QUANTUM SEISMIC IMAGING: IS IT POSSIBLE?

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ABSTRACT

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To get an accurate subsurface image from seismic data we need to build a highly accurate velocity model. In most cases this goal is difficult to achieve due to the ill-poseness of the inverse problem. Numerous tomography schemes are suggested and most of them are based on the common image gather flattening. Another scheme, named "full waveform inversion" is connected to data fitting. There are various reasons why exact velocity knowledge is impossible. A fundamental problem in velocity estimation is related to the erroneous measurements and the stochastic nature of the subsurface velocity. In this case the velocity model should be represented by a probability density function, rather than a unique deterministic value and a single velocity model generally does not exist.

In this paper we discuss an alternative way to look at seismic imaging using the quantum mechanics concept and path integral idea. The method computes the image by summing contributions of individual signals propagated along all possible paths between the source and observation points. In fact it samples different paths between the source and receiver instead of relying on only one path derived from Fermat's principle. All random ray or wave trajectories between the source and receiver within this volume are, in principle, taken into account. The focusing mechanism is achieved by a weighting function (probability amplitude), which is designed to emphasize contributions from trajectories close to the stationary one and to suppress contributions from unlikely paths. The presented examples demonstrate principles and feasibility of the new concept. There are many issues still needed to be investigated.

KEY WORDS: imaging, velocity model, path integral, quantum mechanics, stationary phase.

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INTRODUCTION

What do we need to achieve an accurate subsurface image? The answer to this question would be: an accurate velocity model or a fundamentally new imaging method which does not require precise velocity information. The first option is dominating today and it is common to think that to get a correct image of the subsurface requires a highly accurate velocity model and that depth and velocity are inevitably linked. This is why velocity model building traditionally attracts attention of seismic imaging research. Numerous travel time tomography schemes are suggested, different criteria for velocity updating are used. Most of them are based on the so-called common image gather flattening and they are using optimization methods to find optimum of an objective function indicating correct velocity model. Another scheme nowadays popular is "data fitting" full waveform inversion which considered by some people as the only one meaning of velocity inversion. But according to Weglein et al. (2012) "the so-called 'full wave inversion' methods are inverting the wrong and fundamentally inadequate P to P data, with wrong algorithms, and with a wrong earth model".

There are various reasons why exact velocity knowledge is impossible. A fundamental problem in velocity estimation is related to the stochastic nature of the subsurface velocity. In this case the velocity model should be represented by probability density function, rather than a unique deterministic value (Tarantola, 2005; Koren et al., 1991). In other words, a single velocity model generally does not exist. A collection of many models, which are useful for obtaining a focused image, should be considered. The imaging velocity may not be identical to the stacking, RMS or interval velocity at all length scales and therefore may not be physically meaningful. There is also a problem with the computation and display of the 'best solution' model. The solution of the problem based on interpretation of observations should consist of a proper display of all (or many) solutions that are consistent with the observations (Tarantola, 2006).

Exceptionally, in this chorus of 'the correct model' thinking Arthur Weglein and his co-authors proposed a method to compute an accurate depth image without the velocity model (Weglein et al., 2000). They use the inverse scattering series (ISS) and argue that ISS imaging algorithm can directly output the correct subsurface spatial configuration without the velocity model.

In this paper we introduce and discuss another way to look at model-independent seismic imaging using the quantum mechanics concept. It is not a new that in principle the quantum mechanics "can explain all the phenomena of the physical world except the gravitational effects and radioactivity" (Feynman, 1988). So, there is a good chance that it can be used for explaining seismic wave propagation in the real earth as well. Path integrals which are so useful in quantum mechanics have been introduced in seismic wave modeling (Lomax, 1999; Schlottmann, 1999). The path-integral method computes the wavefield by summing the contributions of individual signals (wave functions in quantum mechanics) propagated along all possible paths between the source and observation points. It samples different paths in a large volume of paths between the source and receiver instead of relying on only one path derived from Fermat's principle. All random ray or wave trajectories between the source and receiver within this volume are, in principle, taken into account. The phase contribution for each path is defined by the Lagrangian of the system and the summation of all phase contributions forms the complete seismogram, by constructive and destructive interference. Attempts using quantum ideas for seismic imaging can be found in Landa (2004), Keydar (2004) and Landa et al. (2006). Kelamis et al. (2006) used the path-summation for optimizing land multiple attenuation procedure. Schleicher and Costa (2009) show how the multipath summation can be modified to extract a meaningful velocity model together with the final image. By executing the path-summation imaging twice and weighting one of the images with the used velocity value, the stationary velocities that produce the optimal image can be extracted by a division of the two images.

FEYNMAN PATH-INTEGRAL FORMULATION OF QUANTUM MECHANICS

To start, let me introduce few basic quantum electrodynamics principles using Richard Feynman's way he looked at the world. According to his famous path integral approach the world is kind of tapestry in which all kind of things can happen. To predict the future one needs to start with a known state in the past, allow everything to happen in the intermediate time in all possible ways, when every field and every particle can move around as much as it wants in all directions. And at the end you simply sum up the contributions from all the histories in between. Each history contributes certain probability amplitude. The amplitude is just an integral of the Lagrangian over time and space volume between past and the future. In Feynman's path-integral approach a particle does not follow a single trajectory x(t). It is assumed that it follows every possible path in the space-time domain when each of the trajectories has its own amplitude and phase. Thus each trajectory contributes a different phase to the total amplitude of the wave function. The phase of the contribution from a given path is equal to the action S for this path in units of the Plank's constant.

Let us consider an arbitrary path x(t) between two points a and b. The quantum mechanics rule for computing the probability of the particle going from point a to point b is (Feynman and Hibbs, 1965):

$$P(b,a) = \sum_{\substack{\text{over all}\\ \text{possible}\\ \text{trajectories}}} \varphi[x(t)] \quad , \tag{1a}$$

where

$$\varphi[x(t)] = const\exp\{iS[x(t)/\hbar]\} \quad . \tag{1b}$$

In the classical physics case it is not absolutely clear how only one trajectory (Fermat's) will give the most important contribution. From (1), it follows that each trajectory makes a different phase contribution. The classical Newton's physics represents the case when mass, time interval, and other system parameters are large and the action S is much greater than the constant $\hbar(S/\hbar \rightarrow \infty)$. In this case, the real part of the function φ (the cosine of angles) can assume negative and positive values. If we now change the trajectory by a small amount, the change in action S will also be small. But these small trajectory changes will lead, in principle, to very large changes of the phase and to very rapid oscillations of cosine. Thus different trajectories give positive and negative contributions, and they cancel each other out. On the other hand, for a stationary Fermat's trajectory, small perturbations do not lead in practice to changes in the action S. For the classical case (1) can be schematically written as:

$$P[x_0(t)] = F[x_0(t)]\exp\{iS[x(t)/\hbar]\} \quad , \tag{2}$$

where F is a smooth functional of the path x(t) and $x_0(t)$ is the Fermat path with stationary action:

$$\nabla_x S = 0 \quad .$$

QUANTUM SEISMIC IMAGING

Can Feynman's path integral idea be used for seismic imaging? In analogy to the path integral method we can construct the seismic image by summation over the contributions of elementary signals (wave functions in quantum mechanics) propagated along a representative sample of possible paths between the source and receiver points. It is precisely this mechanism, namely, summation and cancellation, which can be applied in seismic imaging. Clearly, such imaging reminds us of conventional Kirchhoff type migration, however with the difference that it does not assume one particular travel time path for each contribution. Instead, it represents the seismic wave as taking any possible path between the two points. All random trajectories between the source and receiver are, in principle, taken into account. The summation of all contributions generates the complete image, by constructive and destructive interference. It is interesting to note also that eq. (2) has the same form as expression in asymptotic ray theory, where the wave equation solution is written as a smooth amplitude function multiplied by the exponential of a stationary travel time.

On the face of it, imaging based on the path integral in an unknown velocity model does not make sense, and integration over arbitrary random trajectories does not lead to a focused image. But as quantum mechanics views classical physics as limiting case, the path integral seismic imaging method provides a new theory that views the conventional seismic imaging as a special case. The new imaging framework reduces to conventional imaging algorithms when the velocity model is known exactly and adequate to reality.

The path integral imaging can be considered in both time and depth domain. There are three important applications: stacking to zero-offset, time migration and depth migration. In all cases, the path integral consists of integration over many trajectories, rather than an optimization for one single trajectory over which the data is finally to be stacked. For a stack to zero-offset, the path integral consists of a summation of prestack seismic data along many stacking trajectories (hyperbolic and non-hyperbolic) instead of only along a single hyperbola corresponding to the stacking velocity in the conventional zero-offset imaging. For prestack time migration (PSTM) or prestack depth migration (PSDM), path-integral imaging consists of a summation of the data over possible diffraction travel time curves, instead of only along a trajectory corresponding to the estimated migration velocity (like it is done in the Kirchhoff-type migration).

I introduce a heuristic construction, based on the path integral idea of seismic imaging without knowing or estimating the velocity model. Let us consider the classical image V_0 for a subsurface location $\mathbf{x}$ in the form

$$V_0(\mathbf{x}) = \int d\xi \int dtU(t,\xi)\delta[t - t_\eta(\xi,\mathbf{x},\mathbf{v}_0)] \quad , \qquad (3)$$

where $\mathbf{x}$ corresponds to (x,t_0) for stack to zero-offset or time migration and to (x,z_0) for depth migration. $U(t,\xi)$ is the recorded input seismic data for an arbitrary source-receiver configuration parameterized by the position ξ, $\mathbf{v}_0$ is the optimal summation path parameter (the optimal stacking or migration velocity), t_η is the summation path over the reflection (for stacking) or diffraction (for migration) travel time curve.

The data is integrated over the observation aperture ξ. The conventional imaging procedure requires a known imaging velocity model $\mathbf{v}_0$. Usually it is achieved by estimation process based on an optimization procedure which results in searching the signal semblance maximum along reflection or diffraction curves or maximizing the flatness of CIGs.

Now let us think in model-independent context. From eq. (3) we can conclude that imaging can be regarded as a function of the summation travel time paths rather than the velocity. In fact, velocity role in the imaging is only helping to define the optimal (stationary) summation paths. Following the path integral concept introduced above, we consider a set of possible time trajectories $t(\xi,\mathbf{x})$ and use these trajectories for image construction. Again, we do not optimize parameters or models, but integrate over a representative range of trajectories. The integration is weighted by a function which is designed to attenuate contributions from unlikely trajectories and emphasize constructive contributions. In this case (3) can be re-written as follows:

$$V_q(\mathbf{x}) = \int d\beta W(\mathbf{x},\beta) \int d\xi \int dt U(t,\xi)\delta[t - t_q(\xi,\mathbf{x},\beta)] \quad , \tag{4}$$

where $t_q(\xi,\mathbf{x},\beta)$ represents all possible trajectories dependent on multi-dimensional parameter β, and $W(\mathbf{x},\beta)$ denotes the weighting factor. Integration is done over all possible values of parameter β. Subscript q stays here for "quantum". For the sake of simplicity, vector β can be considered as stacking or interval velocity.

I choose the weighting function W in form used in the path integral:

$$W(\mathbf{x},\beta) = \exp[i\lambda S(\mathbf{x},\beta)] \quad , \tag{5}$$

where S is some functional of the data U which indicates action and can be computed for any trajectory $t_q(\xi,\mathbf{x},\beta)$, λ is a large number playing the role of the Plank's constant in quantum mechanics.

Function S can be some coherency measure (semblance or differential semblance), CIG flattening etc. In this case we get

$$V_q(\mathbf{x}) = \int d\beta V_0(\mathbf{x},\beta)\exp[i\lambda S(\mathbf{x},\beta)] \quad , \tag{6}$$

where $V_0(\mathbf{x},\beta)$ is the classical image computed for stationary summation path $t_q(\xi,\mathbf{x},\beta_0)$ and β_0 is the optimal/stationary parameter vector.

Eq. (6) describes a new seismic imaging and it has a structure of the path integral where vector represents all possible trajectories. Each trajectory contributes a different phase to the total amplitude of the image. The phase of the contribution from a given path is proportional to the action S for this path. I refer to this new imaging algorithm as quantum seismic imaging. If we choose an oscillatory weighting function (5) in eq. (4), the imaging algorithm has a form of the Feynman path integral (1). For an exponential weighting function,

$$W(t,\xi,\mathbf{x},\beta) = \exp\{-\lambda S[t_q(\xi,\mathbf{x},\beta)]\} \quad , \tag{7}$$

we have the Einstein-Smoluchovsky path integral, which was first introduced in the theory of Brownian motion (Einstein and Smoluchovsky, 1997). Note that it only differs from (5) in that $i = \sqrt{-1}$ in the exponent is replaced by -1.

It is straightforward to show that the path-integral image $V_q(\mathbf{x})$ converges to the classical limit $V_0(\mathbf{x})$ in an asymptotical sense. This can be done by a stationary-phase approximation (Bleistein, 1984; equation 2.7.18), under the assumptions that for stationary value of $\boldsymbol{\beta}$,

$$S'(\beta_0) = 0 \text{ and } S''(\beta_0) \neq 0 \ ,$$

$$V_q(\mathbf{x}) \approx KV_0(\mathbf{x}) \ .$$

The stationary-phase approximation shows that the quantum seismic image approaches the classical stack up to a constant factor K.

Numerical implementation of the path-integrals in general and quantum seismic imaging [eq. (6)] in particular, is a challenging and difficult task. Among the difficulties are:

- the choice of possible paths/trajectories;
- computation of the probability amplitude ("wave function") for summation of the elementary contributions;
- the choice of integration limits;
- the choice of a proper value for the parameter λ;
- the integration step size.

All this requires a complicated mathematical apparatus and enormous computer power which exceeds theoretically possible modern computer performance and may be possible only with future quantum computers (Feynman, 1982). In fact, it should not be surprising that quantum imaging requires quantum computing!

In this paper I will only be showing the feasibility of the proposed approach and illustrating it by making several simplifications and assumptions. Firstly, I replace the parameter the vector $\boldsymbol{\beta}$ with vector $\mathbf{v}$, where $\mathbf{v}$ has a meaning and dimensionality of velocity (stacking, RMS, interval, average), and I use this velocity to compute all possible travel time trajectories $t_q(\xi,\mathbf{x},\mathbf{v})$ for imaging. This trick allows essentially reducing the number of possible trajectories and makes it possible to run the imaging procedure (4). At the same time it does not compromise on the model-independent concept. It merely provides an effective selection of realizable travel time trajectories and excludes unrealistic ones. Secondly, the action measure S from (5) is chosen as flatness (semblance) along the horizontal direction on the CIGs computed for each set of trajectories $t_q(\xi,\mathbf{x},\mathbf{v})$.

In this context, I emphasize again that there is not necessarily a single velocity model that results in an optimally focused image. Instead, a weighting function $W(\mathbf{v})$ which depends smoothly on $\mathbf{v}$ does not require any optimization or precise knowledge of the correct velocity. Note that optimization for velocity implies a choice for a single optimum, whereas the used weighting function allows taking several optima into account, as well as the uncertainty in any trial velocity. Computing possible summation paths via possible velocity models can be computationally efficient for time imaging where class of stacking/migration velocity models can be described by only few parameters. For depth migration we still do not know what an effective set of parameters could be, this is topic of current and future research.

EXAMPLES

A simple example of imaging without precise velocity is presented in Jedlicka (1989). He considered a case of zero-offset imaging (stack) and assumed that the NMO velocity is estimated with uncertainties. In this case it is considered as random function and can be characterized by a probabilistic density function (PDF) which is assumed to be known. The author introduces a concept of stochastic moveout correction (SMOC) which consists of stacking CMP traces along a range of velocities:

$$image = \sum_{v=v_{min}}^{v=v_{max}} p(v)\,data(NMO_v) \quad , \tag{8}$$

where *image* is the stacked trace, is the probability that an event of the non-perfect hyperbola is an event with the velocity v, v_{min} and v_{max} are possible minimum and maximum velocities, respectively, and *data*(NMO_v) means NMO with velocity v.

By construction, the described imaging algorithm can be included in the class of quantum imaging algorithms. Fig. 1 shows application of the SMOC to a real data set. One CMP gather (left) was used to construct velocity stack panels [the inner product in (8)]. Comparison between conventional velocity stack panel with the SMOC velocity panel (right) shows strong filtering effects of the proposed procedure and much cleaner resulting picture. The resulting image trace (horizontal summation of the central panel) of the SMOC processing takes into account velocity uncertainties and can be considered as quantum image.

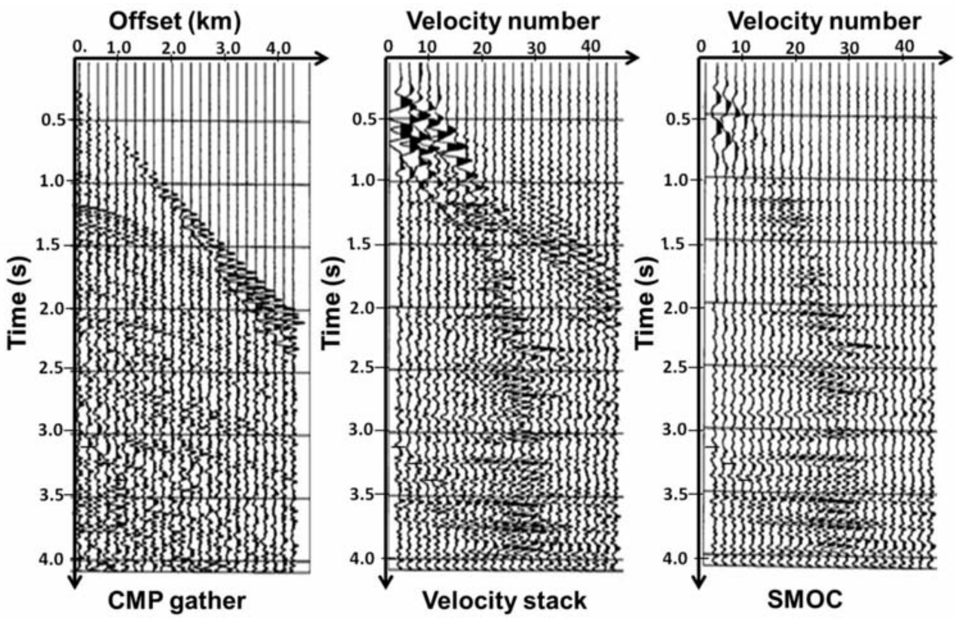

Fig. 1. Left: CMP gather. Center: conventional velocity stack. Right: velocity stack using SMOC. The right picture is much cleaner. (Reproduced from Jedlicka, 1989).

Fig. 2 shows a near-offset section extracted from a 3D marine data (Y = 3 km). Full prestack data were used to compute a 3D stacked cube by the quantum imaging algorithm as it described above. Imaging process did not include any velocity analysis or parameter estimation procedures. 500 hyperbolic trajectories were used for weighted summation [eq. (4)] at each CMP position and each time sample. The resulting cube is displayed in Fig. 3a. For comparison, Fig. 3b shows the extracted stacked section of the same line as in Fig. 2. There is practically no difference between the two images confirming the fact that quantum imaging can create a correct subsurface image without a priori known velocity model. Note that our procedure takes into account possible azimuthal changes in the stacking parameters. It is important to emphasize that in principle non-hyperbolic summation trajectories can be used essentially increasing the validity of the CMP stacking procedure.

Fig. 4 shows the quantum time migration of part of the Sigsbee data set. For each imaging sample 600 different hyperbolic travel time trajectories $t_q(\xi,\mathbf{x},\mathbf{v})$ were used in eq. (4). This is equivalent to scanning of 600 different migration velocity models in a range of 1500 m/sec to 3000 m/sec with step of 25 m/sec. Let me emphasize again that there is no velocity optimization and/or

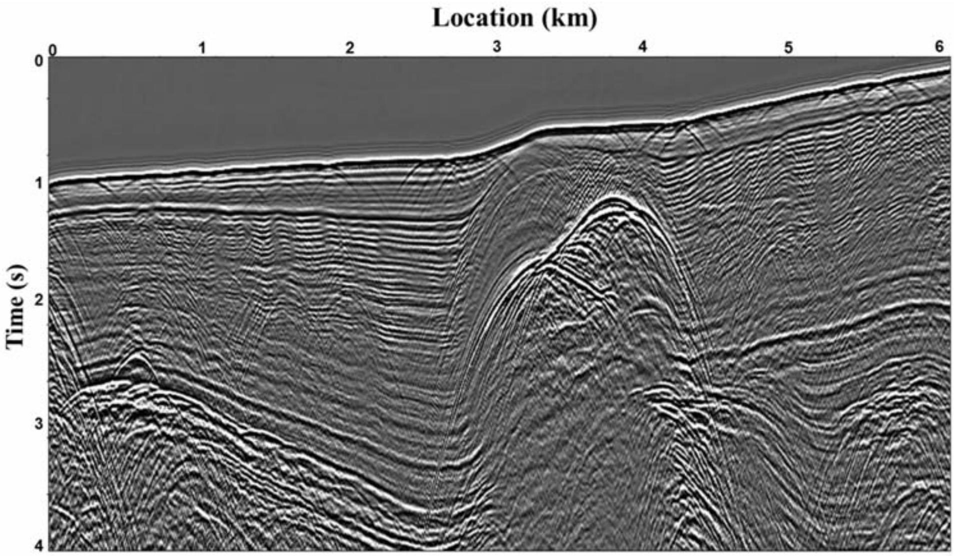

Fig. 2. Near-offset section extracted from 3D marine data set.

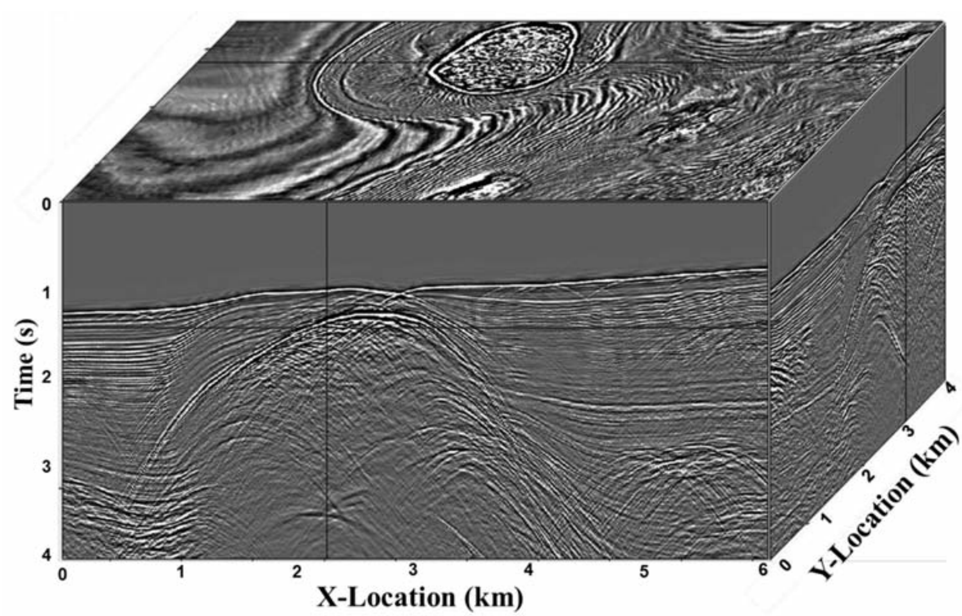

Fig. 3a. Quantum stacked cube.

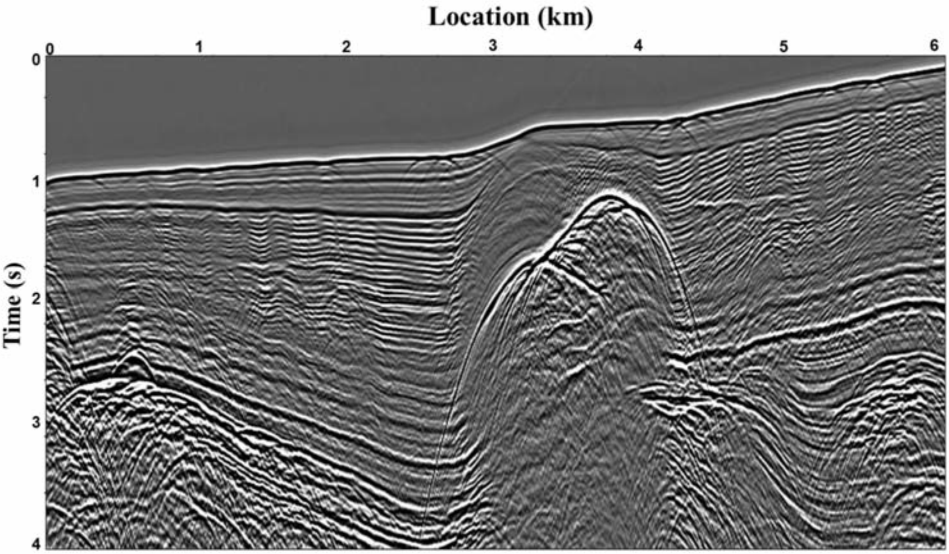

Fig. 3b. Seismic line extracted from the quantum cube (compare with Fig. 2).

semblance maximization in the imaging procedure. Each possible trajectory contributes to the final image whereas each contribution is weighted by the probability amplitude computed by eq. (7). The measure S was taken as the CIGs flatness computed for each value of v. As it is predicted by theory in this example the results is practically equivalent to the standard PSTM with optimal migration velocities. Continuous reflectors as well as diffractors and faults are perfectly focused and correctly positioned.

The difficulty with depth imaging, compared to time, is that for a given travel time trajectory $t_q(\xi,\mathbf{x},\beta)$ the image can be located at very different depth positions. It is interesting, that this happens even for constant velocity models and horizontal reflectors. Correct focusing and positioning in this case require careful choice of the constant λ in the expression (7). In addition, the travel time trajectories for depth imaging are often non-hyperbolic and require a large number of parameters to describe all possible travel time trajectories. To avoid the problem of having tremendously huge number of possible time trajectories, in this paper I choose to use hyperbolic approximation for time trajectory which is equivalent of using average velocity in depth migration. This practically corresponds to moderate complexity assumption.

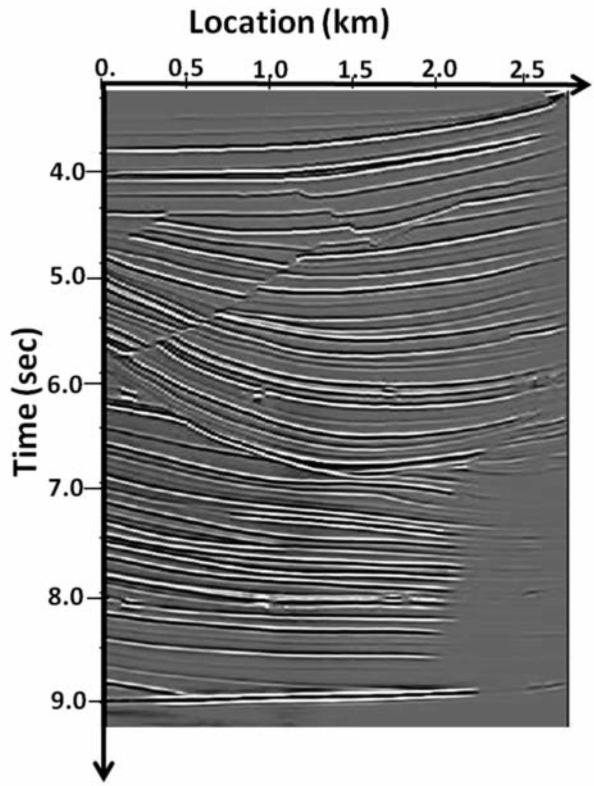

Fig. 4. Quantum time migrated image of the Sigsbee model (after Landa et al., 2006).

Fig. 5 shows a model, consisting of dipping and curved reflectors, and 4 diffractors. The velocity contains vertical gradient and is defined by the function v = 1500 + 0.3z. Four hundred shot gathers located along seismic line with inter shot distance of 25 m served as an input to the imaging procedure. I used 101 different constant velocity values between 1500 m/sec to 2500m/sec to generate a sufficiently general sampling of the set of physically realizable traveltime curves for each imaging sample. These traveltimes are used to produce CIGs and to measure the "action" function S [eq. (7)].

Fig. 6 shows the quantum depth image which is comprised of the contributions of possible trajectories for each image sample. Parameter λ in the weighting function (7) was chosen equal to 20 by 'trial and error'. Four shallow reflectors and diffraction well focused and correctly positioned. Two deepest reflectors are positioned with an error of about 1% and 2%, respectively. It happens because a simple hyperbolic approximation for the travel times I used in this example is not accurate enough for very long offset (8000 m in this example).

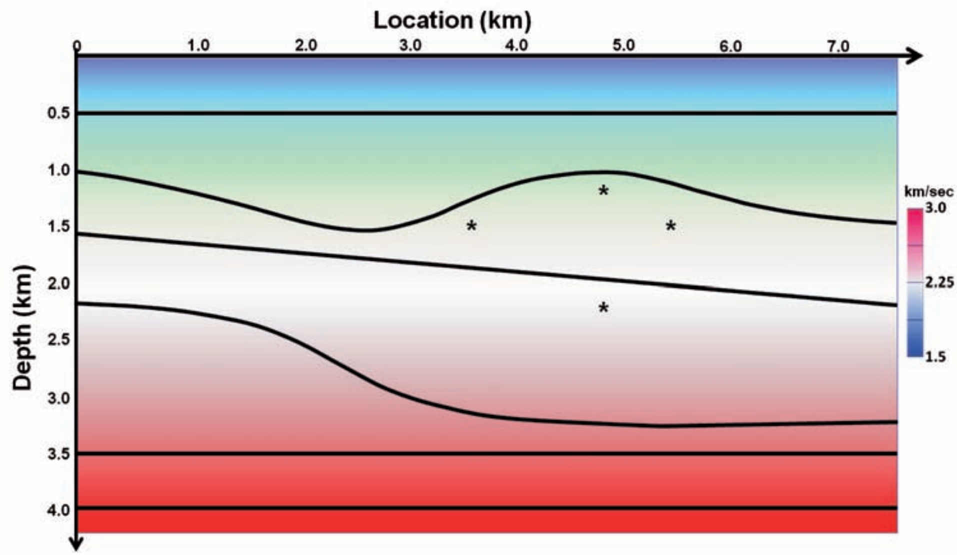

Fig. 5. Synthetic model with velocity function $v = v_0 + kz$.

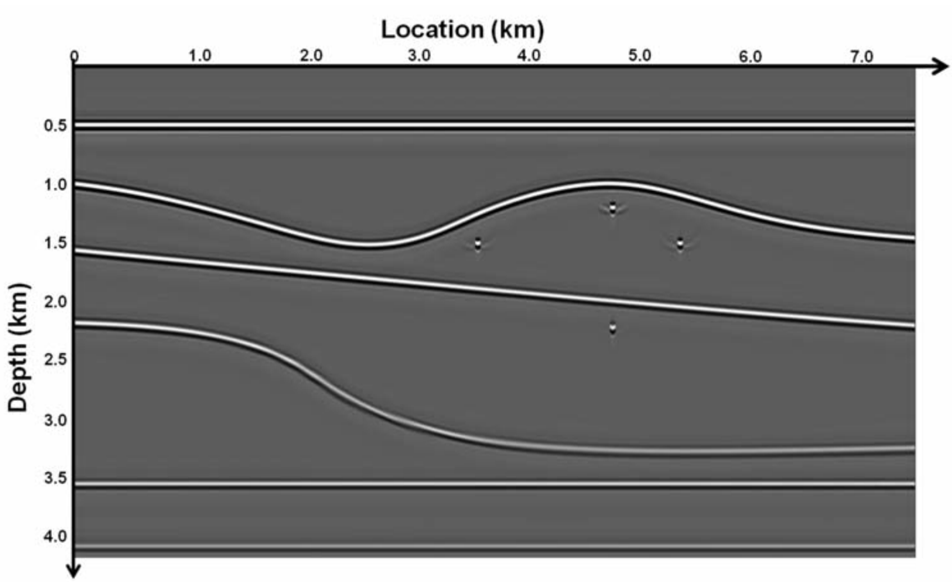

Fig. 6. Quantum depth image obtained without knowledge on velocity model.

But even with this inaccuracy the example illustrates that the quantum depth imaging with a well-designed weighting function is capable of positioning the reflectors and diffractors properly without knowing the correct velocity model.

More examples of quantum imaging on synthetic and real data can be found in Landa et al. (2006).

DISCUSSION AND CONCLUSIONS

Quantum seismic imaging method provides a new and promising framework for subsurface imaging without precise knowledge or selection of a velocity model. In this framework the conventional step of velocity analysis can be avoided. Quantum seismic imaging can be considered as a model-independent technique, since it does not involve any velocity or parameter estimation in a common sense. The image is constructed by summation over many (ideally all) possible travel time trajectories. The focusing mechanism is achieved by a weighting function (probability amplitude), which is designed to emphasize contributions from trajectories close to the optimal one and to suppress contributions from unlikely paths.

The quantum imaging converges to a standard imaging procedure only in trivial situations of a deterministic and known velocity model. But what happens when the model is unknown, random or estimated with uncertainties (which usually the case in practice), or even worse, the model does not describe adequately the wave propagation process in the real earth? In my opinion, it happens more often that we think (examples: wrong parameterization of model description, randomness of the velocity function, velocity dispersion etc.). In this case a single stationary path does not describe adequately ray/wave propagation process and conventional imaging does not produce a correct focused subsurface image. In contrary, quantum imaging using all possible trajectories accounts for multiple stationary paths and take into account model uncertainties.

In this paper I did not propose or describe completely developed and efficient algorithm or computational scheme for quantum seismic imaging. Rather I introduced a new and unconventional view on seismic imaging problem. This view is based on analogy to fundamental and general ideas of quantum mechanics and provides an appealing framework for new way of thinking and acting. Applications to synthetic and real data, both in time and in depth, show the feasibility and potential of the proposed method and represent progress in seismic imaging.

Of course, the algorithm described does not pretend to be a present-day efficient operational tool and is used only for illustration purposes. There are many issues still needed to be investigated: the implications of the choice of a weighting function, quality control of images, amplitude control, efficient implementation of path integral summation etc.

Presenting the quantum seismic imaging idea to different people and at different occasions, I've heard sometimes a skeptical comment: "What's new here? Practically we do the same in Kirchhoff migration by summing contributions from elementary signals". First, there is no contradiction between these two things. I already showed and emphasized that quantum mechanics views classical physics as limiting case. And when the adequate and correct velocity model exists and known the quantum imaging algorithm converges to Kirchhoff migration. But in reality, for the real Earth with its almost continuous spectrum of heterogeneities it is difficult (if possible at all) to expect that we find a way to describe and estimate the true subsurface model. It has been known for a long time that if the objects to be imaged are in a richly scattering environment (which is probably the case of the real Earth subsurface) then most migration algorithms do not work well. This happens because the response from a reflector in the traces recorded has a lot of time shifts or coda that is generated by the inhomogeneous medium. As a result, classical migration leads to unreliable and unfocused images that depend unpredictably on the detailed features of the media. In this case interferometry (which is also called matched field imaging) can be used (Borcea et al., 2005; Schuster, 2009). This procedure in general is statistically stable (self-averaging with respect to the random fluctuations in the medium properties) and can be considered in the path integral framework. Moreover, Bayesian approach, Monte Carlo and simulated annealing methods can also be formulated and interpreted in terms of the Feynman path integral (Lemm et al., 2005; Lee et al., 2000).

In reality we can compute only probabilities of an event. "Does this mean that physics, a science of great exactitude, has been reduced to calculating only the probability of an event, and not predicting exactly what happens? Yes. That's a retreat, but that's the way it is: Nature permits us to calculate only probabilities. Yet, science has not collapsed" (Feynman, 1988).

And finally a few words to skeptics ... A remarkable property of physics is that nature can be described in different ways. Richard Feynman referred to this during his Nobel Prize address. He wrote: "It always seems odd to me that the fundamental laws of physics, when discovered, can appear in so many different forms that are not apparently identical at first, but, with a little mathematical fiddling you can show the relationship .... There is always another way to say the same thing that does not look at all like the way you said it before .... I think it somehow a representation of the simplicity of nature .... Perhaps a thing is simple if you can describe it fully in several different ways without immediately knowing that you are describing the same thing".

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