SEISMIC DATA INTERPOLATION WITH CURVELET DOMAIN SPARSE CONSTRAINED INVERSION

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ABSTRACT

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To improve the accuracy of seismic data processing, the irregular, missing seismic data need interpolation. Retrieving the missing data can decrease the error in the prediction of multiple and the aliasing of image etc. The iterative threshold method based on l_1 norm optimization inversion with sparsity constraint can achieve better result with increase of the sparsity of the inversion parameters. Take the advantage of the sparse representation of seismic data in the Curvelet domain, we can get a better result when performing a l_1 norm optimization problem in seismic data analysis. In this paper, we apply this method in interpolation of missing seismic data, and using the Curvelet threshold iterative method after NMO correction, which make the data sparser. By comparing the results of the interpolation with and without NMO correction, we confirm that the Curvelet threshold iterative method can get a better effect in seismic missing data interpolation. Its results are more close to the initial model as well after the process of NMO correction.

KEY WORDS: Curvelet transform, interpolation, threshold iteration, NMO correction.

INTRODUCTION

Seismic data processing requires the density and the regularity of the seismic data itself. However, data acquisition is affected by the geological environment of the field and the economic condition. The acquisition geometry grid of the seismic data are often irregular and not complete (Liu et al., 2011). Moreover, the irregular, discrete and missing seismic data may increase additional alias in multiple prediction and imaging. The seismic data interpolation is required to handle these issues. The interpolation of seismic data

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plays an important role in seismic data processing and interpretation such as multiple attenuations, wave equation migration, and seismic data regularization. The interpolation of the missing data can make the seismic data more accurate and regular under the circumstance of the increasing of the signal to noise ratio, which establishes a better foundation for the further steps. In recent years, seismic data regularization has become increasingly popular, the scholars propose many effective methods, such as the largest coherent dip interpolation, the tilted coherent power spectrum difference interpolation, the prediction filtering interpolation in f-x domain (Spitz, 1991) and the Curvelet threshold iteration combined with focal transform (Felix et al., 2008a,b).

The Curvelet transform is widely used in seismic data processing and interpretation in the last few years (Felix et al., 2008a,b; Wang et al., 2008). Curvelet transform has the property of multi-scale, multi-direction and locality, and has significant advantage compared with wavelet transform and Ridgelets transform in solving sparsity constrained norm optimization problems. The application of the Curvelet transform, within recent years, has developed into every field from computer image processing to seismic data processing and interpretation. Actually, this mathematical transform method can be used not only in the attenuation of the multiple (the correlated noise) and random noise but also in the area of the interpolation and migration of seismic data processing and interpretation. The Curvelet transform is kind of anisotropic wavelet transform, whit the multi-scale and multi-angles properties. It's primary function can make a sparse representation of graphics with segmented smooth edge. It takes the advantage of these properties to produce an optimal decomposition of the seismic reflection events. The superiority of the Curvelet transform in that it can show the feature of the function with two or more dimension, which can achieve the optimal result handling the singularity of the curve of seismic travel time. In this article, we applied the Curvelet threshold iteration method into missing trace interpolation of seismic data. More important, we conduct the interpolation process for pre-stack missing trace after NMO correction, which make the data sparser and got better results with l_1 norm optimization.

BASIC THEORY

The basic principles of interpolation

In brief, the interpolation is to fit a curve with series discrete point control. The function indicating the curve can be called the interpolation function. We can find the function value of other points besides these finite numbers of points. In seismic data interpolation, this means we can get the missing seismic data from the known traces. We assume that the array of the given sampling point is $\{m_i, i = 1,2,3,\ldots,N\}$, here $m_1,m_2,m_3,\ldots m_N$ is called the interpolation nodes. By inserting some sampling points into this array, it make the data become a Q sampling sequence $\{n_j, j = 1,2,3,\ldots Q\}$, and $Q \geq N$. The order of the sampling points we get from the field is usually with equal distance. The interpolation process can be shown as following (Chen et al., 2005):

$$y(m) = \sum_{n=0}^{N-1} C_N h(m - m_n) \quad , \tag{1}$$

where h is the core of the interpolation, C_N is the weight characteristic. Usually function h(m) is a symmetric function, we can evaluate the character of the function in the frequency domain. Generally, when we interpolate the missing seismic data, the simplest way is to interpolate between two sampling points, namely the linear interpolation, as can be shown in the form of the interpolation:

$$y(m_i) = y_0 + [(m_i - m_0)/(m_1 - m_0)](y_1 - y_0) \quad . \tag{2}$$

There are many interpolation methods for the missing trace of the seismic data in practice, such as the largest coherent dip interpolation, SINC function interpolation, prediction filtering interpolation in the f-x domain and the tilt coherent power spectrum difference interpolation, etc. (Zhang, 2010). Normally these methods can get a satisfactory result when the missing data has high quality (with high signal to noise ratio , with small gap, etc.), otherwise these methods does not work well.

Seismic data interpolation with Curvelet sparsity-promotion inversion

The Curvelet transform has the feature of the multi-scale, multi-direction and locality. It has significant advantages compared with the wavelet transform and the Ridgelet transform in missing traces interpolation. The Curvelet transform is a kind of anisotropic wavelet, with multi-scale and multi-angle properties. Differing from wavelet transform, the Curvelet transform also includes direction parameters. By taking the advantage of these properties it can produce a more sparse representation of the seismic reflection events than traditional Fourier and wavelet transform.

The Curvelet transform possesses the special capacity that it can focus the seismic data energy on the fraction of the Curvelet coefficient. Generally, more signal energy concentrates on the area with the larger Curvelet coefficient, while the noise spreads into the area with the smaller one instead. Consequently, we can remove the noise by thresholding the original data with a small value in the Curvelet domain, and take an inverse Curvelet transform. The Curvelet transform has the feature of the sparse representation for the seismic data, therefore, we can indicate the noise attenuation problem of seismic data under the sparse constrained condition using the following constrained optimal problem (Wang et al., 2010):

$$P_\varepsilon : \begin{cases} \tilde{x} = \arg\min_x \| x \|_1 = \sum_{i-1}^{N} |x_i| \quad \text{st.} \quad \| Ax - y \|_2 \leq \varepsilon \\ \tilde{f} = A\tilde{x} \end{cases} , \qquad (3)$$

where y represents the initial seismic data, x is the Curvelet coefficient, and $A = C^T$, C is the forward Curvelet transform and C^T is the inverse Curvelet transform, ε is the value that can be infinitely small, and $\tilde{f}$ is the final output result. The solution of this optimization problem is to invert the Curvelet coefficient with the smallest l_1 norm.

The Curvelet threshold method combined with the l_1 norm optimization problem can enhance the signal and suppress random noise, which is its major advantage for the interpolation by using the coherence of the seismic signal between the data and the missing gap. As a result, we can achieve basic principle of the iterative Curvelet thresholding algorithm for missing trace interpolation. We combine the Curvelet transform with the threshold iteration method and add a missing operator in the optimization problem discussed above. This approach of missing trace interpolation can be represented as solving the optimization problem of l_1 norm as the following (Tong et al., 2009):

$$P_\varepsilon : \begin{cases} \tilde{x} = \arg\min_x \| x \|_1 = \sum_{i-1}^{N} |x_i| \quad \text{st.} \quad \| Ax - y \|_2 \leq \varepsilon \\ \tilde{f} = C^T\tilde{x} \end{cases} . \qquad (4)$$

In eq. (4), y is the original missing seismic data, $A = RC^T$, C^T is the inverse Curvelet transformation, R is the missing operator, which function is to take out the traces according to original missing data. Where x represent the Curvelet coefficient, ε is an arbitrarily small positive quantity, and $\tilde{f}$ is the interpolated seismic data.

The l_1 norm optimization problem of P_ε described above can be substituted with a series of some simple optimization problems:

$$P_\lambda : \begin{cases} \tilde{x}_\lambda = \arg\min \| y - Ax \|_2^2 + \lambda \| x \|_1 \\ \tilde{f}_\lambda = C^T x_\lambda \end{cases} . \qquad (5)$$

The solution of P_ε is reached by solving P_λ by descending λ starting from $\lambda_\varepsilon = \sup_\lambda\{\lambda: \| y - A\tilde{x}_\lambda \|_2\} \leq \varepsilon$, and where sup{} denotes the upper limit function (Wang et al., 2010).

The optimization problem of the l_1 norm as given in eq. (5) can be solved by the threshold iteration method. To solve this problem, we need to depend on the inversion process on the smallest l_1 morn of the Curvelet coefficient, which satisfies the condition that close to the primary data for the interpolation under the circumstance of the 2-norm restraint, by removing the missing data with the R operator after the inverse Curvelet transform. $\tilde{f}_\lambda$, given in eq. (5) is the final output, which represents the interpolated data.

Interpolation with the precondition of NMO correction

The Curvelet transform has a better directional characteristic due to the multi-scale principle, in addition to the scale and location. The structural element is consisted of directional parameter. Since the Curvelet function has better property of direction selection and anisotropy, it has a preferable sparsity. The reconstruction error in the Curvelet domain will decrease if the data has a better sparsity. In this paper we apply NMO correction to get a better sparsity for the data to be interpolated. Fig. 1 shows the comparison of the reconstruction error with and without NMO correction for original data, and with 25%, 50% and 75% missing traces, respectively.

(a) Reconstruction error for the original no missing traces data

(b) Reconstruction error with 25% missing traces

(c) Reconstruction error with 50% missing traces

(d) Reconstruction error with 75% missing traces

Basically, NMO correction can make a hyperbolic travel time curve become a straight line. That means in Curvelet domain we can use smaller number of Curvelet coefficients to represent a straight event than that of a hyperbolic event. After the NMO correction, the CMP gather's sparsity has been enhanced in Curvelet domain as shown in Fig. 1.

In practice, we use a larger NMO velocity to avoid the effect of NMO stretch in shallow and large offset area, the NMO velocity used is usually larger than the real NMO velocity for each even, that means a deficient NMO correction is applied.

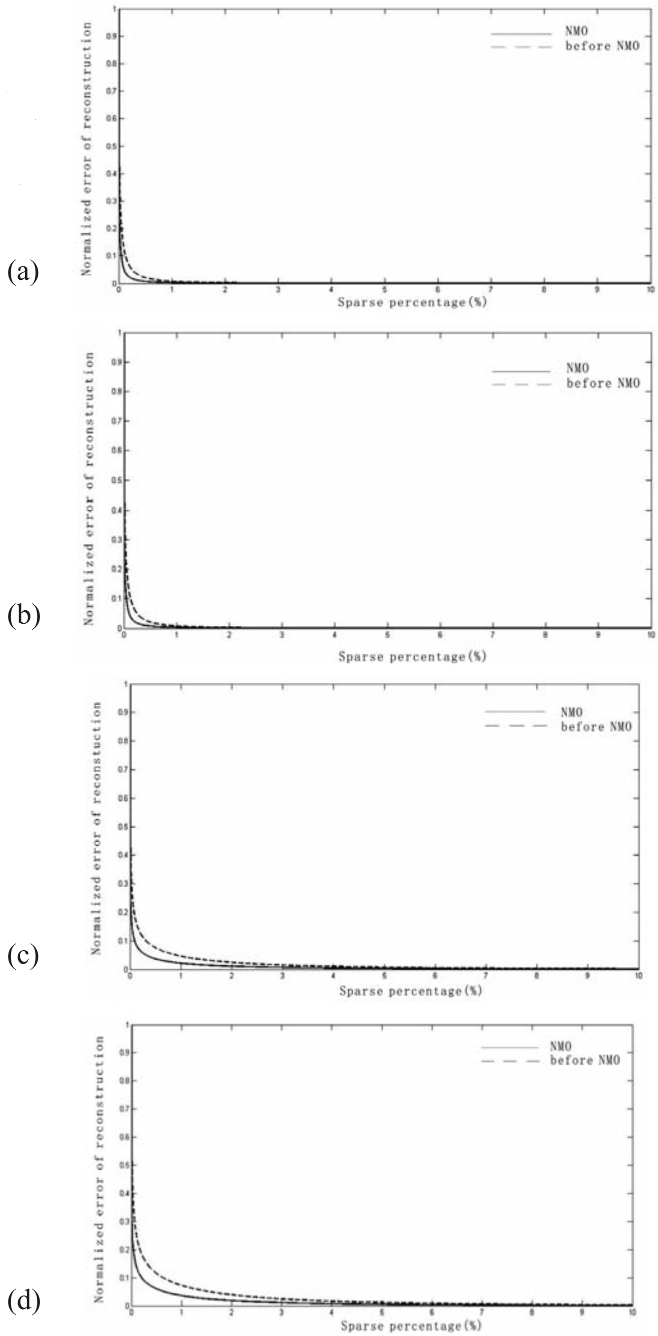

Fig. 1. The comparison of reconstruction error in Curvelet domain with and without NMO correction for (a) original data, (b) with 25%, (c) with 50%, (d) with 75% missing traces. The horizontal axis represents the percentage of preserved coefficients and the vertical axis represents the normalized coefficients reconstruction error.

NUMERICAL EXAMPLES

In Fig. 2, (a) is a shot record, Fig. 2 (b), (c) (d), is the data with 25%, 50% and 75% missing traces, respectively. We apply the Curvelet threshold iterative method to the interpolation of these three missing record, the results are shown in Fig. 3.

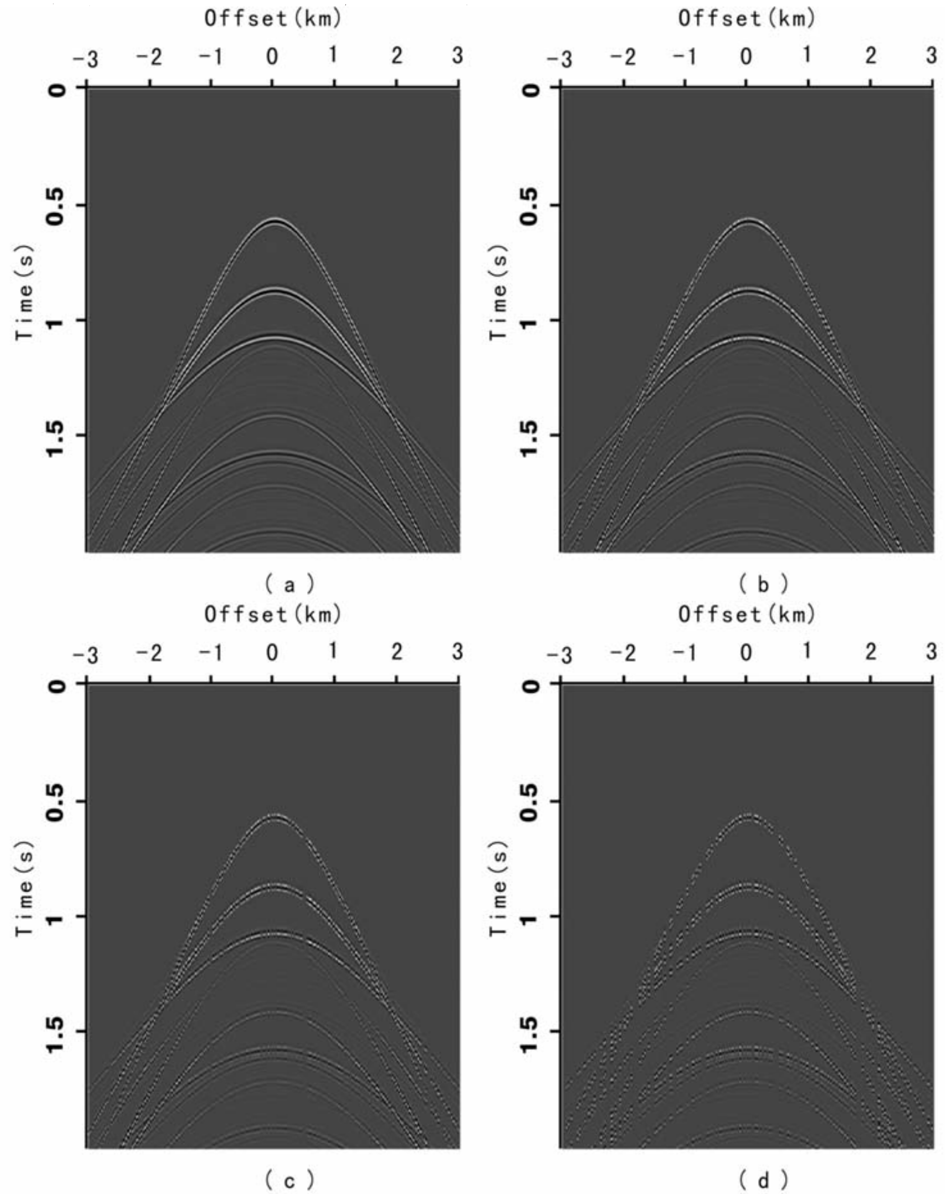

Fig. 2. (a) the original model; (b), (c), (d) data with 25%, 50%, 75% of the traces missing.

Fig. 3 shows that the interpolation result of the data with 25% missing traces is ideal,almost the same as the original model. When the percentage of the missing traces is increased to 50%, the interpolation results are still acceptable. But for the data with 75 missing traces, the missing seismic data in the fan-shaped region cannot be reconstructed completely.

For a homogeneous reflector in isotropic media, the reflection travel time is hyperbolic. Here we interpolate the data after NMO correction and then apply an inverse NMO correction. The comparison results of interpolation with and without NMO correction are shown in Fig. 4, Fig. 5 and Fig. 6. Fig. 7 shows the difference between the model and interpolated results with and without NMO correction for 25%, 50% and 75% missing traces respectively.

Actually, from the comparisons of the interpolation results and the differences, we can find the interpolation results have been significantly improved with NMO correction precondition. A better result can be gained when applying the Curvelet threshold iteration method after NMO correction compared with those without NMO correction according to the differences (shown in Fig. 7), especially in the fan-shaped region where the missing is serious. We apply the proposed method to the field data,the results are shown in Fig. 8, Fig. 9. For the field data we also reconstruct better results by applying the Curvelet iterative thresholding method after NMO correction. The interpolated data is more accurate after NMO correction and is more close to the original data.

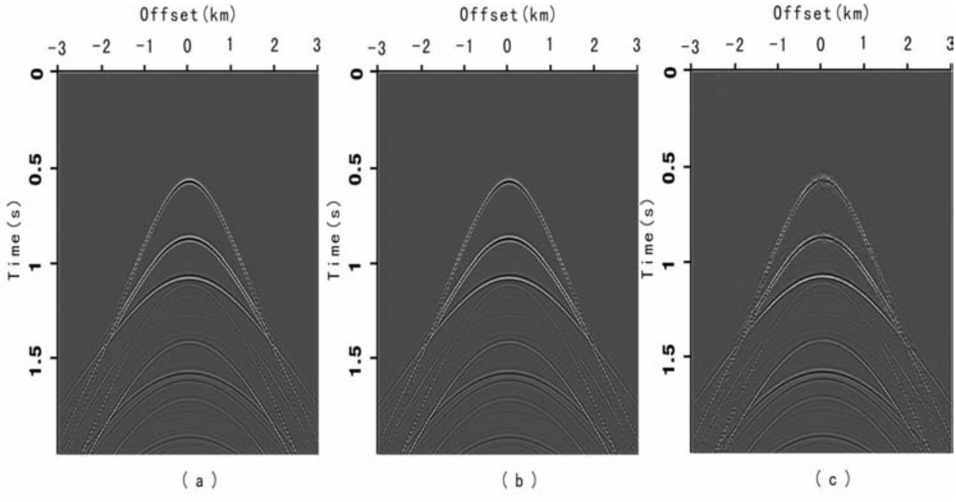

Fig. 3. The interpolation results with Curvelet threshold iterative method for the data with (a) 25%, (b) 50%, (c) 75% missing traces.

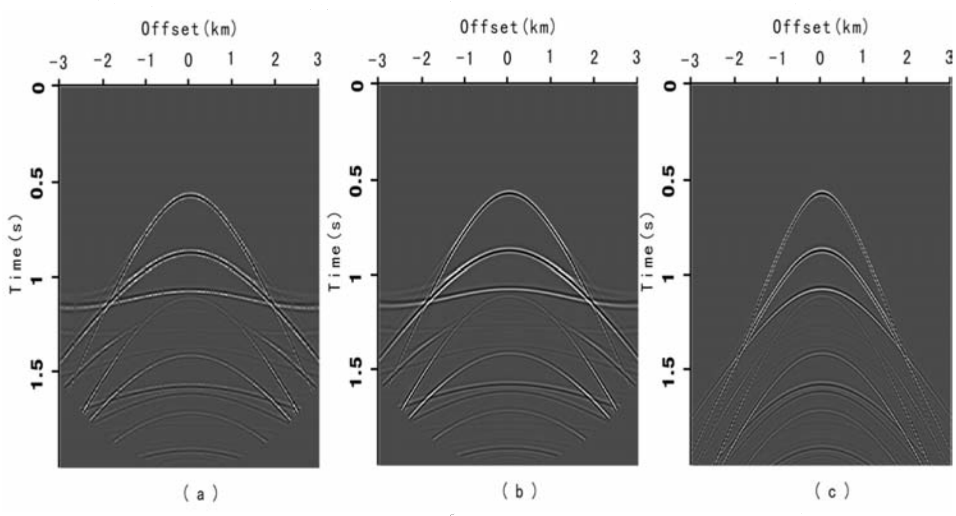

Fig. 4. (a) data with 25% missing traces after NMO correction; (b) data with Curvelet interpolation of (a); (c) the interpolation results after inverse NMO correction of (b).

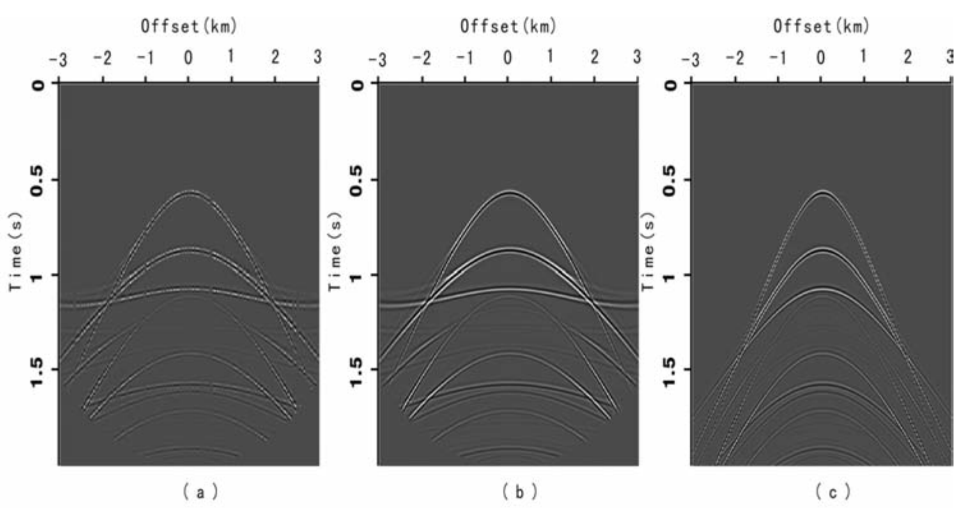

Fig. 5. (a) data with 50% missing traces after NMO correction; (b) data with Curvelet interpolation of (a); (c) the interpolation results after inverse NMO correction of (b).

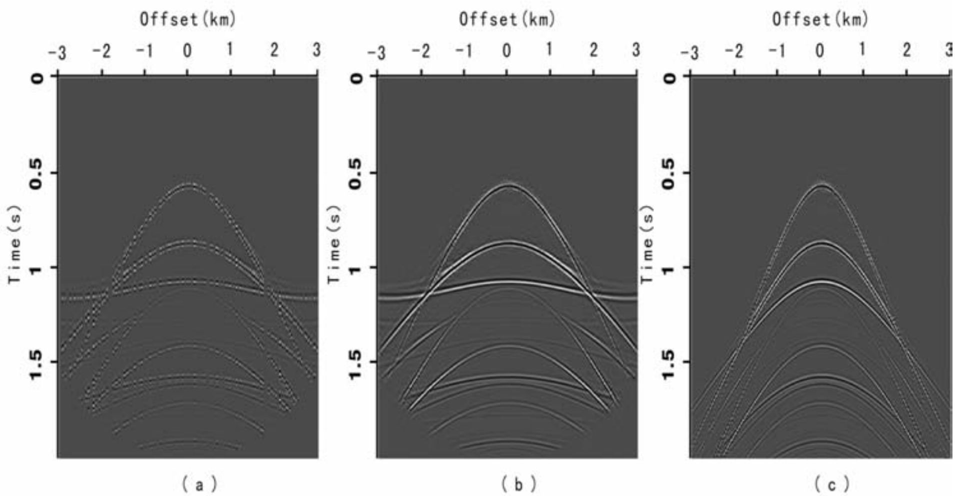

Fig. 6. (a) data with 75% missing traces after NMO correction; (b) data with Curvelet interpolation of (a); (c) the interpolation results after inverse NMO correction of (b).

CONCLUSION

Compared with the wavelet transform and the Fourier transform, the advantages of the Curvelet transform lies in the property of its multi-direction, which can get a better result in solving the feature of the high dimensional function and the Curve singularity. The iterative thresholding method is derived from the sparse constraint optimization inversion, and the interpolation results will be improved as the enhancement of the sparsity of the data to be interpolated. A better result of interpolation can be obtained by combining the Curvelet transform with the threshold iteration method. NMO correction, before the interpolation, can make the missing data sparser in Curvelet domain, and we can get better interpolation results after NMO correction, it can effectively reconstruct the part of the fan-shaped area and the region with large missing traces.

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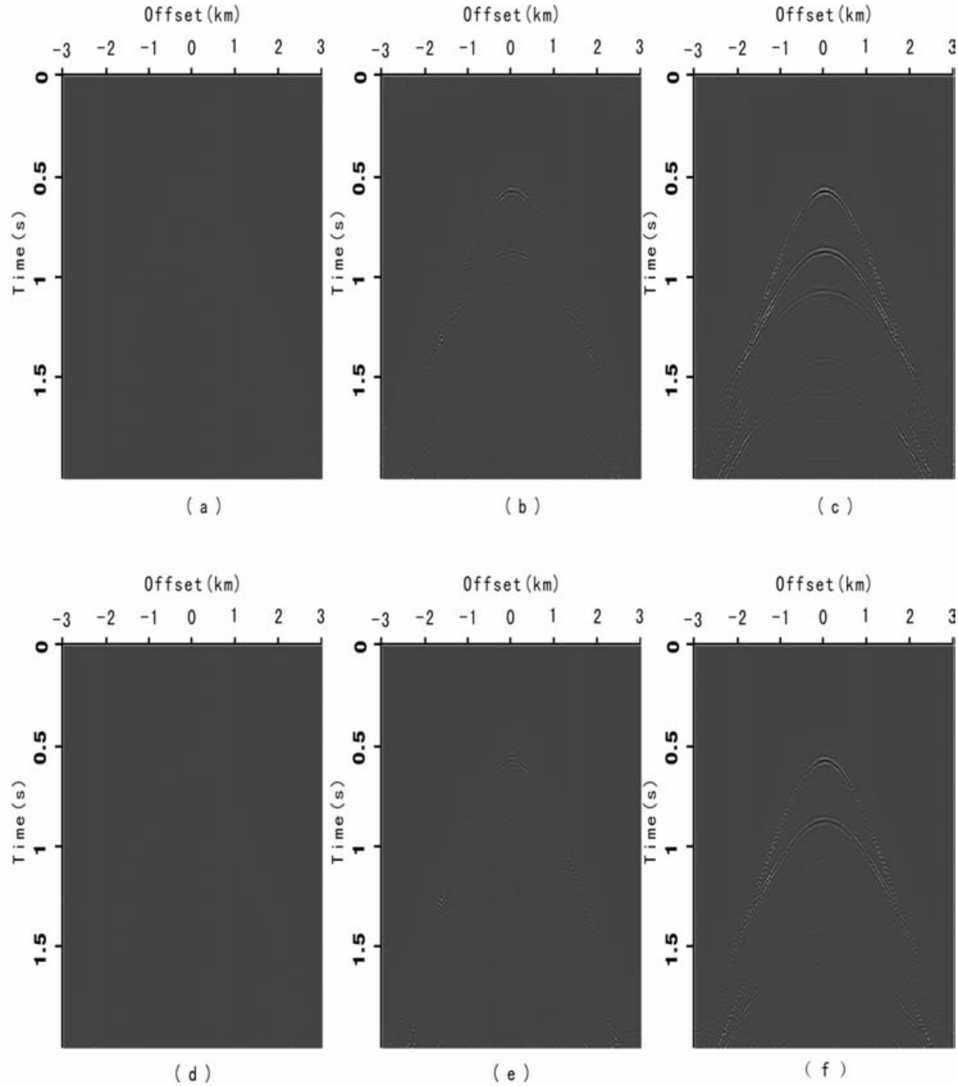

Fig. 7. Difference between the interpolated results and the model. (a), (b), (c) are data with 25%, 50%, 75% missing traces and without NMO correction. (d), (e), (f) are data with 25%, 50%, 75% missing traces and with NMO correction.

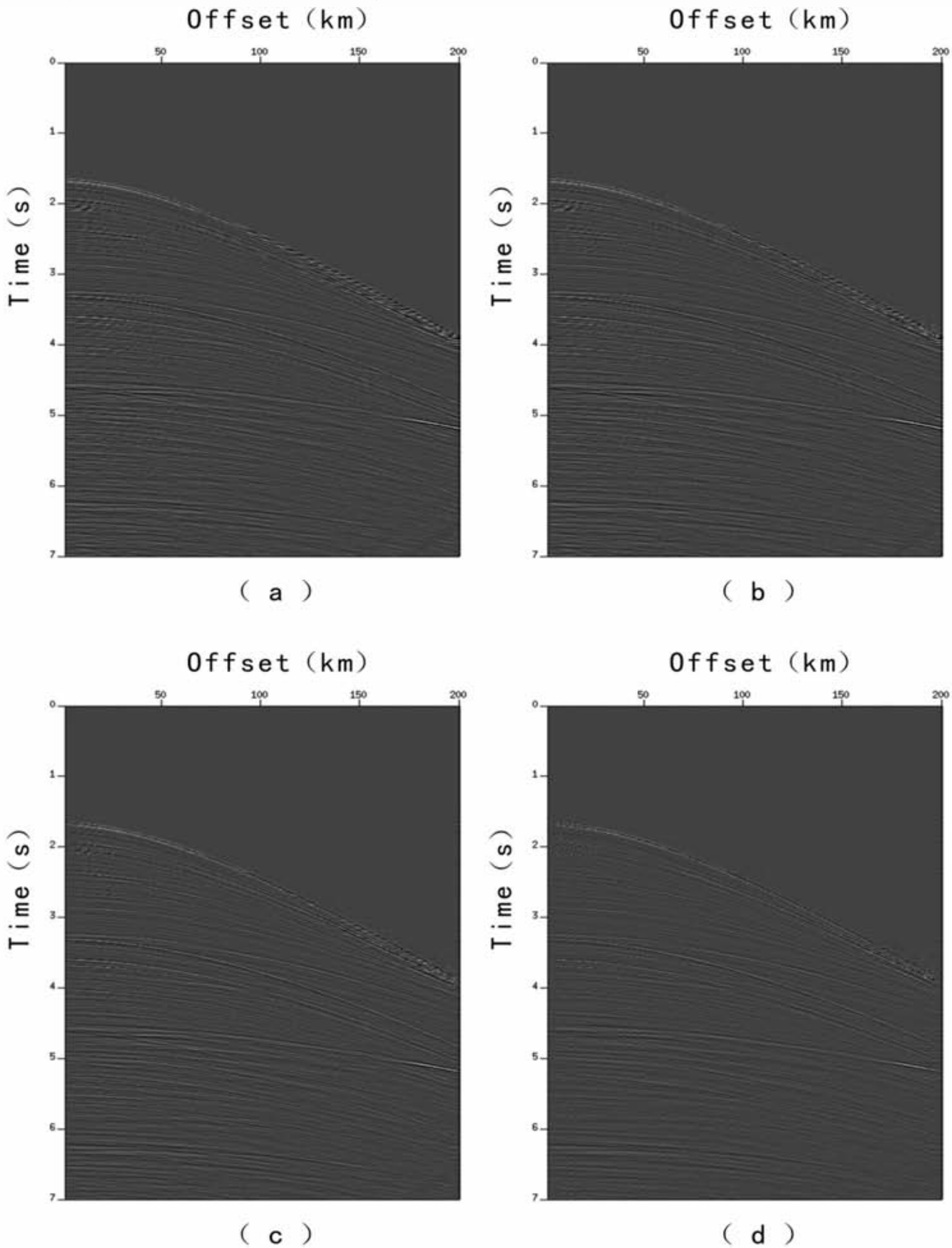

Fig. 8. Field data interpolation results. (a) the original seismic shot data; (b), (c), (d) are interpolated data with 25%, 50%, 75% missing traces and with NMO correction.

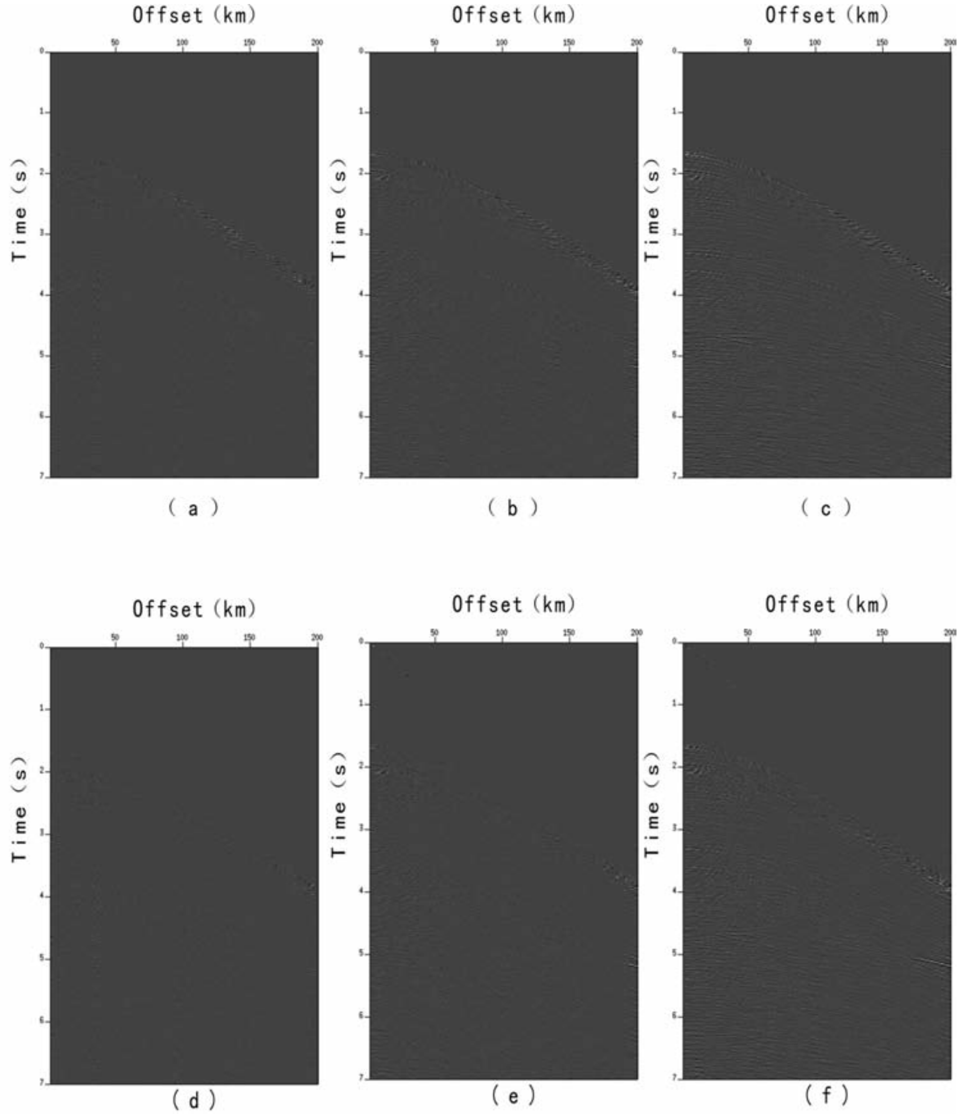

Fig. 9. Difference between the interpolated results and the original field shot. (a), (b), (c) are data with 25%, 50%, 75% missing traces and without NMO correction. (d), (e), (f) are data with 25%, 50%, 75% missing traces and with NMO correction.

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