A NEW GREEN'S THEOREM DEGHOSTING METHOD THAT SIMULTANEOUSLY: (1) AVOIDS A FINITE-DIFFERENCE APPROXIMATION FOR THE NORMAL DERIVATIVE OF THE PRESSURE AND, (2) AVOIDS THE NEED FOR REPLACING THE NORMAL DERIVATIVE OF PRESSURE WITH THE VERTICAL COMPONENT OF PARTICLE VELOCITY, THEREBY AVOIDING ISSUES THAT CAN ARISE WITHIN EACH OF THOSE TWO ASSUMPTIONS/APPROACHES: THEORY AND ANALYTIC AND NUMERIC EXAMPLES

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ABSTRACT

Weglein, A.B., Liang, H., Wu, J., Mayhan, J.D., Tang, L. and Amundsen, L., 2013. A new Green's theorem deghosting method that simultaneously: (1) avoids a finite-difference approximation for the normal derivative of the pressure and, (2) avoids the need for replacing the normal derivative of pressure with the vertical component of particle velocity, thereby avoiding issues that can arise within each of those two assumptions/approaches: Theory and analytic and numeric examples. *Journal of Seismic Exploration*, 22: 413-426.

Green's theorem deghosting requires the pressure and its normal derivative on a cable. Current marine dual measurement deghosting approaches can have issues caused by: (1) using over-under cable pressure measurements to provide a finite-difference approximation to the normal derivative; or (2) using the pressure, P, and the vertical component of particle velocity, V_z; V_z also can have issues at low frequency, and with instrument response differences. The deghosting method provided in this paper avoids both of those issues. Analytic and numerical tests show encouraging results, in comparison with current approaches.

KEY WORDS: Green's theorem, offshore preprocessing, on-shore preprocessing.

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INTRODUCTION

Deghosting is a longstanding seismic objective and problem (Robinson and Treitel, 2008; Amundsen, 1993) that has received considerable renewed attention due to: (1) an interest in so-called "broadband seismology" and the low frequency/low vertical wave-number requirements of linear iterative model matching and updating for velocity analysis, and (2) the requirements for deghosted data in, for example, inverse scattering series multiple removal algorithms for amplitude and phase fidelity. Green's theorem methods for deghosting were first introduced in Weglein et al. (2002); Zhang and Weglein (2005, 2006) and were developed and tested in Zhang (2007) and further tested and evaluated on complex synthetic (SEAM) and Gulf of Mexico field data by Mayhan et al. (2011); Mayhan et al. (2012); Mayhan and Weglein (2013). There were different realizations of this Green's theorem deghosting methodology, with different data requirements and different benefits and deficits. For receiver deghosting, one fundamental and general formulation, required, for each shot record, measurements of the pressure and the normal derivative of pressure (with respect to hydrophone location) on the cable (actually, on the measurement surface). A second formulation, for receiver deghosting, assumed a known and isotropic source signature, and measurements of the pressure on the measurement surface. The latter method employed a Green's function that vanished on the measurement surface and the free surface.

For the source side deghosting the general Green's theorem formulation required the pressure measurements and the normal derivative of pressure with respect to source location, for the entire collection of shot records, where each shot record actually has two separate experiments with over-under sources. An alternative Green's theorem source side deghosting version once again employed a Green's function that vanished on the measurement surface and the free surface, and avoided the need for dual source experiments, and without a need for a source signature. The versions that avoided the need for dual measurements were shown to be effective, except in the vicinity of notches where the dual cable and dual sources for receiver and source deghosting, respectively, could provide a stable and useful result. The results can be summarized as two measurements at one depth, which can at times be preferable and more stable than one measurement at two depths, where the sensitivity to depth enters through the Green's function (in these Green's theorem methods) that vanishes at the cable and the free surface. In Weglein et al. (2013) the Green's theorem deghosting methods derived in the space and temporal frequency domain was shown to derive (as a special case) the P-V_z spatial wave-number and temporal frequency industry standard deghosting approach. Strengths and limitations of the Green's theorem formulations in each domain were described and exemplified, with implications for 3D marine, and on-shore and ocean bottom acquisition and application. The focus then became how best to realize a two measurement at one depth formulation, and the need to recognize and address issues of: (1) a finite-difference approximation for a normal derivative with over-under pressure measurement cables and (2) the low frequency and instrument response differences that can arise with methods that depend on measurements of the pressure and vertical component of particle velocity. The method presented and evaluated in this paper avoids both of those two sets of shortcomings, and tests show encouraging results compared to current approaches.

Green's theorem deghosting has been demonstrated using several acquisitions: (1) a single streamer with hydrophones, an estimate of the wavelet, a "double Dirichlet" Green's function, G_0^{DD}, constructed to vanish on the air/water boundary and the measurement surface, and a causal Green's function, G_0^{d+}, representing direct propagation from the source to the receivers, (2) an over-under streamer, a finite-difference approximation to the normal derivative, dP/dz, G_0^{d+}, and G_0^{DD}, and (3) a dual-sensor streamer, G_0^{d+}, and G_0^{DD} (Weglein et al., 2002; Zhang and Weglein, 2005, 2006; Zhang, 2007; Mayhan et al., 2011, 2012; Mayhan and Weglein, 2013). These approaches illustrate some of the different ways of satisfying the Green's theorem requirement for two measurements on the measurement surface. (i) A single streamer with hydrophones and the air-water boundary can be used to capture one measurement (the pressure wavefield, P) at two different depths. (ii) Similarly, an over-under streamer captures one measurement, P, at two different depths. (iii) A dual-sensor streamer measures P using hydrophones and the vertical component of particle velocity, V_z, using geophones; V_z can be converted into the normal derivative, dP/dz.

This paper proposes the use of an over-under streamer; however, instead of using the two measurements of P to compute a finite-difference approximation to dP/dz, they would be combined with a new Green's function constructed to vanish on both streamers.

THEORY

We show in Weglein et al. (2013) that a two-way wavefield in a homogeneous medium can be written

$$P = Ae^{iqz} + Be^{-iqz} \quad , \tag{1}$$

where $q = +\sqrt{\{(\omega/c_0)^2 - k_x^2\}}$ (2D) or $q = +\sqrt{\{(\omega/c_0)^2 - k_x^2 - k_y^2\}}$ (3D). If an over-under cable at depths a,b is used directly, the upwave (deghosted wave) is

$$P_r = Be^{-iqz}$$

$$= \{[P(b)e^{iqa} - P(a)e^{iqb}]/2i\sin[q(a-b)]\}e^{-iqz} \quad . \tag{2}$$

Eq. (2) is found by using the cable measurements to solve for B in eq. (1):

$$B = \begin{vmatrix} e^{iqa} & P(a) \\ e^{iqb} & P(b) \end{vmatrix} \Big/ \begin{vmatrix} e^{iqa} & e^{-iqa} \\ e^{iqb} & e^{-iqb} \end{vmatrix}$$

$$= [P(b)e^{iqa} - P(a)e^{iqb}]/[e^{iq(a-b)} - e^{-iq(a-b)}]$$

$$= [P(b)e^{iqa} - P(a)e^{iqb}]/2i\sin[q(a-b)] \quad .$$

Eq. (2) assumes sufficient inline and crossline sampling for a Fourier transform from r,ω to k,ω. Eq. (2) has ghost notches when $q(a - b) = n\pi$, where special care must be taken. On the other hand, $a - b$ may be (and typically will be) smaller than the depth of a single cable, which means the ghost notches move out (to higher frequencies).

ANALYTIC EXAMPLE

Analytic data including one primary and one receiver ghost

For a 1D normal incidence case, as shown in Fig. 1, assume the recorded data at z' is

$$P(z',z_s,\omega) = R[e^{ik(2z_w-z'-z_s)}/2ik] - R[e^{ik(2z_w+z'-z_s)}/2ik]$$

$$= R[e^{ik(2z_w-z_s)}/2ik](e^{-ikz'} - e^{ikz'}) \quad , \tag{3}$$

where z', z_s and z_w are the depths of receiver, source and water bottom, respectively, and $k = \omega/c_0$.

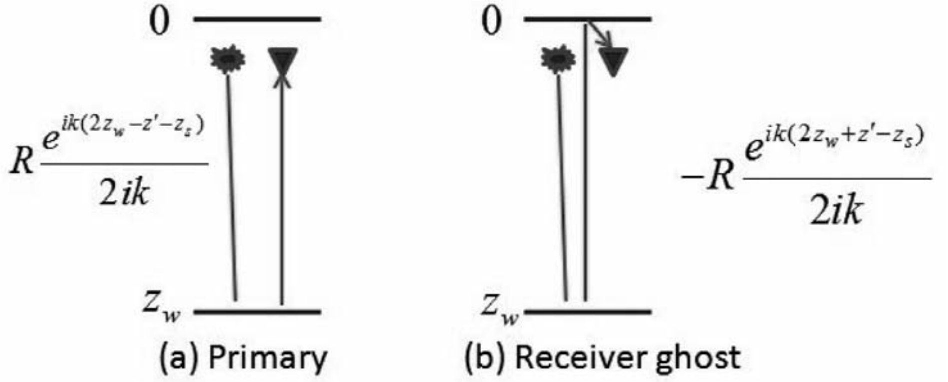

Fig. 1. 1D normal incidence analytic data including one primary and one receiver ghost.

Therefore, the exact derivative of the wavefield at z′ is

$$P'(z',z_s,\omega) = R[e^{ik(2z_w-z_s)}/2ik](-ike^{-ikz'} - ike^{ikz'})$$
$$= -(R/2)e^{ik(2z_w-z_s)}(e^{-ikz'} + e^{ikz'}) \ . \quad (4)$$

Up-down separation using two measurements at two depths

Consider a simple 1D normal incidence example, where in the vicinity of the (towed streamer) cable the pressure field P satisfies:

$$[(\partial^2/\partial z^2) - (1/c_0^2)(\partial^2/\partial t^2)]P = 0 \ , \quad (5)$$

where c_0 is the wave speed in water, and

$$[(d^2/dz^2) + (\omega^2/c_0^2)]P = 0 \ , \quad (6)$$

is the temporal Fourier transform of eq. (5).

The solution of eq. (6) is

$$P = \underbrace{Ae^{(ikz)}}_{\text{down}} + \underbrace{Be^{(-ikz)}}_{\text{up}} \ . \quad (7)$$

Using two measurements at two depths

$$P(a) = Ae^{ika} + Be^{-ika} \ , \quad (8a)$$

$$P(b) = Ae^{ikb} + Be^{-ikb} \ . \quad (8b)$$

By multiplying eq. (8a) with $e^{ik(b-a)}$ and subtracting eq. (8b), we get

$$e^{ik(b-a)}P(a) - P(b) = Be^{ik(b-2a)} - Be^{-ikb}$$

$$B = [e^{ik(b-a)}P(a) - P(b)]/[e^{ik(b-2a)} - e^{-ikb}] \ . \quad (9)$$

Therefore, the upgoing wave at $z' = a$ is

$$P_r(a) = Be^{-ika}$$
$$= [e^{ik(b-2a)}P(a) - e^{-ika}P(b)]/[e^{ik(b-2a)} - e^{-ikb}] \ . \quad (10)$$

From eq. (3), we can get the two measurements, i.e., $P(z' = a,z_s,\omega)$ and $P(z' = b,z_s,\omega)$, as follows:

$$P(a,z_s,\omega) = R[e^{ik(2z_w-z_s)}/2ik](e^{-ika} - e^{ika}) \;, \tag{11a}$$

$$P(b,z_s,\omega) = R[e^{ik(2z_w-z_s)}/2ik](e^{-ikb} - e^{ikb}) \;, \tag{11b}$$

and then substituting eqs. (11a) and (11b) into eq. (10), we get

$$\begin{aligned}
P_r(a) &= Re^{ik(2z_w-z_s)}[(e^{-ika}-e^{ika}) \times e^{ik(b-2a)}P(a) - (e^{-ikb}-e^{ikb}) \times e^{ika}] \\
&\quad /2ik[e^{ik(b-2a)} - e^{-ikb}] \\
&= Re^{ik(2z_w-z_s)}[e^{ik(b-3a)} - e^{ik(b-a)} - e^{-ik(b+a)} + e^{ik(b-a)}] \\
&\quad /2ik[e^{ik(b-2a)} - e^{-ikb}] \\
&= Re^{ik(2z_w-z_s)}[e^{ik(b-3a)} - e^{-ik(b+a)}] \\
&\quad /2ik[e^{ik(b-2a)} - e^{-ikb}] \\
&= Re^{ik(2z_w-z_s)}e^{-ika}[e^{ik(b-2a)} - e^{-ikb}] \\
&\quad /2ik[e^{ik(b-2a)} - e^{-ikb}] \\
&= Re^{ik(2z_w-z_s)}e^{-ika}/2ik \\
&= R[e^{ik(2z_w-a-z_s)}e^{-ika}/2ik] \;.
\end{aligned} \tag{12}$$

This is exactly the upgoing wave recorded at $z' = a$ with source at z_s (in this case, the primary).

Up-down separation using wave-field and the exact derivative of wave-field at one depth

For eq. (7), if we have the wave-field and the exact derivative of the wave-field at one depth, then

$$P(a) = Ae^{ika} + Be^{-ika} \;, \tag{13a}$$

$$P'(a) = ikAe^{ika} - ikBe^{-ika} \;. \tag{13b}$$

By multiplying eq. (13a) with ik, and subtracting eq. (13b), we have

$$ikP(a) - P'(a) = ikBe^{-ika} + ikBe^{-ika} \;,$$

$$B = [ikP(a) - P'(a)]e^{ika}/2ik \;. \tag{14}$$

Therefore, the upgoing wave at $z' = a$ is

$$P_r(a) = Be^{-ika} = [ikP(a) - P'(a)]/2ik \quad . \tag{15}$$

Substituting P(a) and P′(a) [using eqs. (3) and (4)] into eq. (15), we have

$$\begin{aligned} P_r(a) &= [ikP(a) - P'(a)]/2ik \\ &= \{ikR[e^{ik(2z_w-z_s)}/2ik](e^{-ika} - e^{ika}) \\ &\quad - [-(R/2)e^{ik(2z_w-z_s)}(e^{-ika} + e^{ika})]\}/2ik \\ &= [Re^{ik(2z_w-z_s)}e^{-ika}]/2ik = Re^{ik(2z_w-a-z_s)}/2ik \quad , \end{aligned} \tag{16}$$

which is the same result as eq. (12).

Up-down separation using wave-field and approximate derivative of wave-field at one depth

If we have two measurements at two depths, we can also get the approximate derivative of the wave-field using finite difference, for example,

$$P'(a) = [P(b) - P(a)]/(b - a) \quad . \tag{17}$$

Substituting P(b) in eq. (11b) and P(a) in eq. (11a) into eq. (17), we can have

$$\begin{aligned} P'(a,z_s,\omega) &= [Re^{ik(2z_w-z_s)}/2ik] \times [(e^{-ikb}-e^{ikb}) - (e^{-ika}-e^{ika})]/(b - a) \\ &= [Re^{ik(2z_w-z_s)}/2ik] \times [-2i\sin(kb) + 2i\sin(ka)]/(b - a) \\ &= -Re^{ik(2z_w-z_s)} \times [\sin(kb) - \sin(ka)]/k(b - a) \quad . \end{aligned} \tag{18}$$

In comparison, the exact derivative of the wave-field in eq. (4) at $z' = a$ can be rewritten as

$$\begin{aligned} P'(a,z_s,\omega) &= -(R/2)e^{ik(2z_w-z_s)}(e^{-ika} + e^{ika}) \\ &= -(R/2)e^{ik(2z_w-z_s)}\,2\cos(ka) \\ &= -Re^{ik(2z_w-z_s)}\cos(ka) \\ &= -Re^{ik(2z_w-z_s)}[d\sin(kz')/d(kz')]\big|_{z'=a} \quad . \end{aligned} \tag{19}$$

And, we have

$$\mathrm{dsin}(kz')/\mathrm{d}(kz') = \lim_{(b-a)\to 0} [\sin(kb) - \sin(ka)]/k(b - a) = \cos(ka) \quad . \quad (20)$$

Hence, if the two depths (b and a) are close enough, eq. (18) will reduce to eq. (19), and using the wave-field and the approximate derivative of the wave-field at one depth in eq. (17) can give a reasonable result; otherwise, using the approximate derivative in eq. (17) can produce an error.

The method developed in this paper can be derived from Green's theorem, where the closed surface consists of the over/under cables and the Green's function is arranged to vanish at each cable. In the (k_x,ω) domain the Green's theorem method comes closest to the 1D analysis in this paper, and would allow deghosting on the cable.

NUMERICAL EXAMPLES

In this section, some numerical examples will be presented to compare the two deghosting methods: (1) one method uses the wave-field at two depths to compute a finite-difference approximation of the derivative of the wave-field at one depth, and (2) the second (and new) method uses a single measurement of the wave-field at two depths.

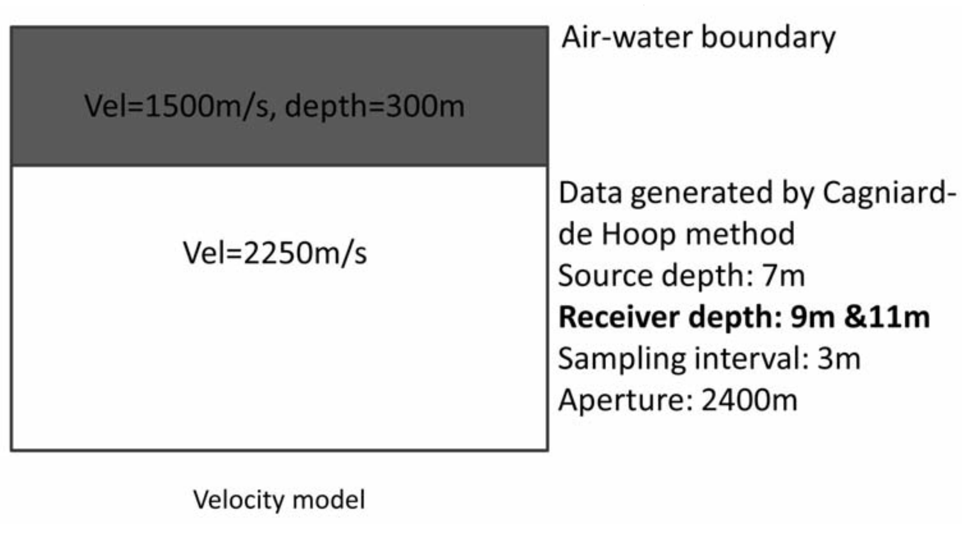

Fig. 2. Model with receivers at 9 m and 11 m.

The synthetic data is modeled using the Cagniard-de Hoop method. We tested two models here, with different depth differences between the over/under cables. Fig. 2 shows the model of two streamers located at 9 m and 11 m. Fig. 5 shows the model with two streamers located at 11 m and 21 m. Figs. 3 and 6 show the numerical test results in the case of zero offset, while Figs. 4 and 7 are showing far offset traces. We compared the test results using the two deghosting methods with the exact up-going waves computed by Cagniard-de

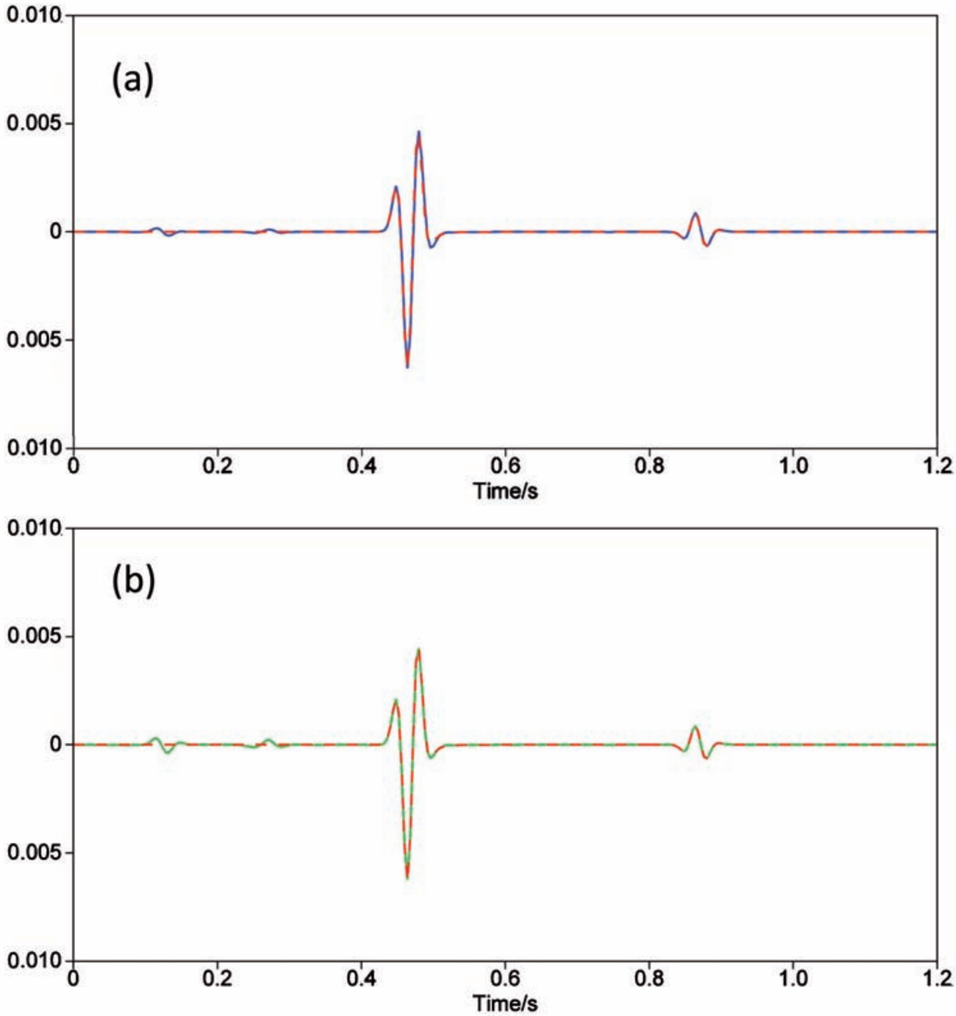

Fig. 3. Zero offset traces: (a) Red: Exact data without receiver ghosts, Blue: Data receiver deghosted using P and dP/dz at the same depth (dP/dz computed using finite-difference), (b) Same as (a) except Green: Data receiver deghosted using P at two depths. Both deghosting methods give satisfactory results.

Hoop modeling without a free surface. Red lines here represent the upgoing wave generated using the Cagniard-de Hoop method, which can serve as a standard to evaluate the accuracy of those two deghosting results. Similarly, blue lines represent the receiver side deghosted results using the wave-field P and a finite-difference approximation of dP/dz, while the green lines are the results from using the wave-field at two depths. When the blue lines or the green lines match well with the red lines, the result is satisfactory.

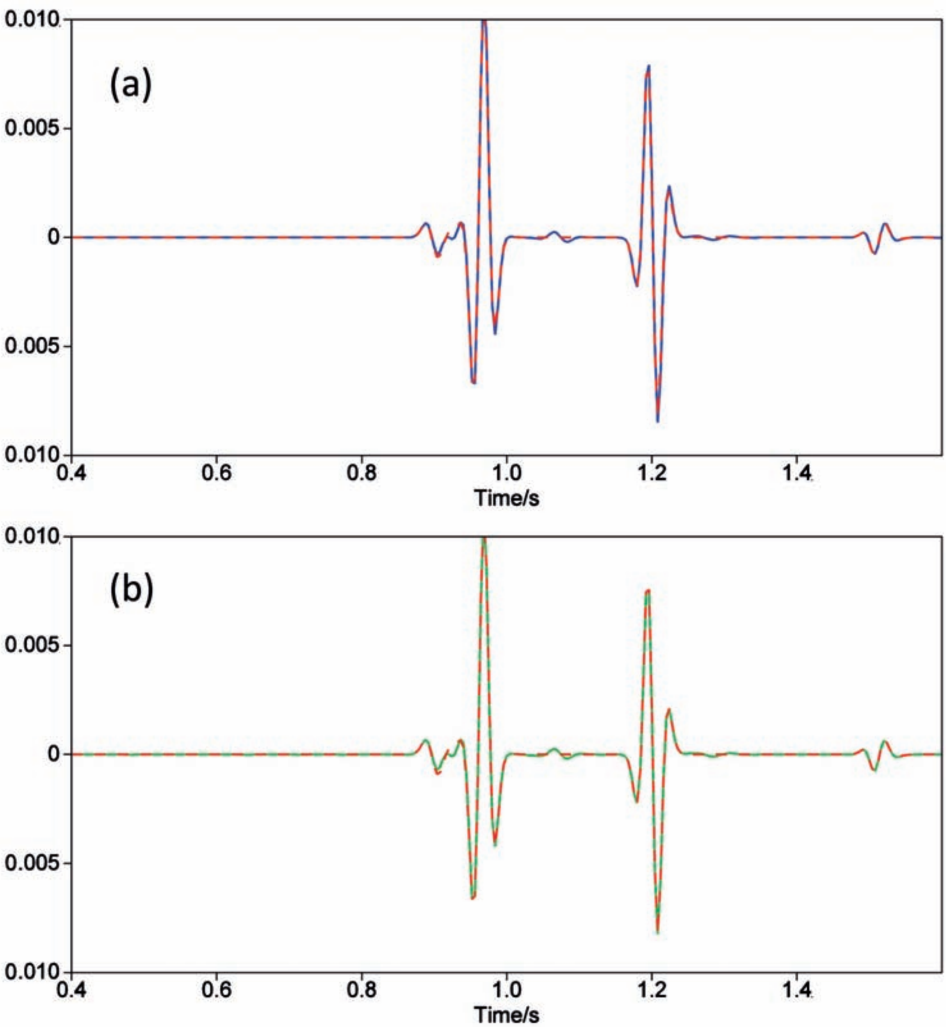

Fig. 4. Far offset traces (1200 m): (a) Red: Exact data without receiver ghosts, Blue: Data receiver deghosted using P and dP/dz at the same depth (dP/dz computed using finite-difference), (b) Same as (a) except Green: Data receiver deghosted using P at two depths. Both deghosting methods give satisfactory results.

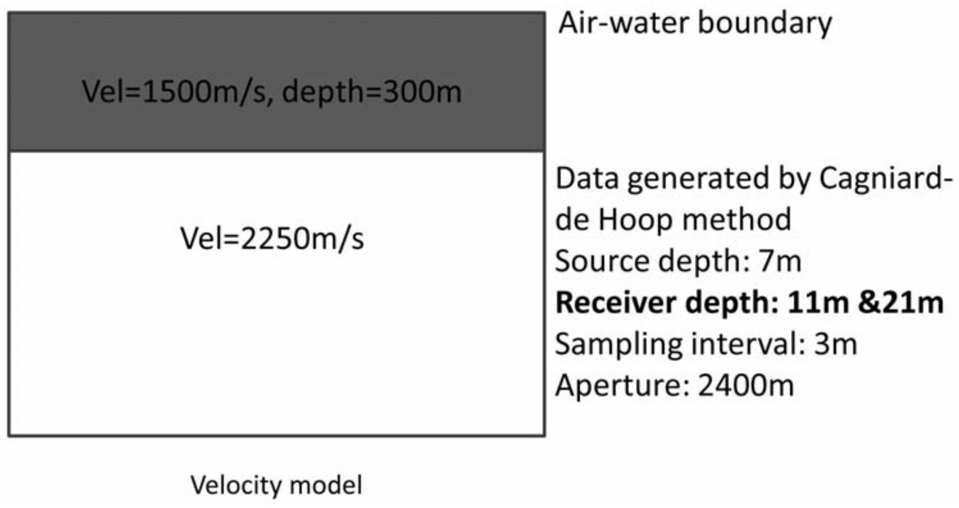

Fig. 5. Model with receivers at 11 m and 21 m.

In the first model we tested (Fig. 2), where the two cables are separated by 2 m, both of the two deghosting methods can give satisfactory results, in both zero offset and far offset. Since here the two cables are only separated by 2 m, the finite-difference approximation of dP/dz is relatively accurate. In the second model (Fig. 5), where the two cables are separated by 10 m, it can be clearly seen that the deghosting result calculated from the two measurements at two depths is satisfactory, but the method of using finite-difference approximation of dP/dz gives more artifacts, especially at the far offset. Since in the second model the two cables are further apart, the approximation of the derivative of P is not accurate enough, and causes artifacts in the deghosting result.

SUMMARY

A new deghosting method is proposed, for over-under cable measurements, that addresses shortcomings in current dual measurement and over-under cable approaches. For over-under cables, a finite-difference approximation of the pressure can be a source of errors. Also, measurements of the pressure and the vertical component of particle velocity at one depth can have issues with different instrument responses, and low frequency issues for the particle velocity measurement. The method proposed in this paper avoids these shortcomings of those current approaches. The method depends on knowing the distance between the cables, which is well known in practice, and more reliable than the depth to either cable. In addition, the notches are much further out in comparison with methods that depend on the depth of the cable. The proposed method requires over-under cables. We evaluate the effectiveness of the new approach in comparison with other methods for over-under cables. Analytic and numerical examples are encouraging, and further tests are planned.

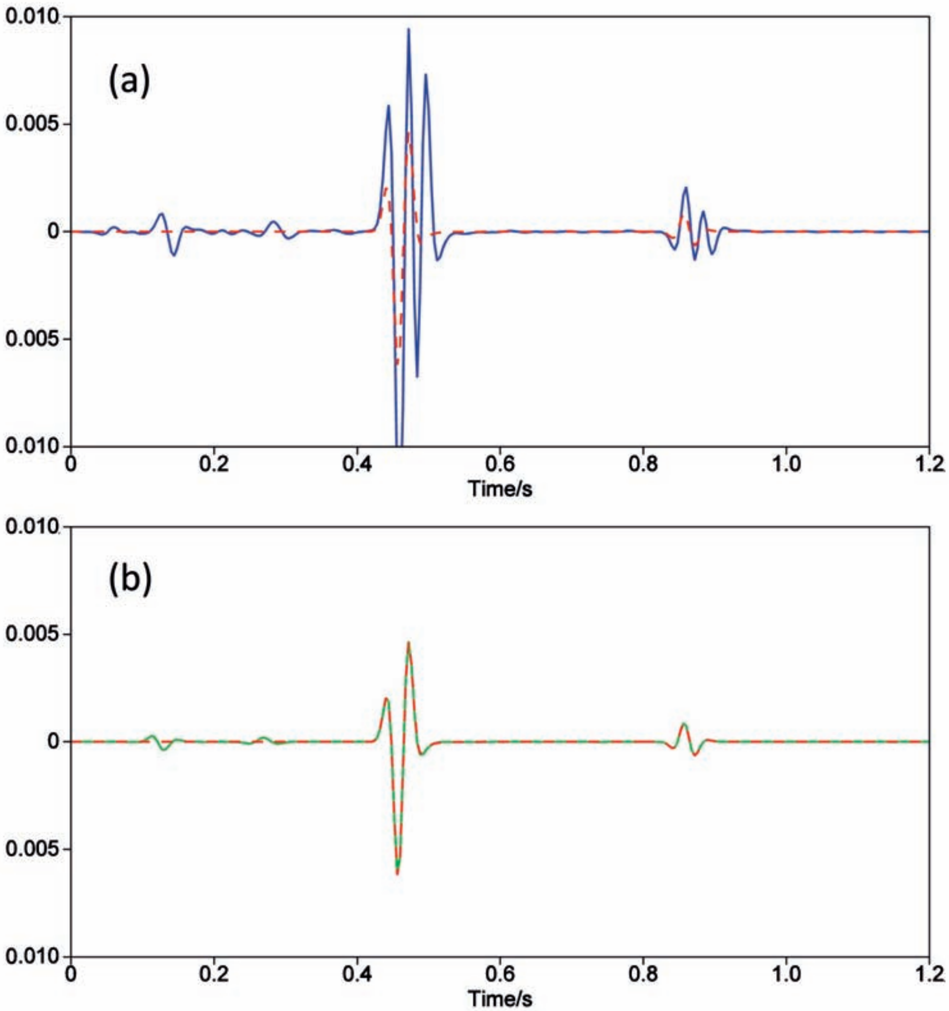

Fig. 6. Zero offset traces: (a) Red: Exact data without receiver ghosts, Blue: Data receiver deghosted using P and dP/dz at the same depth (dP/dz computed using finite-difference), (b) Same as (a) except Green: Data receiver deghosted using P at two depths. The method in (b) gives a better result than the method in (a).

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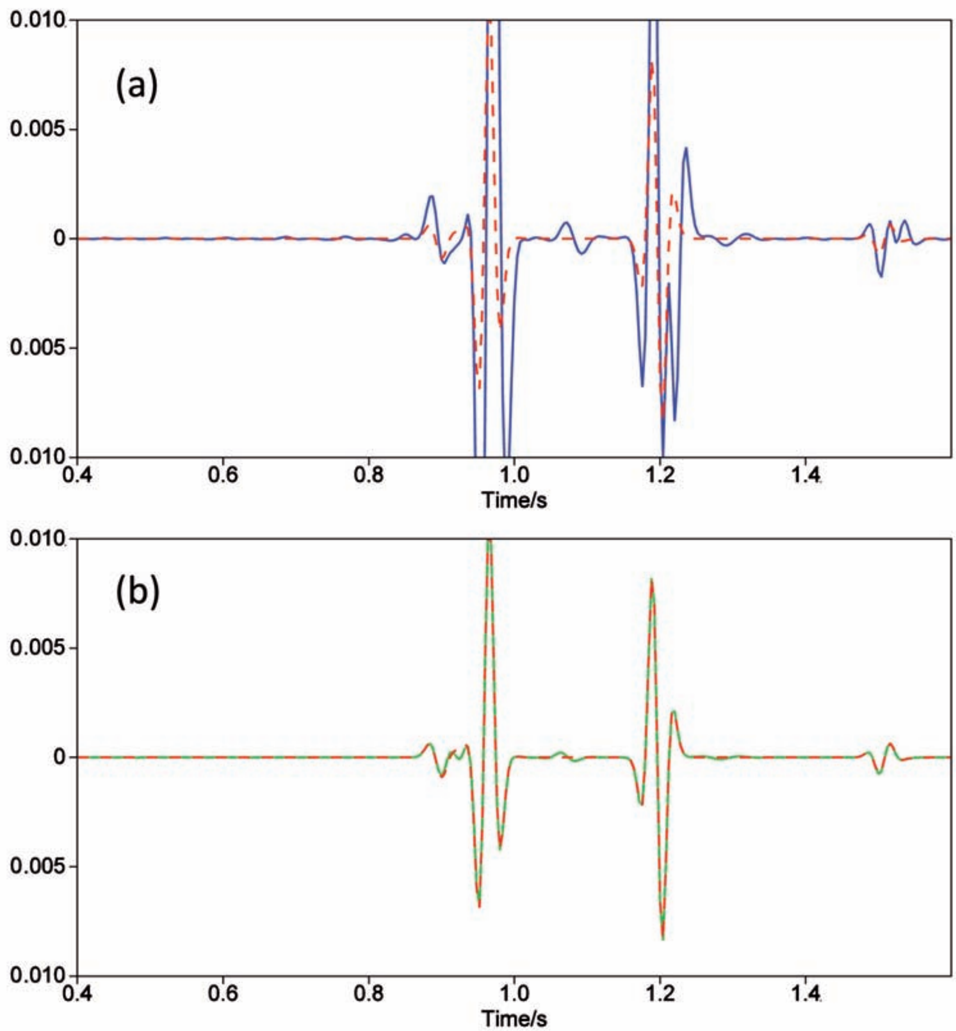

Fig. 7. Far offset traces (1200 m): (a) Red: Exact data without receiver ghosts, Blue: Data receiver deghosted using P and dP/dz at the same depth (dP/dz computed using finite-difference), (b) Same as (a) except Green: Data receiver deghosted using P at two depths. The method in (b) gives a better result than the method in (a).

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