EIGENIMAGE WAVELET TRANSFORM FOR GROUND ROLL ATTENUATION: A CASE STUDY ON AN IRANIAN OILFIELD

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ABSTRACT

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Coherent noises such as ground roll, guided waves, multiples and refractions are usually present in seismic records. In land surveys, the ground roll can cause serious problems since it has higher energy and stronger amplitude than reflected signals. In this case, detection of reflections becomes very difficult and sometimes impossible. Common methods in suppressing the ground roll noise are based on high-pass and frequency-wave number filters. In spite of the fact that these filters are commonly used in attenuating the ground roll noise, they have some limitations, e.g., signal distortion, data aliasing and artifacts. Therefore, we need to consider alternative methods. Wavelet transformation provides a mean to analyze signals simultaneously in both time and frequency. With this approach, we can analyze the time evolution of the signal's frequency content. Therefore wavelet-based filters can easily differentiate between early arriving ground roll signals and late arriving signal of interest. In other words, signal components at higher scales (lower frequencies) and specific time periods consist mainly of noise. Since there is usually some overlap between the (desired) signal and noise, it is necessary to introduce a measure to determine whether a certain time-scale region is dominated by noise or signal.

Here we use singular value decomposition as a criterion to separate signal from noise. Our results show that proper setting of the new filter parameters result in distinct signal and noise frequency bands, arrival times, and energies. This approach causes less signal distortion when compared with conventional 1D wavelet transform or singular value decomposition.

KEY WORDS: eigenimage, wavelet transform, ground roll, time-scale filtering, coherent noise filtering.

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INTRODUCTION

Wavelets were first discussed in an appendix to Haar' thesis (1909). One useful property of the Haar wavelet is its compact support, i.e., it vanishes outside of a finite interval. Haar wavelets are not continuously differentiable which somewhat limits their applications. Daubechies (1992) elegantly constructed a set of wavelet orthonormal basis functions that become the cornerstone of the wavelet applications today.

Wavelet transform have been applied to ground roll attenuation by Deighan and Watts (1997), Abdul-Jauwad and Khene (2000), Corso et al. (2003), Leitea et al. (2008) and Hamidi et al. (2013). Analysis of seismic traces based on wavelet transform produces time-scale wavelet coefficients. The scale in wavelet transform provides a measure of frequency analysis and can be used for the purpose of filtering. Ground roll energy contaminates traces in a time-limited fashion and is represented in higher scales. As a result, wavelet transform provides a basis to separate the undesired ground roll from desired later arriving weak reflections. Moreover, these filters only affect wavelet coefficients that are related to ground roll, leaving other parts of data unaltered.

Singular value decomposition (also known as Karhunen-Loeve transform) is a powerful tool to detect and enhance laterally coherent signals in multi-traces recordings. It has been implemented in a variety of seismic applications like dip filtering, VSP up/down wave-field separation, and residual statics corrections (Freire and Ulrych, 1988). The Singular value decomposition (SVD) is suitable for data where coherent events can be aligned laterally, i.e., shot records, NMO-corrected CMP (common-midpoint) gathers and stacked sections. Coherent signals in multi-trace data are extracted using eigenvalue decomposition of the data-covariance matrix after an initial alignment of events by means of dip steering. This is done by only including the contribution of the largest singular values. These contributions represent the high amplitude laterally coherent signals. Contributions of the smallest singular values are related to the low amplitude background noise. Application of this transform in seismology was introduced by Hemon and Mace (1978). They used a zero-lag KL transform to improve horizontal events. Sacchi (2002) discussed the relationship between the KL and SVD showing that the SVD transformation is another way of viewing the KL transformation.

In this paper, a hybrid filter is developed to suppress the ground roll noise in land seismic data. The proposed method combines the one dimensional wavelet transform (1DWT) with eigenimage analysis; called eigenimage wavelet transform (EIWT). The basic steps of the approach consist of applying 1DWT to the trace and constructing eigenimages of the wavelet coefficient-matrix. At first, a seismic trace is transformed into a time-scale domain by one-dimensional DWT. This step has a great importance in better separating the noise and signal and causes the eigenimages more effectively differentiate signal from noise. The algorithm is implemented in MATLAB environment, using Wavelet Toolbox (Misiti et al., 1996). The EIWT is used to separate the ground roll from reflections in seismic data. The results of applying this method to the synthetic and real data are presented. This approach produces eigenimages of the time-scale coefficients and selects eigenimages of higher energy as representatives of the ground roll.

METHODOLOGY

Wavelet transform

Continuous wavelet transform (CWT) of a function in time, e.g., f(t), was defined as (Daubechies, 1992)

$$W_{a,b} = \int_{-\infty}^{\infty} \psi_{a,b}(t) f(t) dt \quad . \tag{1}$$

This transform is based on the basis function $\psi_{a,b}(t)$ defined as

$$\psi_{a,b} = (1/\sqrt{|a|})\psi[(t - b)/a] \quad , \tag{2}$$

where a and b are real numbers ($a \neq 0$) representing the scale and time shift in the fixed kernel wavelet $\psi(t)$, respectively. The inverse transform is defined by

$$f(t) = (1/C_\psi) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (dadb/a^2) W_{a,b} \psi_{a,b}(t) \quad , \tag{3}$$

where

$$C_\psi = \int_{-\infty}^{\infty} (1/|\omega|) |\Psi(\omega)|^2 d\omega \quad , \tag{4}$$

and $\Psi(\omega)$ is the FT of $\psi(t)$. Aforementioned wavelet function $\psi(t)$ is obtained from what is referred to as the mother wavelet. Wavelet transform is not unique in the sense that there are many possible mother wavelets to choose from. However, each mother wavelet must meet certain conditions, as outlined by Daubechies (1992). In summary, these conditions are:

- The kernel wavelet $\psi(t)$ should be absolutely integrable and have a finite energy, i.e.,

$$\int |\psi(t)|dt < \infty \quad , \tag{5}$$

and

$$\int |\psi(t)|^2 dt < \infty \quad . \tag{6}$$

- The kernel wavelet $\psi(t)$ should be band limited, and the low-frequency behavior of the Fourier transform is sufficiently small around $\omega = 0$, so that

$$\int |\hat{\Psi}(\omega)/\omega| d\omega < \infty \quad . \tag{7}$$

The mother wavelet used in this work is the Daubechies four-coefficient wavelet.

Implementation of the WT for a discretely sampled signal is formulated as discrete wavelet transform (DWT). Continuous wavelet transform (CWT) have much redundancy in the signal analysis and cannot be implemented exactly by digital computers. Since parameters (a, b) take continuous values in CWT, the resulting transform is a highly redundant representation of the signal. At the same time, continuous variations of these parameters make CWT impractical. DWT, on the other hand, varies the scale and shift parameters on a discrete grid of time-scale plane leading to a discrete set of continuous basis functions (Daubechies, 1992). The discretization is performed by setting

$$a = a_0^j \text{ and } b = ka_0^j b_0 \text{ for } j,k \in Z \quad , \tag{8}$$

where $a_0 > 1$ is a dilation step and $b_0 \neq 0$ is a translation step. The family of the wavelets then relate to the main function by

$$\psi_{j,k}(t) = a_0^{-j/2}\psi(a_0^{-j}t - kb_0) \quad . \tag{9}$$

Synthesis of the function f(t), in terms of its wavelet coefficients is then given by

$$f(t) = \sum_j \sum_k D_f(j,k)\psi_{j,k}(t) \quad , \tag{10}$$

in which the two-dimensional set of coefficients $D_f(j,k)$ is the discrete wavelet transform of function f(t).

Singular value decomposition

Singular value decomposition decomposes matrix **D** into a weighted sum of orthogonal rank-one matrices. The SVD of **D** is given by (Lanczos, 1961):

$$\mathbf{D} = \sum_{i=1}^{r} \sigma_i u_i v_i^T \quad , \tag{11}$$

where r is the rank of **D**, σ_i is the i-th singular value of **D**, u_i and v_i are the i-th eigenvectors of DDT and DTD, respectively and superscript T denotes matrix transposition. Singular values, σ_i, are square roots of the eigenvalues of matrices $\mathbf{DD}^T$ or $\mathbf{D}^T\mathbf{D}$. These eigenvalues are always positive owing to the positive definite nature of the aforementioned matrices. In the matrix form, eq. (11) is written as:

$$\mathbf{D} = \mathbf{U\Sigma V}^T \quad , \tag{12}$$

where **D** is a (M × N) matrix, **U** is a (M × M) matrix whose columns consist of eigenvectors of $\mathbf{DD}^T$, **V** is a (N × N) matrix whose columns are eigenvectors of $\mathbf{D}^T\mathbf{D}$ and **Σ** is a diagonal matrix with $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \ldots \geq \sigma_M$ as its diagonal elements (Freire and Ulrych, 1988). Since SVD is used to extract ground roll coefficients in the time-scale domain, it acts as a data-driven, low-pass filter by rejecting highly uncorrelated events.

Eigenimage wavelet transform (EIWT)

Considering eqs. (10) and (11), the EIWT of a function, f(t), can be obtained by:

$$f(t) = \sum_j \sum_k \sum_{i=1}^{r} (\sigma_{i,j,k} u_{i,j,k} v_{i,j,k}^T) \psi_{j,k}(t) \quad , \tag{13}$$

where the used parameters are the same as those used in previous equations.

Since the EIWT is applied to trace-by-trace of a seismic signal a chirp signal is used to describe the process (Fig. 1). The frequency content of this signal is 0-50 Hz, the time sampling interval is 2 ms and the total time is 1 s. The wavelet coefficient matrix (Fig. 2) shows how different frequencies are presented in the wavelet domain. Energy of the low frequency components for the early times is placed in higher scales; whereas the higher frequencies of the latter times are placed in lower scales. Another feature of this figure is the coherency of the higher scales, which is a consequence of the coefficient matrix construction process. The number of the sample points is halved at each level of wavelet decomposition; but as it is seen here, they are the same as the original signal (500 samples) at all scales. This is achieved by repeating each sample as much as required to compensate for the down-sampling. It also brings about more coherent higher scales. Because of the higher amplitude of ground

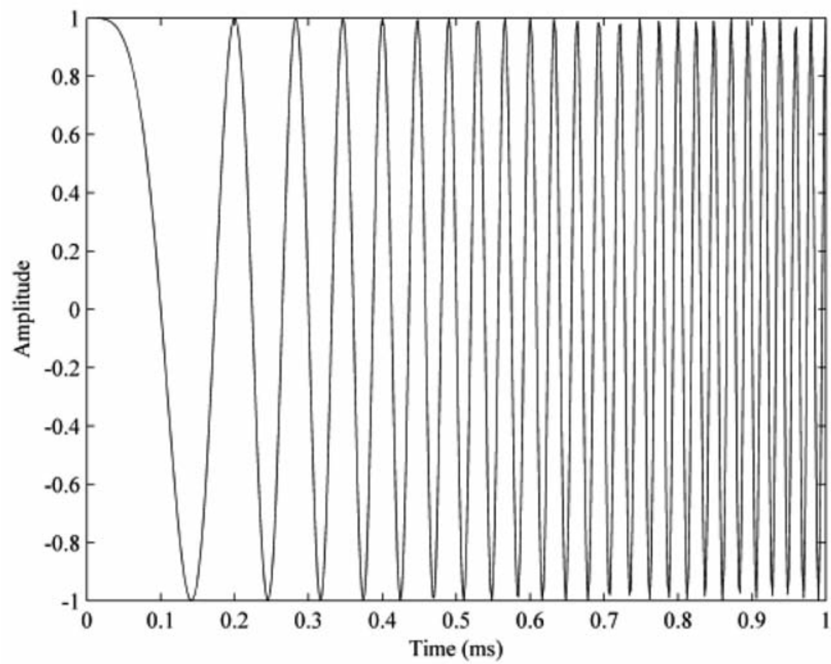


Fig. 1. A chirp signal of frequencies 0 through 50 Hz. The time sampling interval is 2 ms and the total time is 1 s.

roll in the time domain, DWT coefficients associated to ground roll are larger in magnitude than those related to the reflections. That is why the eigenimage analysis is a suitable tool to select the noise-representing coefficients. Since the number of eigenimages is determined by the rank of the coefficient matrix, there would be nine eigenimages (there are 9 scales and so 9 rows in the coefficient matrix) as seen in Fig. 3. Eigenimages are sorted in the descending order of their magnitudes. The i-th eigenimage of the coefficients matrix is a matrix resulted from the outer dot product $u_i v_i^T$ (Andrews and Hunt, 1977) multiplied by its corresponding singular value as a weight.

The following algorithm is used for filtering (Hamidi et al., 2012):

- The maximum possible decomposition level is calculated.
- The time-scale region representing the noise is determined.
- The seismic trace is selected.
- The 1D DWT, applied to the trace, and its coefficient matrix are calculated.

- SVD is applied to the coefficient matrix.
- Eigenimages corresponding to the ground roll coefficients are selected.
- Inverse SVD is calculated.
- The time-scale region corresponding to noise is selected.
- The inverse 1D DWT is used to obtain the extracted ground roll in the time domain.
- The next trace is selected.

The flowchart of this algorithm is shown in Fig. 4. This procedure is performed on all traces and the extracted noise-related part of data is subtracted from the input data to get the filtered data.

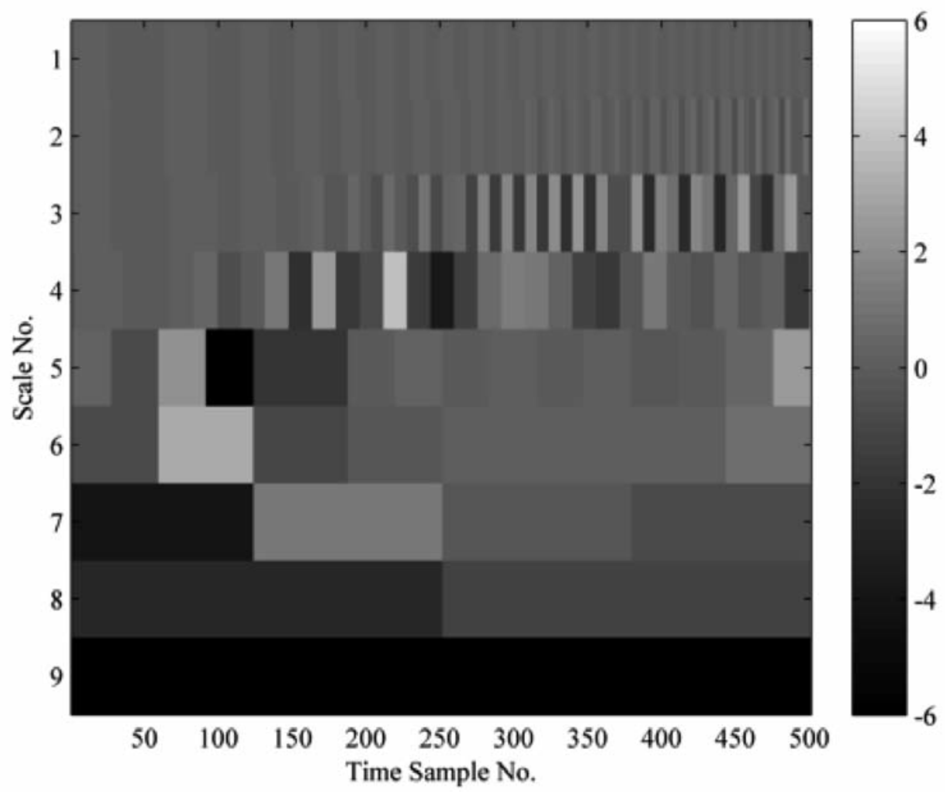
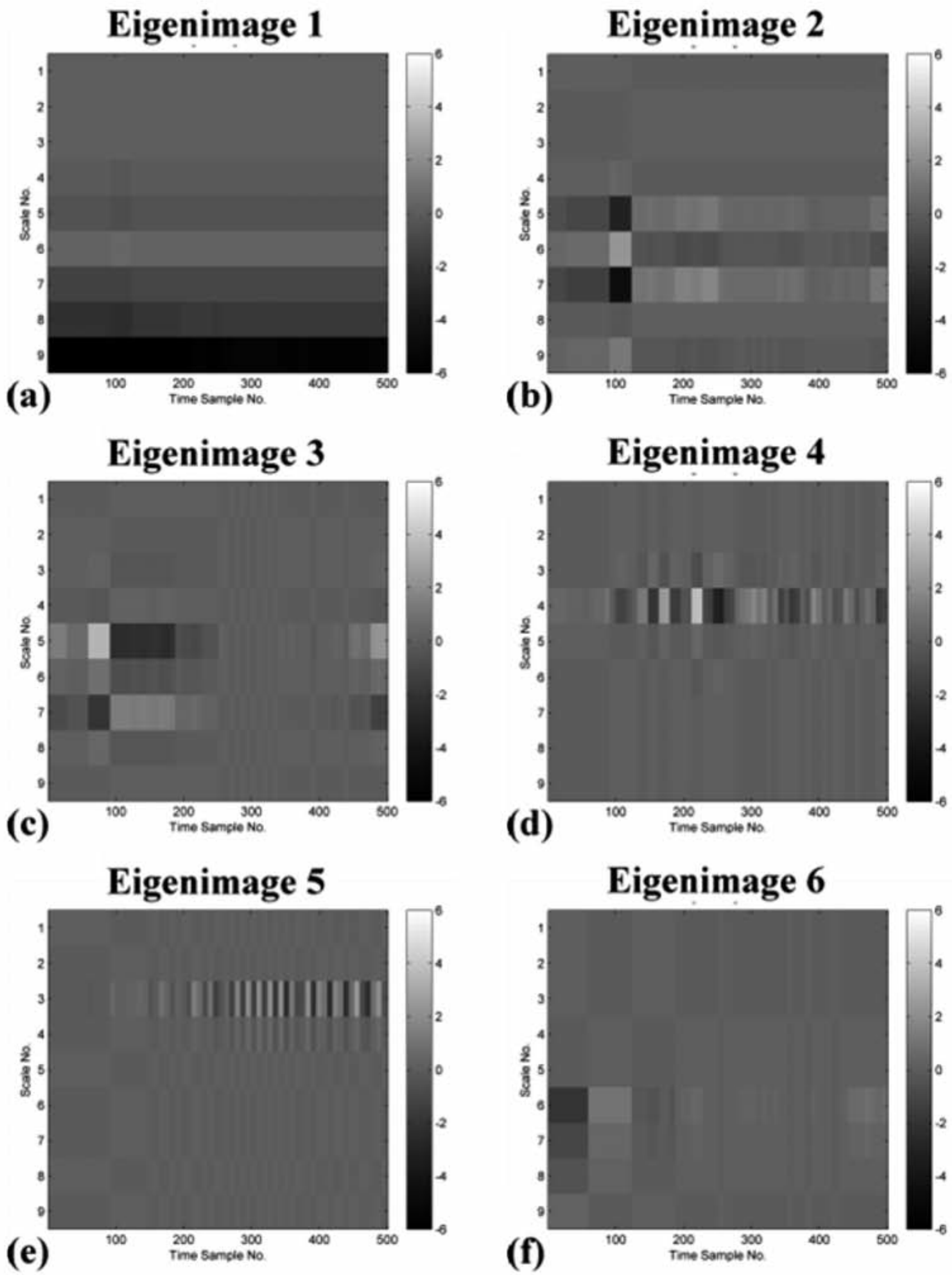


Fig. 2. The 1DWT coefficients matrix of the chirp signal in Fig. 1. Since there are 500 samples in the time domain, the maximum decomposition level of 1DWT as well as the maximum number of scales is nine.


Eigenimage 1
Eigenimage 2
Eigenimage 3
Eigenimage 4
Eigenimage 5
Eigenimage 6
Scale No.
Time Sample No.
(a)
(b)
(c)
(d)
(e)
(f)

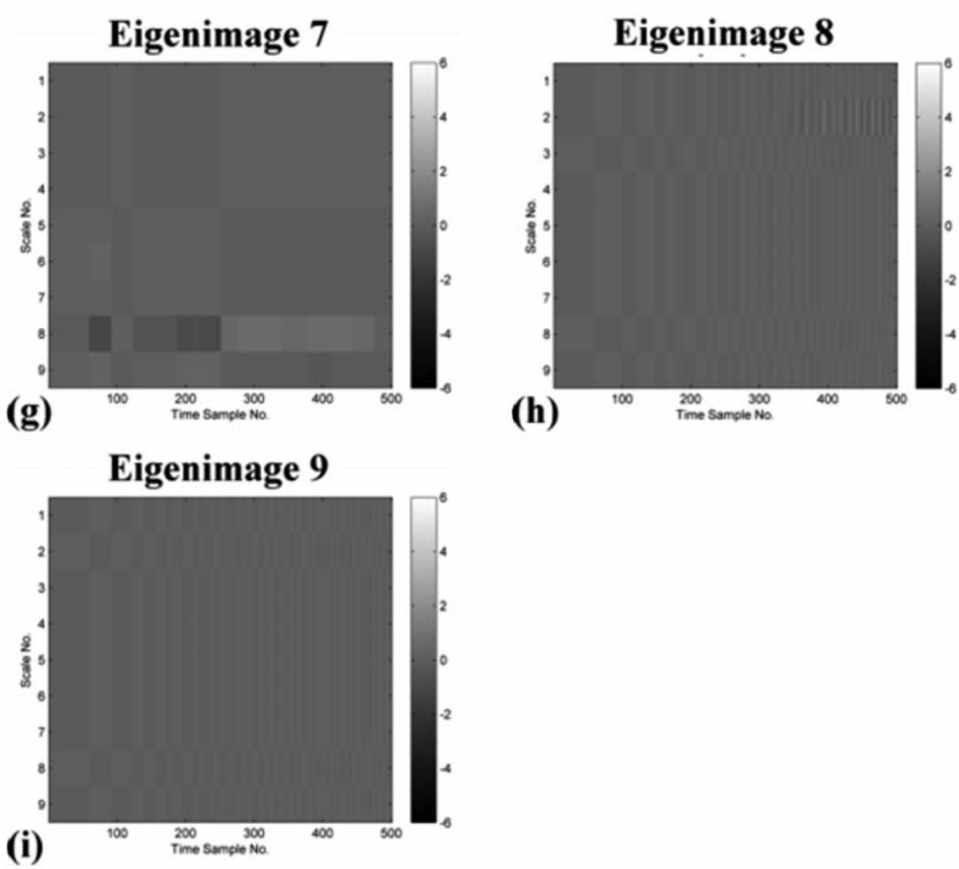


Fig. 3. Different eigenimages of 1DWT coefficients matrix in Fig. 2. Since there are nine scales, there will be a maximum of nine eigenimages.

DATA EXAMPLES

Synthetic data

The synthetic data (Fig. 5) used here contains four dipping events, each with a different frequency content (1-15 Hz, 1-25 Hz, 1-35 Hz, and 1-45 Hz corresponding to the event with the highest dip through the horizontal one) and amplitude (3, 1.5, 1, and 1, respectively). This data set has 100 traces with a time sampling interval of 4 ms and a trace interval of 50 m. Since the ground roll usually has a higher amplitude and lower velocity than reflections, the purpose of filtering synthetic data is to attenuate the event with the highest dip

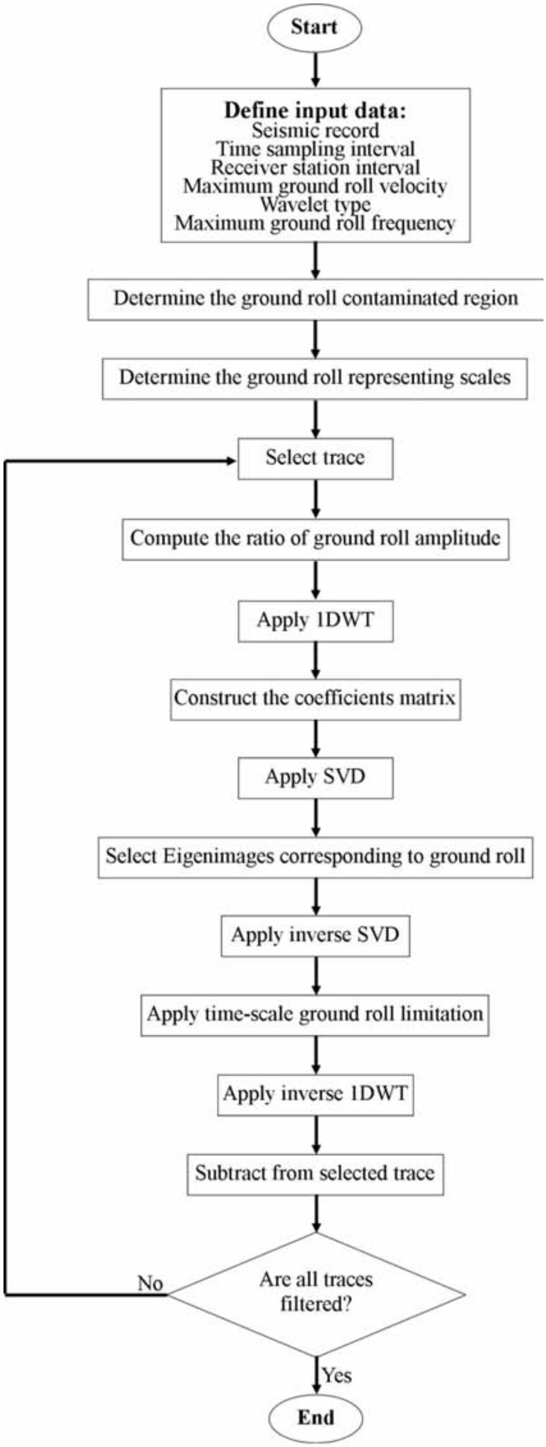


Fig. 4. The flowchart of the EIWT filter.

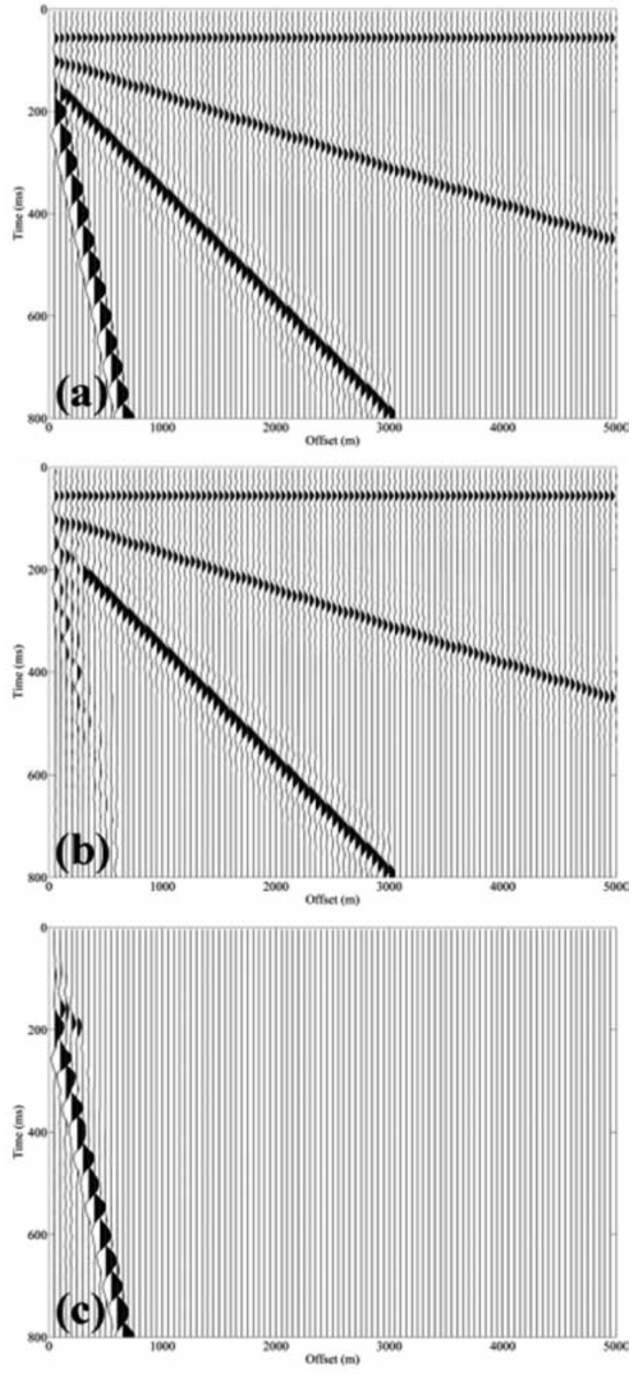


Fig. 5. (a) Synthetic data with 100 traces and total time of 100 ms. There are four different dipping events, containing 1 through 10, 25, 35, and 45 Hz frequency band widths, (b) filtered synthetic data, and (c) the difference between the input and filtered data using the EIWT method.

and the lowest frequency. The result of the filtering is shown in Fig. 5b. It is shown that the filter mostly attenuates the unwanted events while preserving the desired reflections. The difference between the filtered and input data (Fig. 5c) shows that only a little part of the desired events is suppressed in the original data. Figs. 6a through 6g show the result when data is reconstructed by using only one scale at each time. Only four scales (the third, fourth, fifth, and sixth ones) contain the noise coefficients. Considering the third scale, the energy presented by the unwanted coefficients is somewhat similar to those of desired events. Whether these coefficients should be eliminated or not is decided by the SVD on a trace-by-trace basis.

Real data

The real data used in this case is a shot record with a split spread from an Iranian oilfield (Fig. 7a). The true amplitude recovery (TAR) was applied to this data set and the refractions were muted. The high amplitude ground roll is seen in a fan shape part of data below the source location. The black rectangle marks a part of data between traces no. 48 and 236 and the time interval of 3780 through 4940 ms. This section is zoomed and is shown in Fig. 7b. The results of the filtering data by the 1D DWT, the SVD, and the EIWT are shown in Figs. 8a, 8b, and 8c, respectively. For a closer examination of the results, Figs. 9a through 9c show the zoomed version of data in Fig. 7b. Comparison of EIWT with 1D DWT and SVD, shows that the hybrid method has a remarkably better performance when compared with the other two methods. The noise is attenuated more and the desired reflections are not suppressed as much. The average amplitude spectra of data before and after filtering by three discussed methods are shown in Fig. 10.

DISCUSSION

There are three main steps in filters applied to seismic data for suppression of the ground roll noise: (1) data transformation from the t-x domain into the filter domain, (2) separation of the parts representing the desired signal and those representing the ground roll noise in the new domain, and (3) transforming back the filtered data to the original t-x domain. In a filter based on the 1D wavelet transform, traces are transformed from the time domain into the time-scale domain. The signal and noise are separated based on their different arrival times and scales in the wavelet time-scale domain. Dealing with the ground roll as noise and reflections as the desired signal, it is obvious that the desired signal is represented by low scale (high frequencies) coefficients while the noise is represented dominantly by the high scales (low frequencies) coefficients. Each scale is analogous to a frequency band. The frequency band of the ground roll is usually considered to be lower than 18 Hz; therefore, the

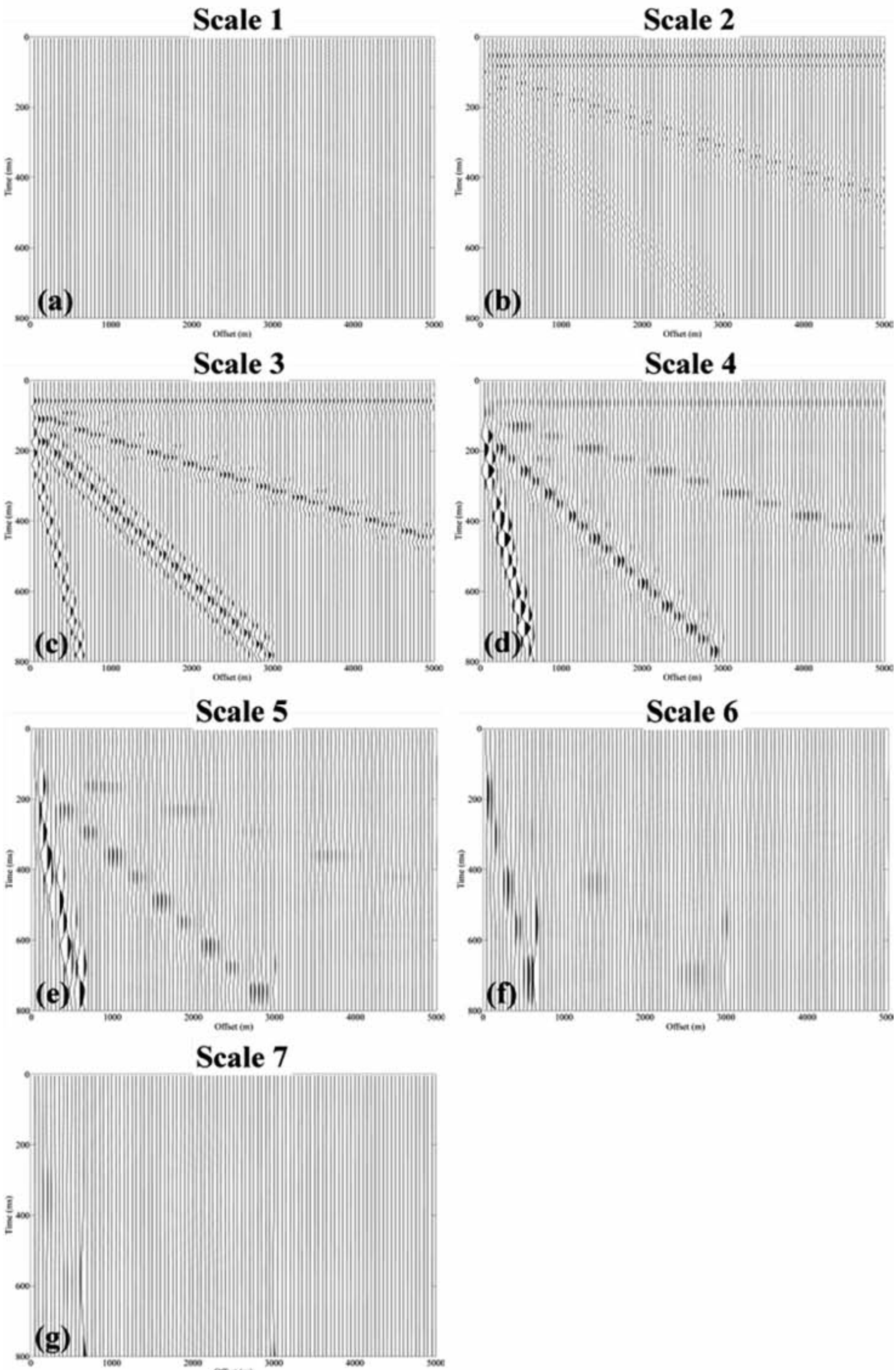


Fig. 6. The first through seventh 1DWT scales of the synthetic data.

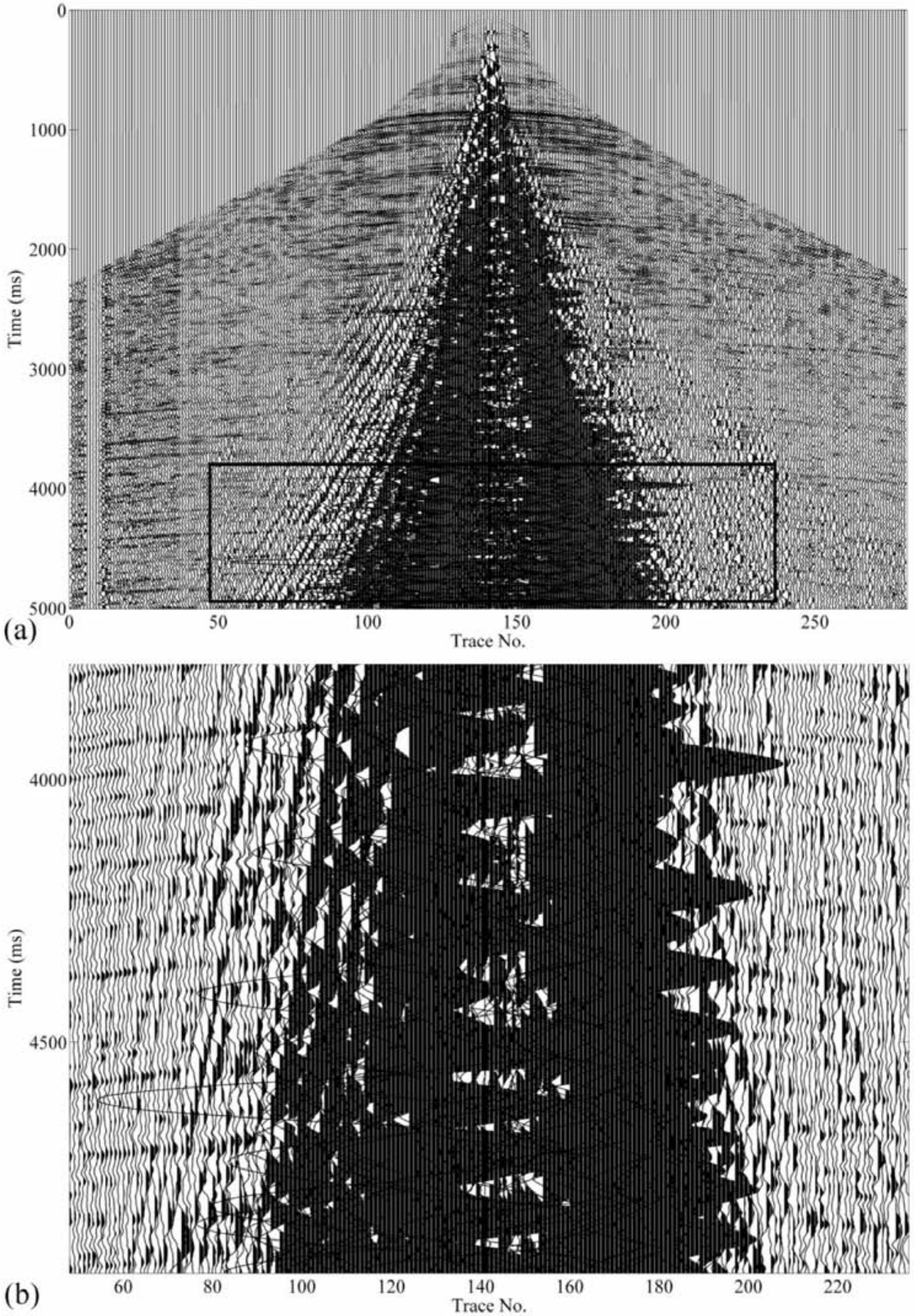


Fig. 7. (a) A shot gather with a split spread. There are 280 receiver stations with a station interval of 25 m, time sampling interval of 2 ms and the total recorded time of 5 s, and (b) the zoomed portion of the data marked by the black rectangle in (a).

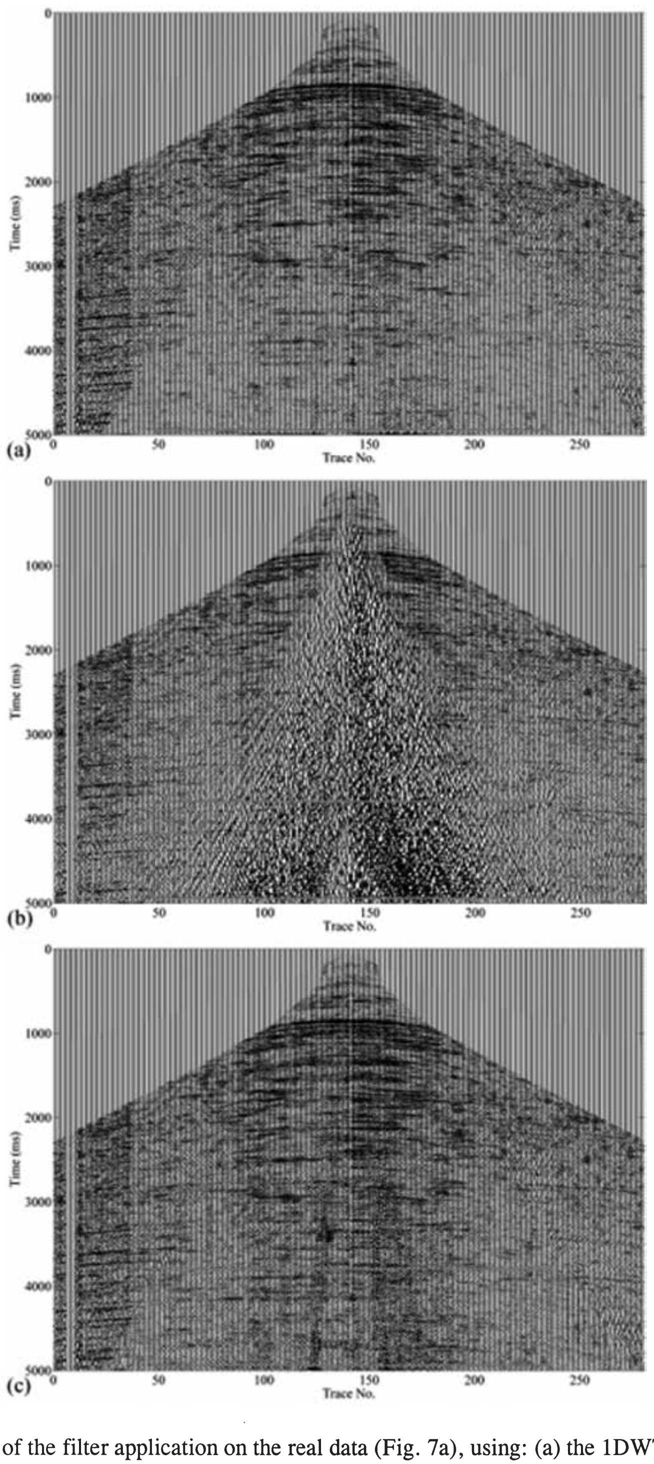


Fig. 8. Results of the filter application on the real data (Fig. 7a), using: (a) the 1DWT, (b) the SVD, and (c) the EIWT methods.

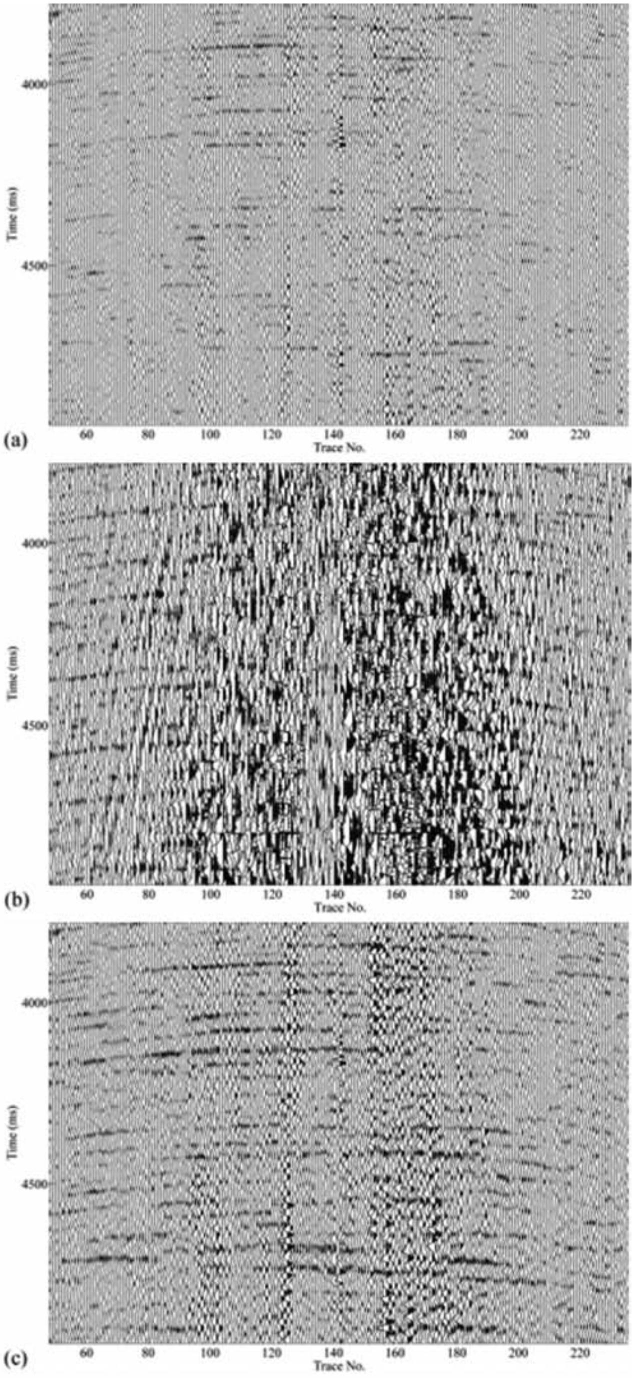


Fig. 9. The zoomed part of real data (Fig. 7b), after filter application using: (a) the 1DWT, (b) the SVD, and (c) the EIWT methods. It is seen that the EIWT method results in better noise attenuation and reflection preservation.

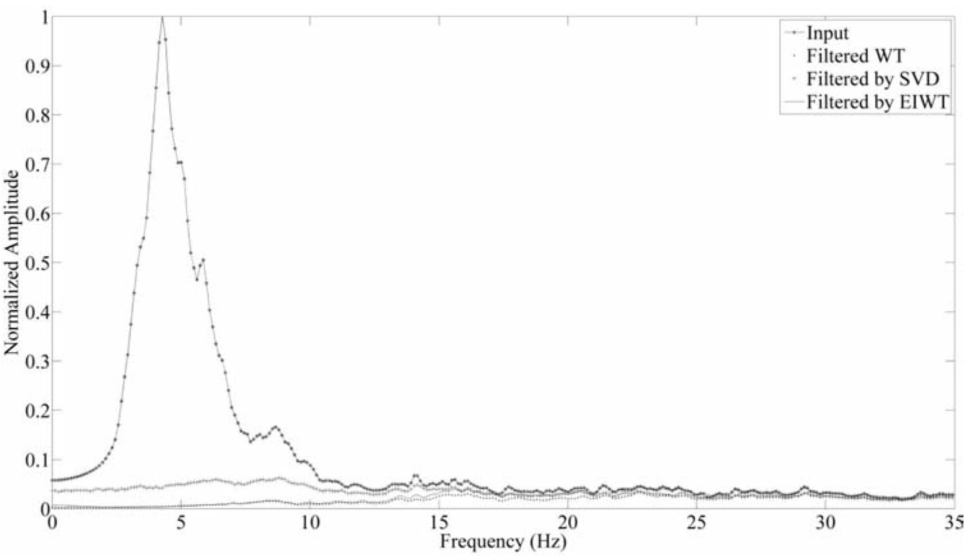


Fig. 10. Normalized average amplitude spectrum of data in the zoomed portion before and after filter application. The top line (with * marks) is the input data, and the dotted (bottom), + (middle) and solid lines are after filtering by 1DWT, SVD, and EIWT methods, respectively.

scales corresponding to the ground roll frequencies can be determined beforehand. The time boundary of the noise can be determined based on the minimum and maximum ground roll velocity, time-sampling interval, trace spacing, and the source location. As a result, the filter is only applied to the noise-contaminated region of data leaving other parts unaffected. With this knowledge, the filtering consists of setting the time-scale coefficients associated to noise equal to zero and transforming the result back to t-x domain.

Although application of filters based on wavelet transform seems promising, however, there is sometimes an overlap between the time durations and scales representing the ground roll and reflections. As a result, it would not be straightforward to consider those coefficients as noise coefficients and eliminate them.

One of the most important properties of ground roll is its higher amplitude relative to reflections; as a consequence, the wavelet coefficients representing the noise have higher energy than the signal coefficients. SVD is a method to decompose data into eigenimages based on the energy content and coherency of the events. The eigenimage analysis of the shot records for the purpose of ground roll attenuation is performed by Karhunen-Loeve transform (Liu, 1999). The ground roll has higher energy and is more coherent (after time stretching) than the reflections. As a consequence, it would appear in the few first eigenimages while the reflections appear in remaining eigenimages. Eliminating the first few eigenimages would suppress the ground roll. While this method seems to be a good tool for denoising data, but the fact is that there are usually eigenimages containing both signal and noise. Therefore, suppressing noise can result in reflection attenuation as well.

In the EIWT method, the results of both previous filters are combined to minimize the signal damage and while having the best possible ground roll suppression. First, the seismic trace is transformed into the time-scale domain. To have a better idea about the noise coefficients, eigenimages are constructed. Therefore, the coefficients are differentiated based on their magnitude. One can choose those coefficients that do not exceed the ground roll energy. Next, the selected coefficients are checked so that they will not be outside the noise time-scale boundary. Finally, these coefficients are transformed to the time domain and subtracted from the input trace.

Considering the ground roll attenuation case, there is a separation of noise and signal in eigenimages of the wavelet domain. The main concern is how to decide which coefficients are representing the noise and which ones are representing the signal. The filter was only applied to the ground-roll contaminated region of data. The required parameters to determine this region are minimum and maximum ground roll velocities, time-sampling interval, trace spacing, and the source location.This information is used to calculate the time boundary for each trace. In a case where a minimum velocity is not determined, we consider it to be zero; therefore, the time boundary would continue to the end of each trace. The next step is to calculate the maximum possible decomposition level. At each level of a wavelet transform, the number of samples in data is halved; consequently, the decomposition could be continued as far as only one sample remains. Accordingly, the initial number of the samples in data is the decisive factor in fixing the upper limit for the number of decomposition levels in the wavelet transform. The maximum number of decomposition level is then equal to the logarithm based 2 of the number of samples. In a 1D DWT, each scale is analogous to a frequency band; therefore, the scales corresponding to the ground roll frequencies are accordingly determined. Therefore, the time-scale coefficients representing the ground roll can be chosen. To determine the SVD criterion, the amplitude spectrum is used. The ratio of the amplitude spectrum in the noise band to the whole spectrum is then calculated. This ratio is used as the energy ratio of the noise representing coefficients.

Comparing the synthetic data reconstructed at different scales, shown in Fig. 6, different amplitudes of noise and signals are apparent at each scale. The EIWT examines the magnitude of eigenimages and selects the few first ones with the energy ratio no greater than the ratio obtained from the amplitude spectrum of the trace. Although the arrival time of the noise is simply different from that of the desired signals, the boundary is selected so that the desired signals are also present in the region that is being filtered. As a result, the effect of the filter when there is a time overlap can be examined. We can also see in Fig.s 5b and 5c, the filter affects the signal with velocity, frequency, and amplitude close to the noise; whereas, the two other events are untouched. This can be explained by the fact that the time resolution decreases when the wavelet decomposition is continued. Therefore, for coefficients representing a larger time band an overlap can occur and the noise and signal amplitudes may be represented by same coefficients.

The real data used in this work is a shot gather with strong ground roll covering the reflections when there is a time overlap. The results of the three filters, 1D DWT, SVD, and EIWT are shown (Fig. 8). In all cases, the filters are only applied to the noise contaminated part of data. This result has the least effect in the filtering process of data. To examine the results more closely, a section of the data marked by a rectangle is zoomed before and after filtering. The ground roll is so strong that completely masks the reflections. This is also obvious in the amplitude spectrum of the input data as shown in Fig. 10. Applying the wavelet transform filter, ground roll is properly attenuated; but the desired reflections are highly distorted. In the case of the SVD filter, ground roll is still present in the filtered data. Since this filter is dependent on the energy difference of the seismic events, very higher amplitude of ground roll at some parts of data relative to other parts resulted in preservation of ground roll at lower amplitudes. Using the EIWT filter, reflections are obvious after ground roll attenuation. The average amplitude spectra of data before and after filtering confirm these results.

CONCLUSIONS

Each scale in the time-scale domain corresponds to a frequency band that relates to the original sampling period of data. As a result, the same scale may contain both desired reflections and ground roll. Then, filtering ground roll will attenuate the desired reflections as well. In the proposed method, data in a 1D DWT domain is decomposed into eigenimages ordered according to the local coherency and their energy content. In a two-dimensional time-scale domain of a trace, seismic events are represented as coefficients in each scale, so that the coefficients regarding different events with different frequency bands are laterally aligned. The coefficients corresponding to the ground roll have higher energy than the ones related to reflections due to their energy differences. Therefore, the ground roll coefficients appear in the first few eigenimages. Introduction of the energy parameter as the third criterion - in addition to the scale and time of arrival - improves the noise suppression and reduces attenuation of the desired reflections. The SVD proves to be a good method to determine the ground roll related coefficients in the time-scale domain. In summary, the proposed method is based on the frequency content, arrival time and energy content of the ground roll and reflections. Hence, these data-dependent variables constitute filter parameters. For proper filtering, at least one of these parameters should have different values for ground roll and reflections.

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