# VELOCITY ANALYSIS BY FOCAL TRANSFORM

SIYAVASH TORABI[1], ABDORAHIM JAVAHERIAN[1,2] and MAJID NABI-BIDHENDI[1]

[1] *Institute of Geophysics, University of Tehran, Tehran, Iran. javaheri@ut.ac.ir*
[2] *Department of Petroleum Engineering, Amirkabir University of Technology, Tehran, Iran.*

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ABSTRACT

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Many steps in seismic processing are, directly or indirectly, affected by the quality of the velocity function obtained through a velocity analysis. Improving the quality of the velocity function can affect the entire processing procedure. Regarding the limited resolution in using semblance velocity analysis, many researchers are trying to introduce new methods of velocity analysis with higher resolution. In this paper, the focal transform is introduced as an effective tool in the velocity analysis. The focal transform can be used as a method to estimate $V_{nmo}$ in terms of time. This method is based on the fact that the shape of any event in the focal domain depends on the similarity between the shape of the event in the operator and the events in the CMP gather. Having this similarity measured, we can determine the velocity of each event in the CMP gather. This new method was tested on a synthetic CMP gather and two real data sets from onshore and offshore. Attributes of the peak quality, velocity resolution and time resolution were introduced in this study to measure and compare the quality velocity panels obtained by the focal transform velocity analysis. A comparison of the results of the focal transform with those of the semblance method demonstrated that a higher resolution in the velocity analysis was achieved when using the focal transform.

KEY WORDS: seismic data processing, velocity analysis, focal transform, semblance.

## INTRODUCTION

For many years, semblance (Neidell and Taner, 1971) has been the most recognized method in the velocity analysis. Most of the velocity analysis methods for seismic gathers scan different values of velocity in different times and generate a semblance of the flatted gather and velocity spectra (Yilmaz, 2001). During the past years, many other methods have been developed by researchers to obtain better results. Eigenvalue methods (Biondi and Kostov, 1989; Key and Smithson, 1990) were developed using the fact that the signal

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covariance matrix is of a low rank in the absence of noise. The results obtained by this method are of a high resolution and need intensive computations. Other methods such as differential semblance method (Symes and Kern, 1994) and more recently AB semblance (Fomel, 2009) have also been developed. The AB semblance tries to solve some problems of the semblance velocity analysis in the presence of an amplitude variation with offsets.

A higher resolution velocity analysis can be achieved using a proper domain for the velocity analysis. A proper domain is a domain in which time or velocity small differences between events in a t-x domain appear more separable in that domain. Using such a proper domain, a high resolution velocity analysis can be achieved. In this study, the focal domain was used as a proper domain for the velocity analysis. This method has a higher resolution and more accuracy in comparison with the semblance method. Similar to the other high resolution methods, higher resolution can be obtained by higher computational cost. Meanwhile, powerful computers that are available in most processing centers make computational cost less important given the resolution that can be achieved. After briefly reviewing the focal transform equations of Berkhout et al. (2004), the methodology of the velocity analysis by focal transform will be discussed. This method was tested on several synthetic and real data sets to check the ability of this method to distinguish different events and sensitivity of this method to the noise level of the input CMP gather. The drawback of this method is the higher computational cost (10 to 15 times) in comparison with the semblance method.

## FOCAL TRANSFORM

Focal transform was introduced by Berkhout et al., (2004) Berkhout and Verschuur, 2006) as a promising tool in the seismic data processing to allow the incorporation of the macro information about the involved wave fields. Application of focal transform introduces a new domain, the - focal domain - which enables the users to accurately separate different events from each other. Forward and inverse focal transforms can be formulated as a matrix multiplication per temporal frequency as follows:

$$\mathbf{Q} = \mathbf{FP} \quad , \tag{1}$$

$$\mathbf{P} = \mathbf{GQ} \quad , \tag{2}$$

$$\mathbf{F} = \mathbf{G}^{-1} \quad , \tag{3}$$

where **P** represents any prestack 3D data volume with one column representing one monochromatic shot record, **Q** represents the focal domain, **G** is a suitable 3D propagation operator and **F** is the inverse operator. In this study, we tried

the above equations in both frequency and time domains. In the time domain, we used the same formulation on a CMP gather in the time-offset domain instead of on a monochromic record. Regarding the synthetic test of the velocity analysis in the frequency and time domains, a better analysis is achieved by applying the above equations in the time domain. Therefore, the above equations can be used in the time domain as follows:

$$\mathbf{q} = \mathbf{fp} \quad , \tag{4}$$

$$\mathbf{p} = \mathbf{gq} \quad , \tag{5}$$

$$\mathbf{f} = \mathbf{g}^{-1} \quad , \tag{6}$$

where, $\mathbf{p}$ represents a CMP gather, $\mathbf{q}$ represents the focal domain, $\mathbf{g}$ is a suitable propagation operator and $\mathbf{f}$ is the inverse operator. Thus the forward and inverse focal transformations can be formulated as a matrix multiplication. Using a weighted least-squares inversion approach, the forward focal operator can be written as

$$\mathbf{f} \approx \mathbf{g}^{H}[\mathbf{g}\mathbf{g}^{H} + \varepsilon^{2}\mathbf{I}]^{-1} = \mathbf{g}^{H}\mathbf{B} \quad , \tag{7}$$

$$\mathbf{B} = [\mathbf{g}\mathbf{g}^{H} + \varepsilon^{2}\mathbf{I}]^{-1} \quad , \tag{8}$$

$$\mathbf{q} = \mathbf{g}^{H}\mathbf{B}\mathbf{p} \quad , \tag{9}$$

where, $\varepsilon$ is a small stabilization constant, $\mathbf{I}$ is the unitary matrix and $\mathbf{q}$ represents the focal domain. In the focal transform, the operator will be a matrix with the same number of samples as the input data. Any event in the t-x domain can be localized in the focal domain using a proper operator in the transform. A proper operator for the event localization will be a synthetic record containing an event similar to the desired event in the input CMP gather. Therefore, the reflection events in common midpoint gathers (CMP) can be localized in the focal domain using the proper operator. The amount of this localization strongly depends on the operator being used in the forward transform. Therefore, measuring the location of the event on the diagonals of the focal domain will help to find out the amount of the similarity between the synthetic event in an operator and the events in the input CMP gather. Fig. 1 illustrates how the operator shapes affect the localization of a single hyperbolic event in the focal domain. Fig. 1b shows the operator containing a reflecting event similar to the reflecting event in the CMP gather in Fig. 1a. The results of converting the CMP gather in Fig. 1a to the focal domain using the operator in Fig. 1b is shown in Fig. 1c. It can be seen that in the cases where the focal operator is equal to the input CMP gather, an event in the input CMP gather will be localized and moved on diagonals in the focal domain (white lines in the focal domain). To study the effect of converting the CMP gather to the focal domain

with an operator not equal to the input data, two different operators are produced. Fig. 1d shows an operator containing a reflection event with a velocity lower than the velocity of the event in the input CMP gather. Fig. 1e shows the results of converting the CMP gather in Fig. 1a to the focal domain using the operator in Fig. 1d. It can be seen that by using such an operator, that event will not move to the main diagonals of the focal domain (white lines in focal domain). The next test will be an operator which contains a reflection event in a shallower depth (Fig. 1f). By applying this operator the event in the CMP gather will not move on diagonals too (white lines in the focal domain). Therefore, high amplitude values on the diagonals of the focal domain can be obtained only if the operator has an event similar to the event in the input CMP gather.

To find the effect of applying a single reflector operator to a multi-event CMP, a CMP gather (Fig. 2a) containing several reflection events was generated. Using the operator g containing the single reflector event in Fig. 2b, the related focal domain in Fig. 2c can be obtained. As can be seen in Fig. 2c, the second reflection event in the input CMP gather has the same velocity and apex time as the reflection event in the operator (the event in the time apex of 0.4 s indicated by an arrow). Therefore, by applying this operator the second event will move on the diagonals in the focal domain. On the other hand, all other reflection events are distributed among other spaces of the focal domain.

## VELOCITY ANALYSIS

Velocity analysis in the focal domain is based on the fact that the amount of the localization of any event on the main diagonal of the focal domain strongly depends on the similarity between that event and the event in the operator used for the forward and inverse transforms (Torabi and Javaherian, 2010). In the extreme case, where the event in operator **g** is equal to one of the events in the input CMP gather, the event will be localized on the diagonals of the focal domain (Fig. 1c). On the other hand, if there is any difference between the event in operator **g** and the event in the CMP gather, that event will not be localized on the main diagonals of the focal domain (Figs. 1e and 1g).

Using this concept, the CMP gather can be scanned with different operators and using summations of the main diagonals in the focal domain to find the events of the best match with the events in the CMP gathers. Each operator is a synthetic CMP record containing a reflection event with a specific velocity ($V_{nmo}$) and an apex time ($t_0$). In other words, if an event in the operator has the same velocity and time as the ones in the events in the CMP gather, then the summation of the main diagonals of the focal domain will have the highest value. The amount of similarity between the event in the CMP gather and the event in the focal domain can be measured by using summation of main

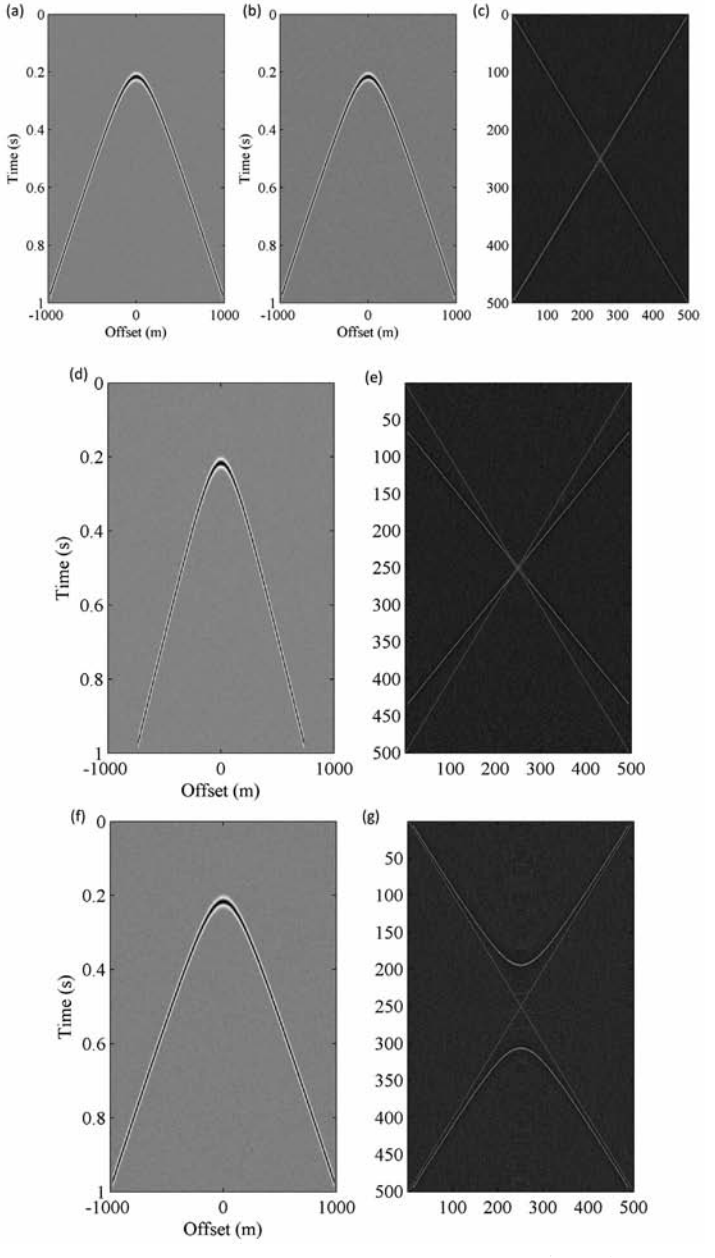


Fig. 1. The effects of the operator shape on the localization of a single hyperbolic event in the focal domain. (a) A synthetic CMP gather containing a single hyperbolic event, (b) the operator containing the similar reflecting event in the CMP gather, (c) the results of converting the CMP gather in Fig. 1a to the focal domain using the operator in Fig. 1b, (d) the operator containing the reflection event with a lower velocity in comparison with the events in the CMP gather, (e) the results of converting the CMP gather in Fig. 1a to the focal domain using the operator in Fig. 1d, (f) the operator containing a reflection event with a lower apex time in comparison with the events in the CMP gather, and (g) the results of converting the CMP gather in Fig. 1a to the focal domain using the operator in Fig. 1e.

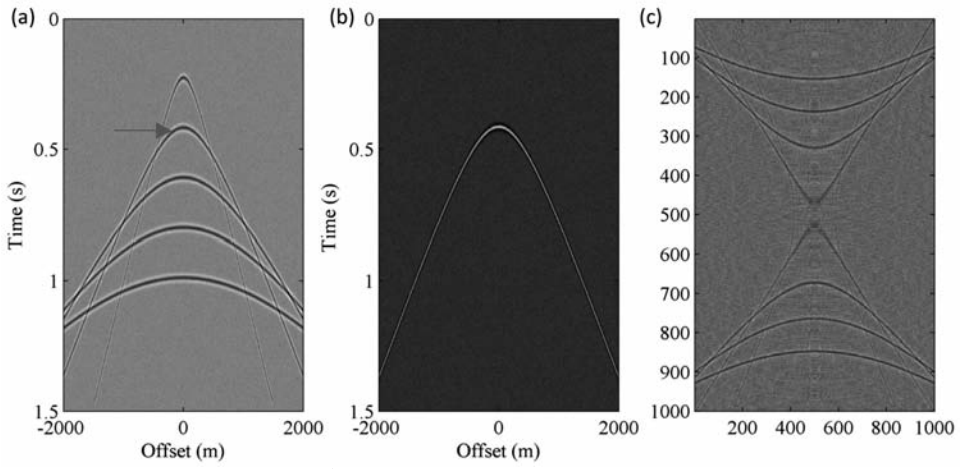


Fig. 2. The effects of the operator on the localization of the hyperbolic events in a multi-hyperbola CMP gather. (a) A synthetic CMP gather containing several hyperbolic events, (b) operator g, and (c) the result of converting the CMP gather in Fig. 2a to the focal domain using the operator in Fig. 2b.

diagonals in the focal domain. Therefore, by plotting the value of the main diagonal summations, beside each other for different times and velocities, velocity panels of related CMP gathers can be created. Fig. 3 depicts the schematic diagrams that show the focal transform velocity analysis methodology. Fig. 3a is schematic view of input CMP gather. For each sample of the velocity panel (Fig. 3d), an operator should be built that contains a reflection event with parameters (such as velocity and time) equal to those of the related sample in the velocity panel. Fig. 3b shows a schematic view of three operators with their related parameters. By applying the mentioned operators to the input CMP gather, the related focal domains are obtained (Fig. 3c). A summation of values on the main diagonal (indicated by a red cross) of each focal domain is shown (indicated by colored circles). The velocity panel of an input CMP gather will be obtained by putting the summation derived from focal domains side by side for operators containing events with different times and velocities (Fig. 3d). Using other functions instead of simple summations on the main diagonals of the focal domain may improve the results of this method. The value of each pixel in the velocity panel matrix is calculated from the operator that contains a reflection event with a velocity and time related to that pixel in the velocity panel. Three of these operators are shown in (Fig. 3b) and the related pixels in the velocity panel are marked with color circles. The iteration of this procedure for all velocities and times results in the velocity panel shown in Fig. 3d. Regarding to the implemented tests, the focal velocity analysis (in the present code which is not optimized) is more expensive (10 to 15 times) than the

semblance, (due to the calculation inverse problem in each iteration) but as the code on the focal transform has not been optimized to run in the fastest mode it may not be fair to compare the run time of the focal transform with that of the of the semblance quantitatively.

In a simple way, each reflection event can be introduced in a hyperbolic equation [eq. (10)] by two parameters, i.e., the velocity ($V_{nmo}$) and the apex time ($t_0$); a more complex definition of a reflection event (of a third order, anisotropy constant) can be used to build the operator of the focal transform. In the same way, calculating the third order velocity panels or even more complex functions can be done by using such functions to build the operator. In this study, we focused on the results from the operator that contained reflection events with a regular hyperbolic NMO equation [eq. (10)].

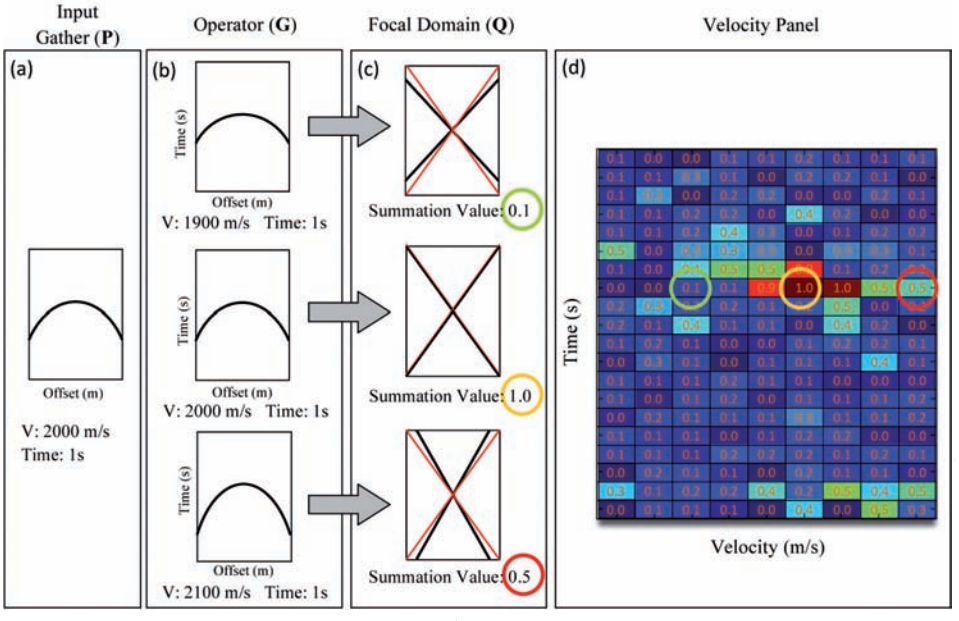


Fig. 3. Schematic diagram that show focal transform velocity analysis methodology. (a) Schematic view of input CMP gather contain reflection event with velocity of 2000 m/s and apex time of 1 s. (b) Schematic view of three of operators that used in focal transform that built contain reflection event with velocities equal to 1800 m/s, 2000 m/s and 2100 m/s, respectively. (c) Schematic view of focal domain related to each of mentioned operator. Summation of values on main diagonal (indicated by a red cross) of each focal domain is shown (indicated by colored circles). (d) Velocity panel of input CMP gather will be obtained by putting summation values that obtained from focal domains side by side for different time and velocity. Value of each pixel in velocity panel matrix calculated from operator that contain reflection event with velocity and time related to that pixel in velocity panel. Three of these operators were shown in (b) and related pixels in velocity panel are marked with color circles.

$$t_x^2 = t_0^2 + (x^2/v_{nmo}^2) \quad . \tag{10}$$

But the results can be expanded easily by using any other equations to build events in the operators. Above procedure of the velocity analysis in the focal domain assumes that there is no amplitude-variation-with-offset (AVO) in the input data. Therefore, the results of this method are not good enough for reflections showing AVO anomaly, particularly class II AVO. But by including the mentioned assumption in the focal velocity analysis such pitfall can be handled.

## EXAMPLES

### Synthetic data

To measure the ability of the focal transform velocity analysis, the method was applied to a set of synthetic CMP gathers containing several hyperbolic events with different levels of random noise. The results were compared to those from the semblance method. Also to compare the resolution of the focal transform velocity analysis results with those of the semblance results, some synthetic data sets with small variations in time and velocity in two separate events were prepared.

Fig. 4a shows a synthetic CMP gather containing several hyperbolic events at various times and velocities. Fig. 4b shows the results of the focal transform velocity analysis. Fig. 4c shows the results of the semblance velocity analysis. Both velocity panels are scaled by dividing all values of each panel by the maximum value of that panel. By comparing the results in Figs. 4b and 4c, it becomes clear that the results of the velocity analysis obtained by the focal transform method have a better peak shape in the velocity panel in comparison with the results obtained by applying the semblance method. For a better comparison, three sections in time directions and three sections in velocity directions were produced from shallow, intermediate and deep parts (indicated by boxes 1-3, respectively) of the focal and the semblance velocity panels (Fig. 4d). To compare the results, three attributes were used. The first attribute, peak quality, was defined as the ratio of the peak amplitude to the background noise amplitude. The second attribute, velocity resolution, was defined as the ratio of the peak amplitude to the width of the peak in the section in the velocity direction at the half amplitude of the peak. The last attribute, time resolution, was defined as the ratio of the peak amplitude to the width of the peak in the section in the time direction at the half amplitude of the peak. Using the sections in time and velocity directions (Fig. 4d) the peak amplitude, background noise amplitude and width of the peaks can be measured in both directions. Therefore, all the attributes mentioned were calculated at shallow, intermediate and deep parts of both velocity panels (Table 1); those parts appear as black boxes

numbered 1-3 on the velocity panels. Using these attributes, it can be seen that the focal transform velocity analysis has a higher peak quality in compare with those of the semblance (Table 1). Also, the peak quality of the focal transform is more stable from the shallow part to deep part of the velocity panel. According to the velocity resolution attribute, the focal transform velocity analysis has a slightly higher resolution in the velocity direction in compare with those of the semblance (Table 1). The velocity resolution in the focal transform velocity analysis decreases in the deeper part of the velocity panel but still has a higher resolution in compare with the semblance. Comparing the results of the time resolution attribute in Table 1 shows that the focal transform velocity analysis has a very high resolution in the time direction in compare with the results obtained by the semblance. Based on the above analysis, it is concluded that the focal transform velocity analysis has a higher resolution, a better peak shape and more concentration in the velocity panel in compare with those of the semblance.

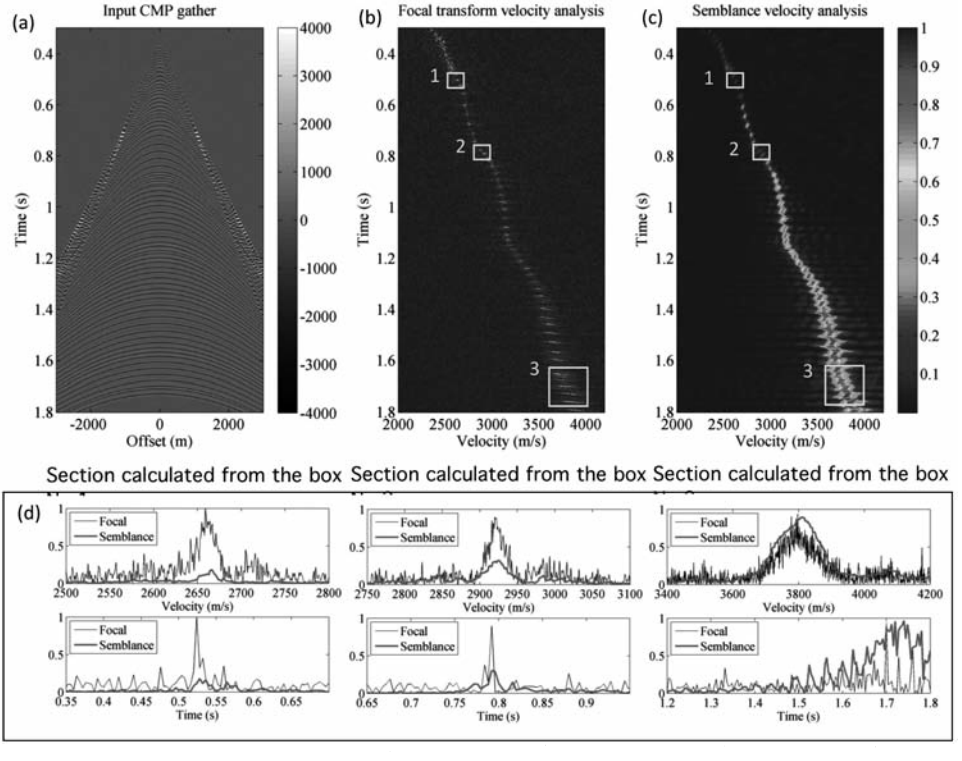


Fig. 4. (a) A synthetic CMP gather containing hyperbolic events, (b) the velocity panel calculated by the focal transform velocity analysis, and (c) the velocity panel calculated by the semblance. (d) Three sections in time directions and three sections in velocity directions were produced from shallow, intermediate and deep parts (indicated by boxes numbered 1-3, respectively) of the focal and semblance velocity panels.

Table 1. Attributes calculated from Fig. 4d (low noise).

| | Focal transform | | | Semblance | | |
|---|---|---|---|---|---|---|
| | Shallow | Intermediate | Deep | Shallow | Intermediate | Deep |
| Peak quality | 10 | 15 | 9.4 | 6 | 10 | 8.9 |
| Velocity resolution | 0.053 | 0.026 | 0.007 | 0.009 | 0.009 | 0.005 |
| Time resolution | 200 | 204 | 218 | 9 | 27 | 28 |

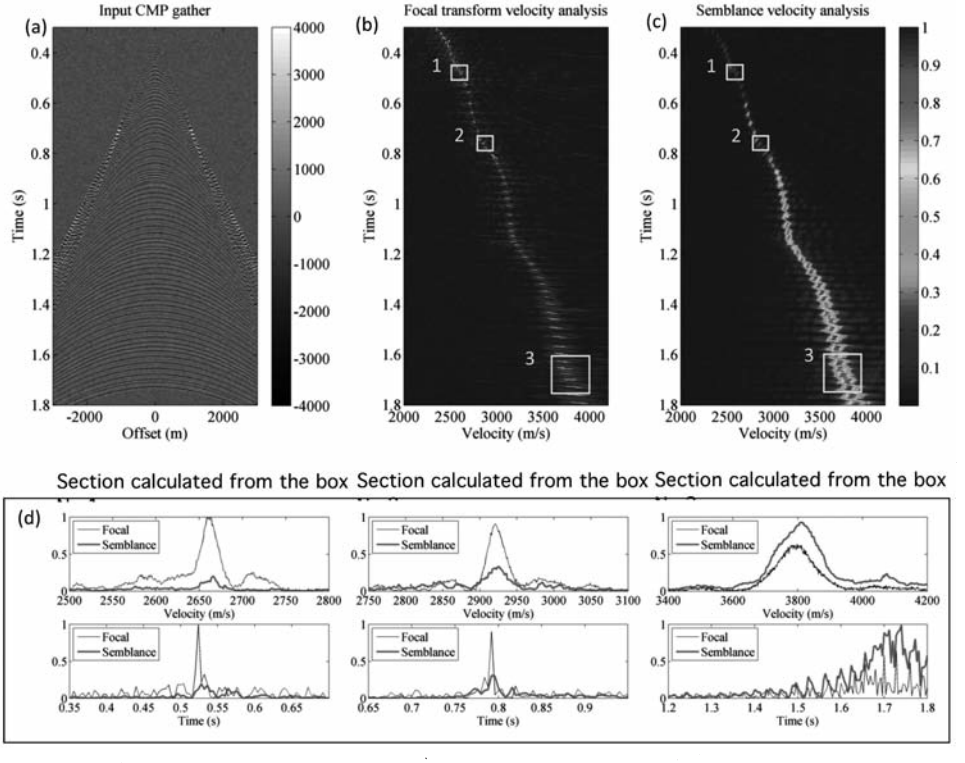


Fig. 5. (a) A synthetic CMP gather in Fig. 4a with a moderate level of random noise, (b) The velocity panel calculated by the focal transform velocity analysis, and (c) the velocity panel calculated by semblance. (d) Three sections in time directions and three sections in velocity directions were produced from shallow, intermediate and deep parts (indicated by boxes numbered 1-3, respectively) of the focal and semblance velocity panels.

To study the sensibility of the focal transform velocity analysis to the noise level of data, two sets of synthetic data were created by adding moderate and high level random noise to the previous synthetic data set. The results of applying the focal transform velocity analysis and the semblance to the above data sets are shown in Figs. 5 and 6. Figs. 5a and 6a show the synthetic data with a moderate and high level of random noise, respectively. Figs. 5b, 5c, 6b and 6c shows the results of applying the focal transform velocity analysis and the semblance velocity analysis to the above two data sets, respectively. Also, three sections in time directions and three sections in velocity directions were produced from shallow, intermediate and deep parts of the focal and the semblance velocity panels (Figs. 5d and 6d). Using the above sections, the peak quality, velocity resolution and time resolution attributes were produced (Tables 2 and 3). All calculated attributes shows that adding noise to input data set has no bad effect on quality of focal transform velocity analysis.

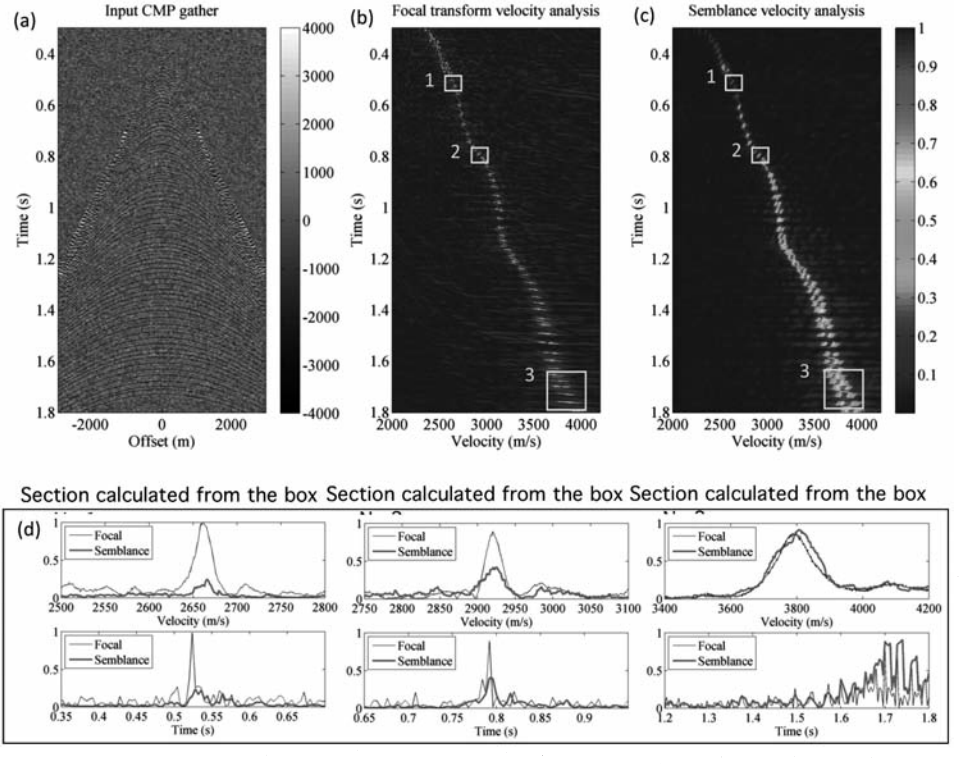


Fig. 6. (a) A synthetic CMP gather in Fig. 4a with a high level of random noise. (b) The velocity panel calculated by the focal transform velocity analysis, and (c) the velocity panel calculated by semblance. (d) Three sections in time directions and three sections in velocity directions were produced from shallow, intermediate and deep parts (indicated by boxes numbered 1-3, respectively) of the focal and semblance velocity panels.

Table 2. Attributes calculated from Fig. 5d (moderate noise).

| | Focal transform | | | Semblance | | |
|---|---|---|---|---|---|---|
| | Shallow | Intermediate | Deep | Shallow | Intermediate | Deep |
| Peak quality | 12.5 | 15 | 12.8 | 6 | 11 | 9.2 |
| Velocity resolution | 0.042 | 0.031 | 0.005 | 0.009 | 0.010 | 0.005 |
| Time resolution | 175 | 200 | 142 | 8.9 | 30.8 | 40 |

Table 3. Attributes calculated from Fig. 6d (high noise).

| | Focal transform | | | Semblance | | |
|---|---|---|---|---|---|---|
| | Shallow | Intermediate | Deep | Shallow | Intermediate | Deep |
| Peak quality | 13.3 | 17.8 | 10.9 | 9 | 8.4 | 11.5 |
| Velocity resolution | 0.043 | 0.036 | 0.006 | 0.014 | 0.012 | 0.006 |
| Time resolution | 172 | 197 | 197 | 16 | 38 | 46 |

Considering the results of this comparison, we can see that the background noise in the velocity panel computed by the focal transform is slightly increased in some areas by adding random noise to the input data set. But focal transform still have high peak quality and good resolution in time and velocity. Therefore, we can conclude that the focal transform velocity analysis is not sensitive to the noise level of data.

To study the ability of the focal transform velocity analysis in order to differentiate between two similar events, the first data set contained two hyperbolic events with the same velocities and a 2% difference in the apex times (Fig. 7a). Figs. 7b and 7c show the results of applying the focal velocity analysis and the semblance velocity analysis to the above dataset, respectively. In view of the above-mentioned figures, it can be seen that the focal transform velocity analysis can distinguish between the exact times of these two reflectors as two separate peaks in the velocity panel. However, the semblance method cannot separate these two reflectors.

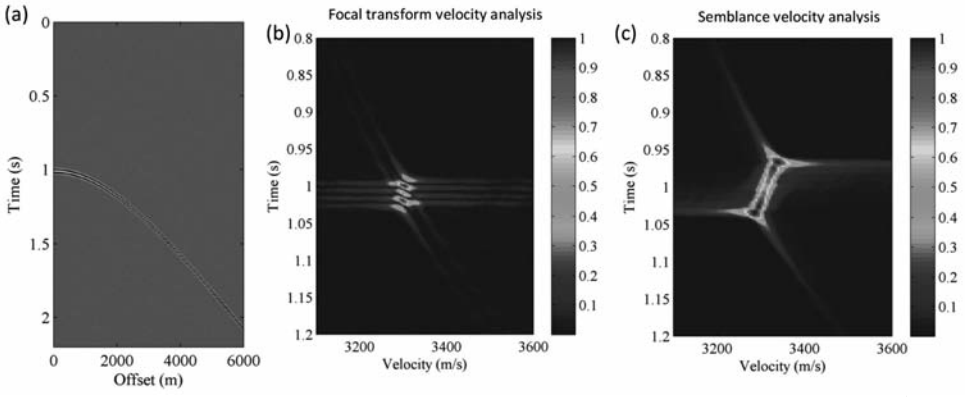


Fig. 7. (a) The synthetic data set containing two hyperbolic events with the same velocities and a 2% difference in the apex times. (b) The result of applying the focal transform velocity analysis. (c) The result of applying the semblance method.

The second data set contains two hyperbolic events with the same apex times and a 2% difference in velocity (Fig. 8a). Figs. 8b and 8c show the results of applying the focal velocity analysis and the semblance velocity analysis to the above dataset, respectively. Similar to the previous dataset, the focal transform can distinguish between these two events as two separate peaks in their right positions. On the other hand, the semblance method cannot show the proper velocity panel.

The ability of distinguishing events in time and velocity by the focal transform is caused by a good peak shape and more concentration in the velocity panel in comparison with the semblance method.

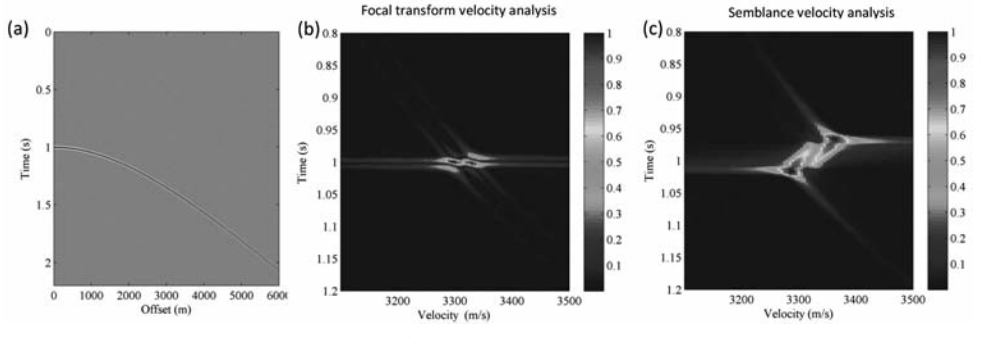


Fig. 8. (a) The synthetic data set containing two hyperbolic events with the same apex times and a 2% difference in velocities. (b) The result of applying the focal transform velocity analysis. (c) The result of applying the semblance method.

## Real data

The focal transform velocity analysis was applied to two sets of real data. The first data set, Fig. 9a, shows a deep marine data set containing strong long period sea bed multiples. The results of the focal transform velocity analysis and the semblance of this CMP are shown in Figs. 9b and 9c, respectively. The results show that the focal transform velocity analysis is less effected by the presence of high-amplitude multiples. Fig. 10a shows a real land CMP gather. The results of applying the focal transform velocity analysis and the semblance to this CMP are shown in Figs. 10b and 10c, respectively. Clearly, the results of focal transformation method have a higher resolution, especially in the time axis, in comparison with the semblance results.

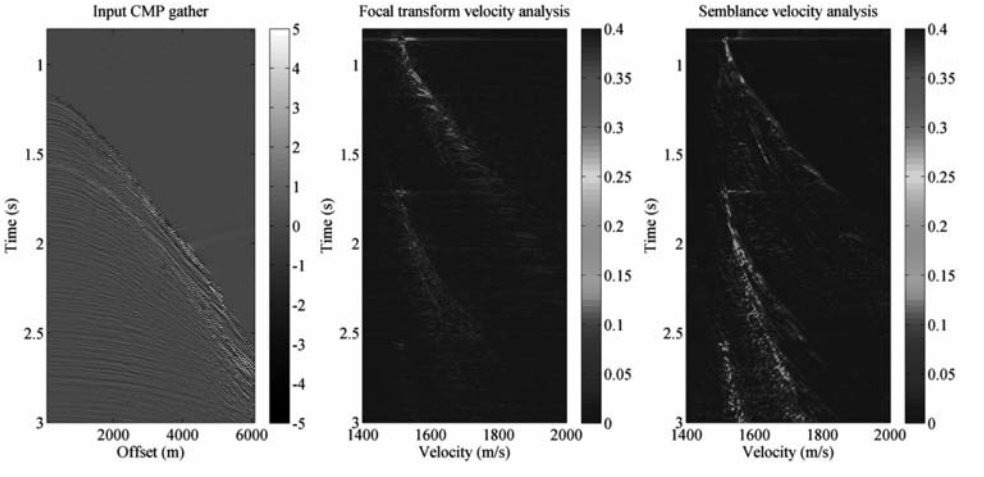


Fig. 9. The deep marine data set containing strong long period water bottom multiples. (a) The velocity panel calculated by the focal transform velocity analysis, and (b) the velocity panel calculated by the semblance method.

## CONCLUSIONS

According to the results of applying the focal transform velocity analysis method to synthetic and real data, it was concluded that the focal transformation could be used as an effective tool in the velocity analysis. The results of the attributes (peak quality, velocity resolution and time resolution) on the velocity analysis obtained by the focal transform method had higher values in comparison with those obtained by applying the semblance method. Based on these three attributes and a comparison of the results of the velocity analysis obtained by the focal transform with those obtained by the semblance, it was demonstrated that a high resolution velocity analysis could be achieved by the focal transform

velocity analysis. Of course, it is worth mentioning that the resolution improvement is more significant in the time direction. Also, based on the mentioned attributes, the focal transform is not much sensitive to the amount of the random noise in the input dataset. A drawback of the focal transform (in the present code which is not optimized) is the higher computational cost (10 to 15 times) in comparison with that of the semblance method. Also the focal transform is assumed that there is no amplitude variation with offset. Therefore, the results of this method are not good enough for reflections that show AVO anomaly, particularly class II AVO.

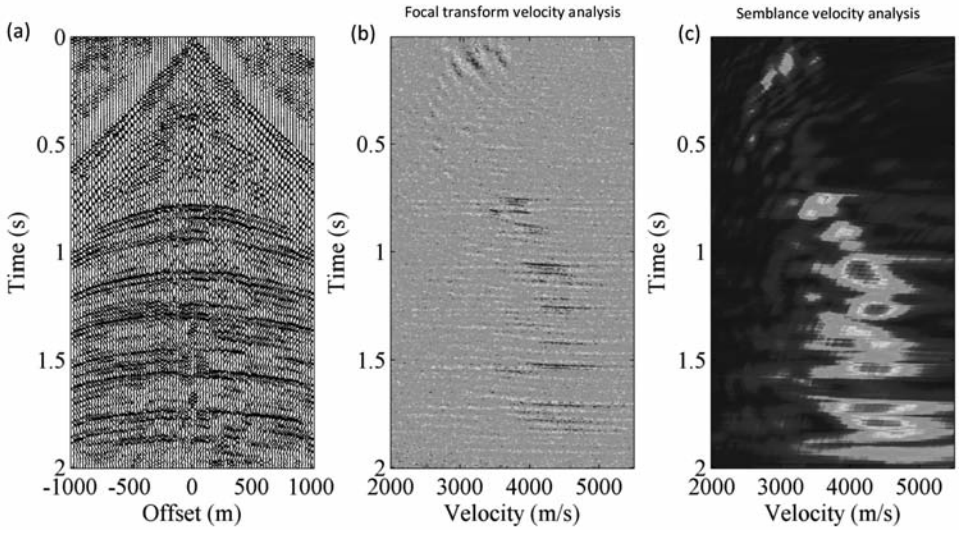


Fig. 10. (a) The real land data set. (b) The velocity panel calculated by the focal transform velocity analysis, and (c) the velocity panel calculated by the semblance method.

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