 m^3.

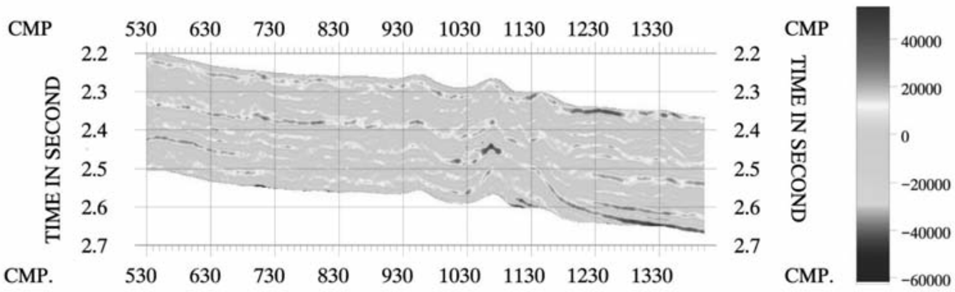

Fig. 6. The stack section of real seismic data.

We obtain the wavelet and the background model using the commercial software "Deliver". Fig. 7 is the P-wave velocity profile, S-wave velocity profile, density profile and Poisson's ratio profile retrieved using the proposed algorithm. We can clearly see that the Poisson's ratio of the reef reservoir is lower than the Poisson's ratio (the rectangle highlight areas) of the surrounding rock that agree well with the drilling results. Therefore, the proposed inverse algorithm for AVO inversion is feasible using in the real seismic data.

CONCLUSION

Because geophysical inverse problems are often ill-posed and sensitive to noise, l_1 norm shows considerably less sensitivity to outliers and yields far more stable model estimations than the classical least-squares (l_2 norm) method. However, the singularity of the l_1 norm leads to instability of IRLS solution in numerical minimization. A novel inversion algorithm called the VSS-NSGA is proposed in this paper. The proposed algorithm is obtained by minimizing the l_1 norm of the *a posteriori* error vector with a minimum disturbance constraint, ensuring that the two adjacent solutions do not change greatly. Compared with the conventional l_1 norm-based IRLS inversion algorithm, the VSS-NSGA reduces computational complexity and averts the instability brought by infinite weighting at the zero point. Furthermore, the variable step size leads to a faster convergence rate and a smaller steady-state error than those of fixed step size. Synthetic and field seismic data tests show that the proposed VSS-NSGA performs better than the conventional l_1 norm-based IRLS algorithm in both the robustness and the convergence performance.

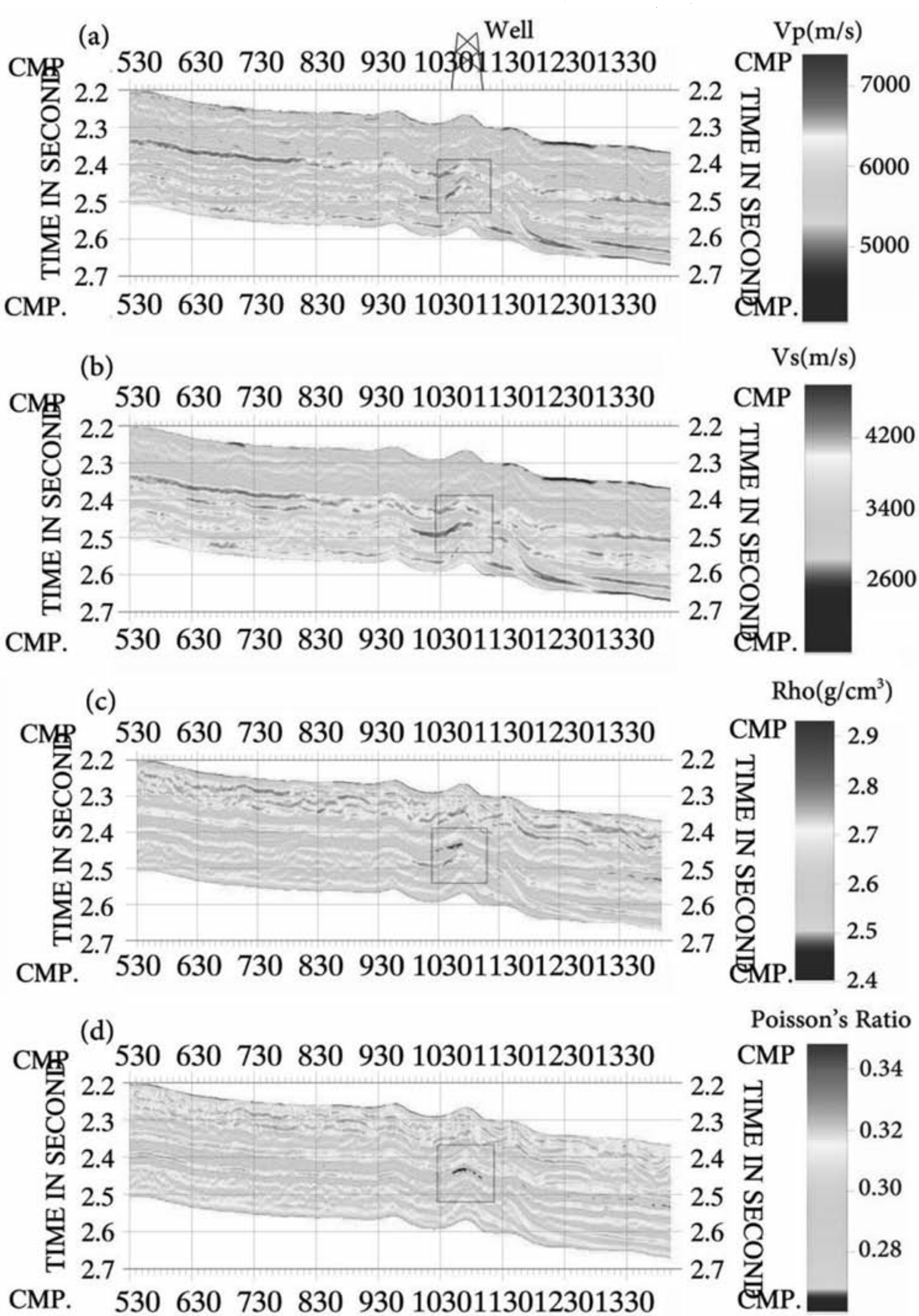

Fig. 7. Inversion results of (a) P-wave velocity, (b) S-wave velocity, (c) density and (d) Poisson's ratio. The rectangle highlights the reef reservoir in this work area.

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