PRE-STACK INVERSION OF ANGLE GATHERS USING A HYBRID EVOLUTIONARY ALGORITHM

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(Received August 24, 2011; revised version accepted February 21, 2012)

ABSTRACT

Saraswat, P. and Sen, M.K., 2012. Pre-stack inversion of angle gathers using a hybrid evolutionary algorithm. *Journal of Seismic Exploration*, 21: 177-200.

Inversion of pre-and post-stack seismic data for acoustic and shear impedances is a highly non-linear and ill-posed problem. A deterministic inversion of band-limited seismic data produces smooth models that are devoid of high frequency variations observed in well logs. The objective of this paper is two-fold, i.e., to develop an efficient scheme to explore and exploit the model space, and to efficiently sample broadband models statistically. We demonstrate that the use of starting models from fractal based a priori pdfs helps us to derive elastic models of very high resolution. We also introduce a new hybrid inversion algorithm that takes advantage of both deterministic and stochastic methodologies. A deterministic inversion based on conjugate gradient (CG) method produces smooth models while a stand-alone stochastic method based on differential evolution (DE) produces high-resolution models of nearly the same accuracy. A hybrid algorithm that uses CG solution as a starting model converges much faster than a standalone DE to very good solutions. We demonstrate our results with application to a field seismic dataset. The hybrid algorithm can also be used to sample the most significant parts of the model space rapidly resulting in estimates of uncertainty.

KEY WORDS: global optimization, pre-stack, inversion, differential evolution.

0963-0651/12/$5.00

INTRODUCTION

Inversion of seismic data plays a vital role in reservoir characterization. High-resolution inversion methods add significant value to the inversion results and increase the confidence level in interpretation of seismic data. Well logs present the most accurate information about the petrophysical properties of a subsurface reservoir. However, a spatially continuous description of a reservoir at the well log scale is not available due to limited well data. Results from seismic inversion are usually integrated with well log data to derive reservoir models in 3D. A typical deterministic seismic inversion procedure derives blocky or coarse subsurface models well below the resolution of the well logs. A stochastic inversion that combines well logs with seismic inversion has the potential to estimate subsurface models at the well log resolution in 3D (Sen, 2006). It is well recognized that some of the low and high frequency parts of the subsurface model reside in the null space of the seismic data and can only be incorporated through a priori information (Francis, 1997). Common stochastic inversion methods employ Gaussian probability density function to describe prior impedance models. Most recently Srivastava and Sen (2009a,b) made use of fractal based a priori models in post-and pre-stack seismic inversion. They showed that geologically realistic acoustic impedance models can indeed be estimated by this approach. Srivastava and Sen (2009a,b) employed very fast simulated annealing, VFSA (Sen and Stoffa, 1995) in the search for optimal models. Our approach here is very similar to that used in Srivastava and Sen (2009a,b) in that we also use fractal based a priori pdf for initial model generation. Unlike Srivastava and Sen (2009a,b), we employ a new global optimization method based on a differential evolution algorithm (Storn and Price, 1997; Mishra, 2006) and increase the computational efficiency by using the result from a local optimization method as one of the members of the initial population in DE.

We demonstrate the feasibility and usefulness of our method with application to 2D field seismic data. Further, the confidence level of stochastic model estimates is determined by a statistical analysis.

METHOD

Forward Problem

For a layered earth model, exact synthetic seismograms can be computed using the reflectivity method (Kennett, 1983). For many practical applications, seismograms are generated using simple wave propagation models using exact Zoeppritz or a linearized reflection coefficient (Aki and Richards, 1980) given below in the angle domain as these computations are extremely fast.

$$R(\theta) = \tfrac{1}{2}[(\Delta V_P/V_P) - (\Delta\rho/\rho)]$$

$$- 2(V_S/V_P)^2[2(\Delta V_S/V_S) + (\Delta\rho/\rho)] \times \sin^2\theta + \tfrac{1}{2}(\Delta V_P/V_P)\tan^2\theta \quad , \qquad (1)$$

where V_P is the average P-wave velocity between two uniform half-spaces, V_S is the average S-wave velocity, and ρ is the average density. The assumptions made are that the relative changes of property ($\Delta V_P/V_P$, $\Delta V_S/V_S$, and $\Delta\rho/\rho$) are small, such that the second-order terms can be neglected, and that θ is much less than 90°. Eq. (1) can be rewritten in terms of P-wave and S-wave impedances:

$$R(\theta) \approx (1 + \tan^2\theta)(\Delta I_P/2I_P) - 8(\Delta V_S/V_P)^2\sin^2\theta(\Delta I_S/2I_S)$$

$$- [\![\tan^2\theta - 4(\Delta V_P/V_P)^2\sin^2\theta]\!] (\Delta\rho/\rho) \quad , \qquad (2)$$

where $I_P = V_P\rho$ is the average acoustic impedance, $I_S = V_S\rho$ is the average shear impedance $(\Delta I_P/2I_P) = (\Delta V_P/V_P) + (\Delta\rho/\rho)$ is the zero-offset P-wave reflection coefficient, and $(\Delta I_S/2I_S) = \tfrac{1}{2}[(\Delta V_S/V_S) + (\Delta\rho/\rho)]$ is the zero-offset S-wave reflection coefficient. Fatti et al. (1994) suggested that the third term in ρ only cancels for most V_S/V_P ratios around 0.5 and small angles and, thereby, eq. (2) simplifies to

$$R(\theta) \approx (1 + \tan^2\theta)(\Delta I_P/2I_P) - 8(\Delta V_S/V_P)^2\sin^2\theta(\Delta I_S/2I_S) \quad . \qquad (3)$$

Eq. (3) has been used by Fatti et al. (1994), Goodway et al. (1997), and Ma (2002) to extract P- and S-wave impedance reflectivities by fitting it to the P-wave reflection amplitudes from real common-midpoint (CMP) gathers. The problems in using the above strategy comes when there is no or improper information on the background $\Delta V_S/V_P$ and if there are no appropriate geological models. Then the resulting AVO inversion output may produce unrealistic and biased information (Wang 1999). To overcome this issue, Ma (2002) replaced average $\Delta V_S/V_P$ ratios by the average I_S/I_P ratio, so that the reflection coefficients $R(\theta)$ are only related to three parameters: I_P, I_S, and θ. Among those, the angle of incidence θ can be calculated using a ray-tracing method (Smith and Gidlow, 1987) and is valid only when density changes between adjacent layers are small. Assuming this hypothesis, we use following equation for the reflectivity series:

$$R(\theta) \approx (1 + \tan^2\theta)(\Delta I_P/2I_P) - 8(\Delta I_S/I_P)^2\sin^2\theta(\Delta I_S/2I_S) \quad . \qquad (4)$$

Taking into account eq. (4) we propose a procedure for estimating acoustic and shear impedances from pre-stack seismic angle gathers. The basic assumptions made are that the earth has approximately horizontal layers at each common depth point and that each layer is described by both acoustic and shear impedances and, thus, a synthetic seismogram can be computed using a standard convolution model.

Optimization

Using eq. (4), we can calculate reflection coefficients R(θ) for each angle and at each layer boundary for an earth model defined by impedances I_S^i, I_P^i. A synthetic angle gather, calculated by convolving the reflection coefficients R(θ) with predetermined wavelets, is then compared with the observed data to form a misfit (usually a L_2-norm) function. Inversion schemes based on stochastic methods such as very fast simulated annealing (VFSA) (Sen and Stoffa, 1991; Chundru et al., 1996), and genetic algorithms (Stoffa and Sen, 1991; Kennett et al., 1992; Sen and Stoffa, 1992a,b; Sambridge et al., 1992, 1993; Jervis et al., 1993a; Jin et al., 1993; Sen et al., 1995; Mallick, 1999; Ghazali et al., 2010) are generally preferred over deterministic methods, since they are able to generate multiple realizations of the model parameters and can easily incorporate geological information with seismic to enhance the results (Contreras et al., 2006; Varela et al., 2006).

One serious limitation of seismic data is that the resulting subsurface models have poorer resolution than the well logs and therefore, missing low and high frequency bands must be incorporated as a priori information. It is fairly straightforward to incorporate such priors using global optimization methods. Conventionally, Gaussian based priors are used to generate starting models for inversion process which often contains white noise; this results in inclusion of spurious and undesired frequency in model space leading to unrealistic information about subsurface geology and its properties. Unlike this approach, Srivastava and Sen (2009) made use of a fractional Gaussian process to generate starting models that honor well log statistics accurately and a VFSA approach to search for an optimal model. We follow Srivastava and Sen (2009) to generate fractal based starting models but employ a different optimization scheme based on differential evolution which we describe next.

Differential Evolution (DE)

DE is a simple and efficient adaptive technique for global optimization over continuous spaces, which forms the class of genetic algorithms. Differential Evolution (DE) (Storn and Price, 1997) is a novel parallel direct search method which utilizes NP parameter vectors, $X_{i,G}$, i = 0, 1, 2, ... , NP$-$1, which is the population for each generation G. During the optimization process NP remains static. However, the choice of population is either random or is based on system properties. For example, in our case the population is governed by fractional Gaussian noise instead of a random generator.

The crux of DE lies in a new scheme for generating a trail vector using three parent vectors. This trail vector is generated by adding a weighted difference of randomly chosen second and third parent vector to the first parent (schemes will be discussed later). If the objective function attains lower value with the trail vector as compared to the first parent, the trail vector (child) wins and takes the place of the first parent. The comparison vector can, but need not, be a part of the generation process mentioned above. Further, for every generation the best parameter vector $x_{best,G}$ is computed and is evaluated. So for every generation we have local best models computed using various schemes and these local best are then evaluated to estimate the best of each generation, which is further evaluated to extract distance and direction information generating random deviations. All these properties make DE robust and self organizing adaptive technique with excellent convergence properties. Although, the performance of DE reduces when there is an element of randomness in the objective function or the initial models, it is perhaps the best among the algorithms that may be used to find out the global optimum of a nonstochastic continuous, real-valued, multi-modal function (Mishra, 2006). A detailed version of DE can found in Storm and Price (1997). Below we describe the workflow for DE strategy briefly.

The general definition of an optimization problem is as follows. Given an objective function f(x) in a D dimensional space, the minimization problem is to find

$$x' \in X \quad \text{such that} \quad f(x^*) \leq f(x) \ \forall \ x \in X \ . \tag{5}$$

We will discuss a method that considers a population of models at a time and attempts to update the population with iteration (called generation here) such that we are able to search for better models. Let us consider that we need to optimize a function with D real parameters and the population size is N, Each model in a generation can be represented by a column vector $X_{i,G} = [x_{1,i,G}, x_{2,i,G}, \ldots, x_{D,i,G}]$, where G is the generation number and i takes a value from 1 to N. Following these notations, we briefly discuss below the algorithm for differential evolution.

Step 1: Initialization - Here we define the upper and lower limits of our model parameters $X_j^L \leq X_{i,j,1} \leq X_j^U$. Further we randomly or strategically select initial parameter values.

Step 2: Mutation - It expands the search space; this step involves generation of a child vector from a parent and two other random population members, known as donor vectors. The general algorithm of mutation follows one of the following schemes:

Scheme 1: The first DE variant works as follows: for each vector $X_{i,G}$, i = 1, 2, 3, ..., NP−1 a donor vector is generated according to the following criteria:

$V_{i,G+1} = X_{r1,G} + \alpha(X_{r2,G} - X_{r3,G})$, where $r_1,r_2,r_3 \in [0,NP-1]$ and mutually different from each other and α is called the mutation constant such that $\alpha > 0$ and less than 2, it controls the amplification of differential variation $(X_{r2,G} - X_{r3,G})$, Fig. 1 below shows a two dimensional example that illustrates the different vectors which play a part in DE1 (Storn and Price, 1997).

Scheme 2: This scheme has an additional feature; the donor vector is generated as:
$V_{i,G+1} = X_{r1,G} + \alpha(X_{r2,G} - X_{r3,G}) + \lambda(X_{best,G} - X_{i,G})$, adding an additional feature via scale factor λ, that provides a means to increase the greediness of the algorithm, where $X_{best,G}$ is the current best vector of the results generated. Fig. 2 depicts this scheme (Storn and Price, 1997).

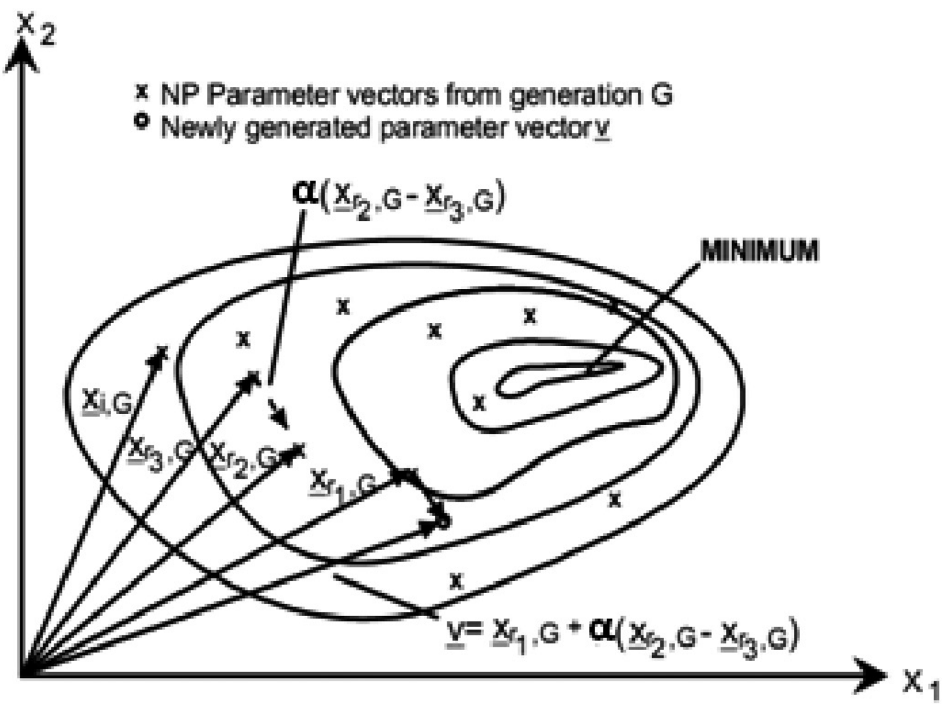

Fig. 1. An example of an objective function in two dimensions showing its contour lines and the process for generating v in scheme DE1 (from Storn and Price, 1997). See text for details.

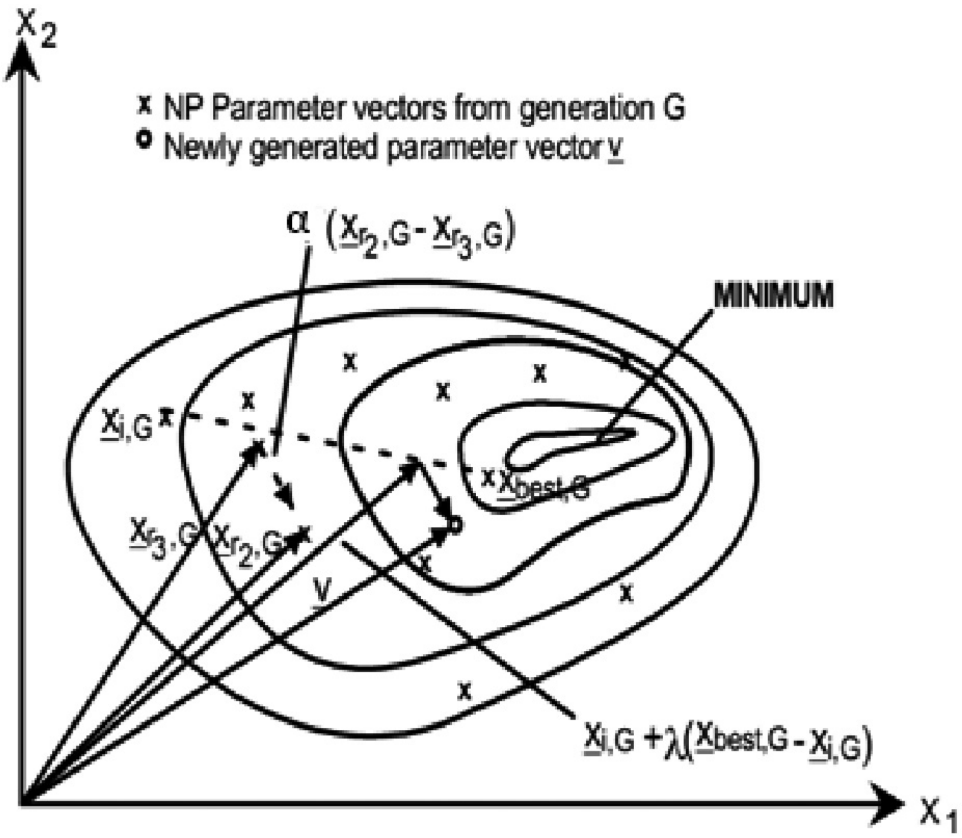

Fig. 2. Two dimensional example of an objective function showing its contour lines and the process for generating v in scheme DE2. (from Storn and Price, 1997).

Step 3: Recombination-incorporates successful solutions from previous generations. The trial vector ($U_{i,G+1}$)is developed from elements of trial vector $V_{i,G+1}$ and elements of target vector $X_{i,G}$. Elements of donor vector enters trial vector with a probability CR

$$U_{i,G+1} = \begin{cases} v_{j,i,G+1} & \text{if } rand_{ij} \leq CR \text{ or } j = I_{rand} \\ x_{j,i,G} & \text{if } rand_{ij} > CR \text{ or } j \neq I_{rand} \end{cases}, \quad (6)$$

$rand_{i,j} \sim U[0,1]$, I_{rand} is a random integer from [1, 2, 3, … , D], ensuring $V_{i,G+1} \neq X_{i,G}$.

Step 4: Selection - The target vector is now compared with the trial vector and the one with the lowest function value is admitted to the next generation.

$$X_{i,G+1} = \begin{cases} U_{i,G+1} & \text{if } f(U_{i,G+1}) \leq f(X_{i,G}) \\ X_{i,G} & \text{otherwise} \end{cases} \quad . \tag{7}$$

All these steps are performed iteratively until a stopping criterion is satisfied.

A hybrid approach

In this paper we propose a hybrid optimization algorithm which exploits the advantages of a Gradient Search approach as well as a stochastic evolutionary algorithm. The motivation behind using a dual approach comes from analyzing the advantages of stochastic and deterministic approaches of inverting seismic data. Our hybrid method incorporates standard conjugate gradient (CG) (Scales, 1987) run along with DE. Given a good starting solution, gradient methods that generally contain model regularization such as smoothing, can obtain very good solutions but are generally band limited. Global search algorithms, on the other hand, are least sensitive to the nature of the starting model and are capable of accepting bad models occasionally; this property helps them escape from local minima, which is a major improvement over deterministic methods. They are, however, computationally expensive.

Hybrid search algorithms (Chunduru et al.,1997) have the potential to make use of some of the important features of both global and local algorithms such that they are computationally fast, work well even with poor starting solutions, reduce randomness in the models, and final models have better resolution for low and high frequencies. The challenge is to find an optimal way to combine the algorithms. Here we use a hybrid model based on CG and DE such that we apply CG as our first step in the optimization process. A starting model based on fractional Gaussian process is used in the CG module. However, the result from CG inversion may not generally be optimal due to limitations of CG but it serves as a very good initial guess when used as one member in the population of the DE (Fig. 3).

The advantages of this algorithm are listed as follows:

- It is capable of simulating characteristics of both deterministic as well as stochastic methods of seismic inversion;
- It has better convergence properties than a single run of either of the two methods.

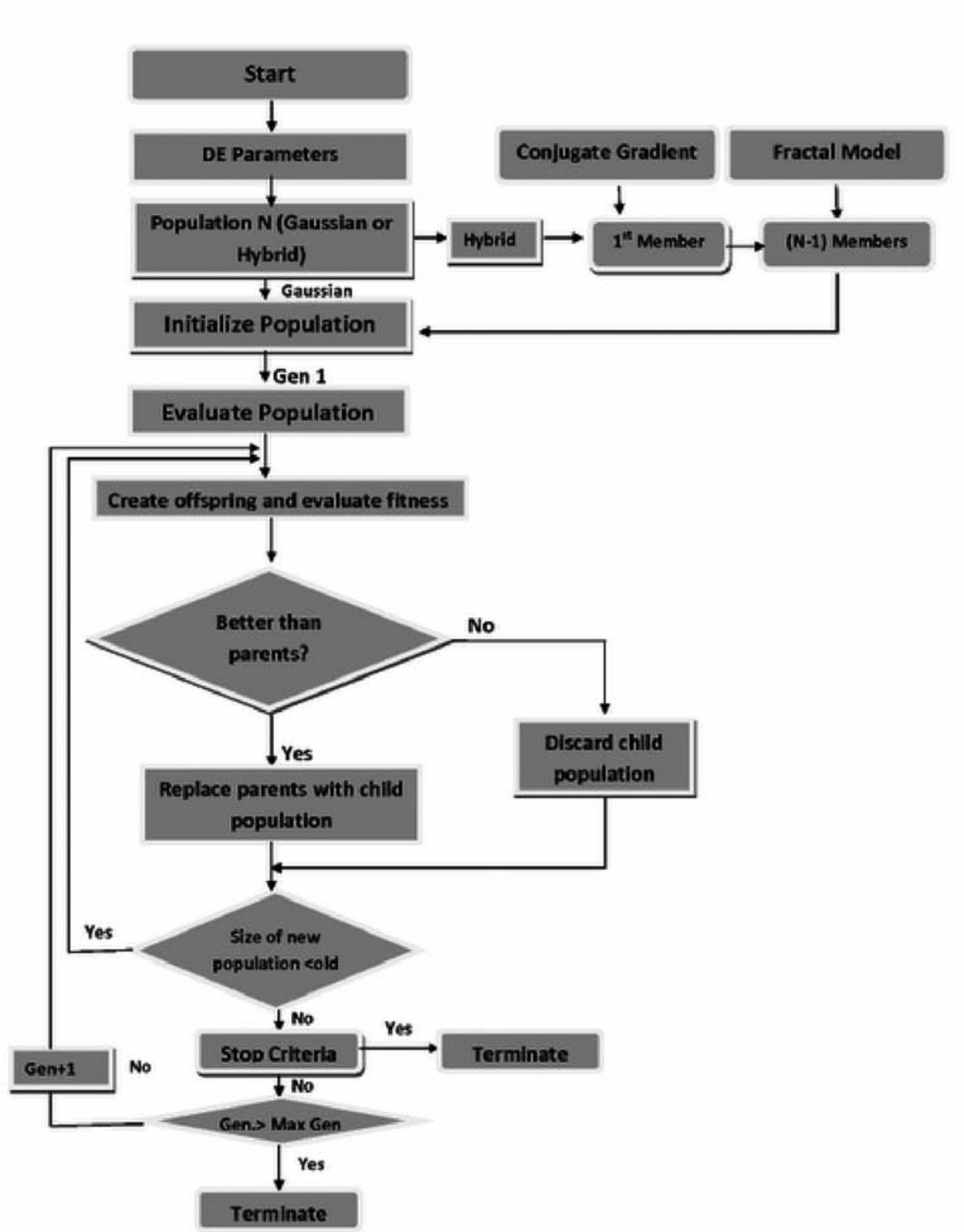

Fig. 3. Flowchart of our hybrid inversion scheme.

Starting Models

As discussed in the preceding sections, we use fractional Gaussian noise distribution to draw our starting models from. Fractal models account for various geological phenomena and have been used to study various natural features and observations. Hurst (1951) made a breakthrough discovery while analyzing a specific phenomenon of geophysical and hydrological time series, which illustrates tendency of wet years clustering in wet periods while dry periods clustering in drought periods - this phenomenon was termed as the Hurst phenomenon, which was further verified in many studies like the flow of the river Nile, wind power variations, annual stream flow records of the continental United States, etc. (Koutsoyiannis, 2002). The idea of fractional Gaussian noise (fGn) was first suggested by Mandelbrot (1965). This process can be defined in discrete time in a similar manner as in continuous time (Saupe, 1988). Thus fGn can be simulated as a time series realization with expected auto-covariance estimated from a sporadic coefficient known as the Hurst coefficient (H). The auto-covariance function for fGn is given by

$$R(\tau) = 0.5\sigma^2[|\tau + 1|^{2H} - 2|\tau|^{2H} + (\tau + 1)^{2H}] \ , \quad (8)$$

where σ represents the standard deviation.

It has been demonstrated in various studies that well-log and seismic data follow a powerlaw relationship (Hewett, 1986; Emanual et al., 1987; Hardy, 1992; Dimri, 2000, 2005). Thus well-log data can be subjected to fractal theory. Generating multiple realizations of well-log information over complete seismic data using fractional Gaussian process (Caccia et al., 1997; Srivastava and Sen, 2009, 2010), requires calculations of some of the statistical parameters such as standard deviation, mean, variance from the well log. The Hurst coefficient can be estimated using various methods such as rescaled range analysis (Turcotte, 1997; Dimri 2005; Chamoli et al., 2007). The time series generated using fractional Gaussian noise corresponding to each seismic trace serves as an excellent starting solution to a seismic inversion process.

INVERSION STRATEGY

Essential steps followed in this study are as follows:

1. Interpolate and extrapolate the P-and S-wave impedances obtained from well-log over the complete zone of interest. Thus, at each CMP location we derive a pseudo well-log.

2. Each interpolated well-log is used to calculate statistical parameters such as mean, variance and Hurst coefficient corresponding to every CMP location.

3. These statistical parameters are then used to generate well-log realizations for every CMP location using a fractional Gaussian noise generator. To make prior information more realistic, low frequency information is included in form of CG estimates, the gradient results are smooth models which serve as an excellent estimation of low frequency information (first parent described above) in conjugation with fGn models. Thus in this way we are able to address both low and high frequency missing bands in our algorithm.

4. We then use this prior information to generate synthetic seismic data by convolving reflectivity series generated using eq. (4) with pre-determined wavelets for near and far offsets.

5. Synthetic gathers are then compared with observed data and the misfit function is minimized using DE to generate P-and S-wave impedance data, which is the best model satisfying the stopping criterion.

EXAMPLES

In this section we will make an attempt to analyze the proposed inversion scheme with its application to a field pre-stack seismic data. Furthermore, we will compare the results using various strategies for inversion followed by an insight on the efficacy of our proposed strategy.

We use a real 2D pre-stack seismic dataset (Fig. 4), together with a well log corresponding to crossline number 71 as shown in Fig. 5, to test our algorithm. The data-set is obtained from the STRATA module of Hampson Russell software. We picked two horizons; top one around 550 ms and bottom one being around 700 ms. Fig. 6 shows the inversion result for P- and S-wave impedance using a conjugate gradient method. As observed in these figures the results are smooth and do not carry information about high frequency components. For example the match between the well-log data and inversion result between 600-620 ms is very poor. To overcome these issues we carry out an inversion process using various methodologies including some hybrid schemes. We show results from inversion using four different strategies listed below.

Method I: A pure DE search using all random members (Gaussian) in the starting population, i.e., pure DE approach (Fig. 7),

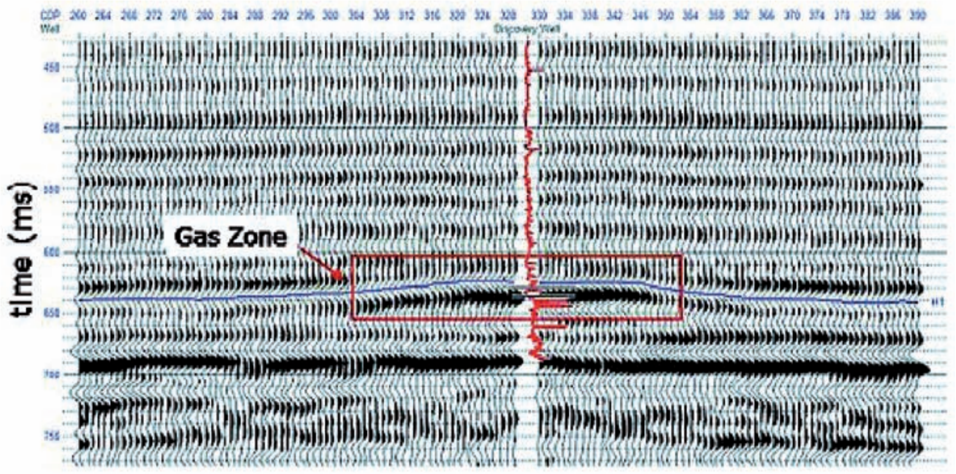

Fig. 4. The complete pre-stack seismic dataset under considerations. (Courtesy Hampson Russell Software Company).

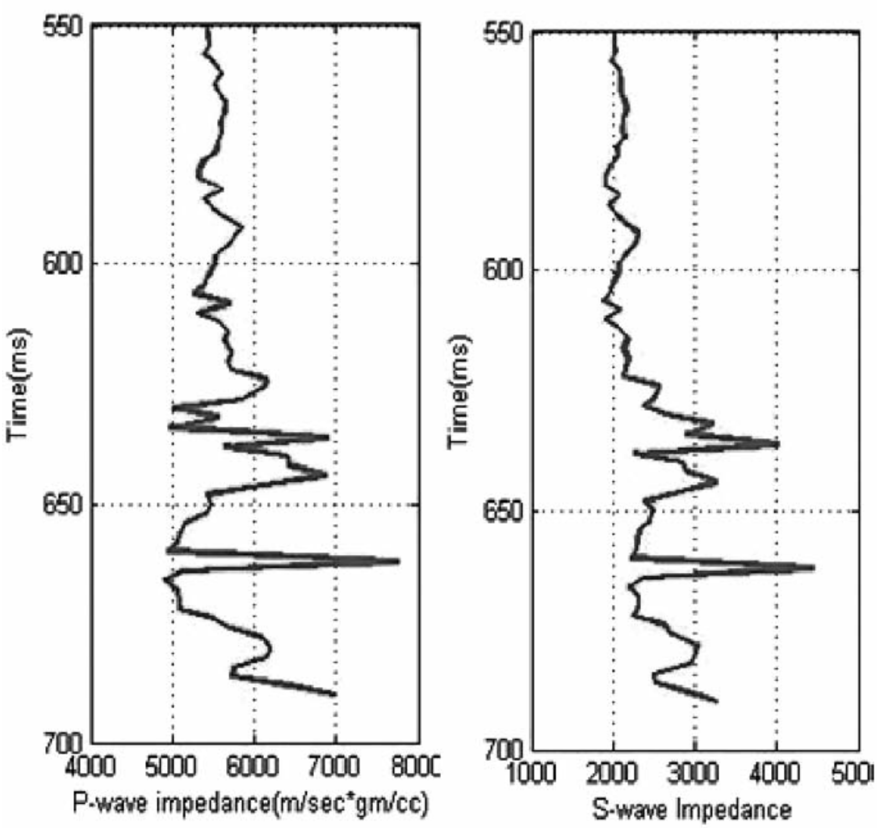

Fig. 5. Computed P- and S-wave impedance at a well location.

Method II: A pure DE search using all fractal based models (fGn) in the starting population (Fig. 8).

Method III: DE in which one of the members of the initial population is the result from CG (as first parent) and others are random Gaussian models (other population members NP−1) - A hybrid approach (Fig. 9).

Method IV: DE in which one of the members of the initial population is the result from CG (as first parent) and others are fractal based models (other population members NP−1) - A hybrid approach (Fig. 10).

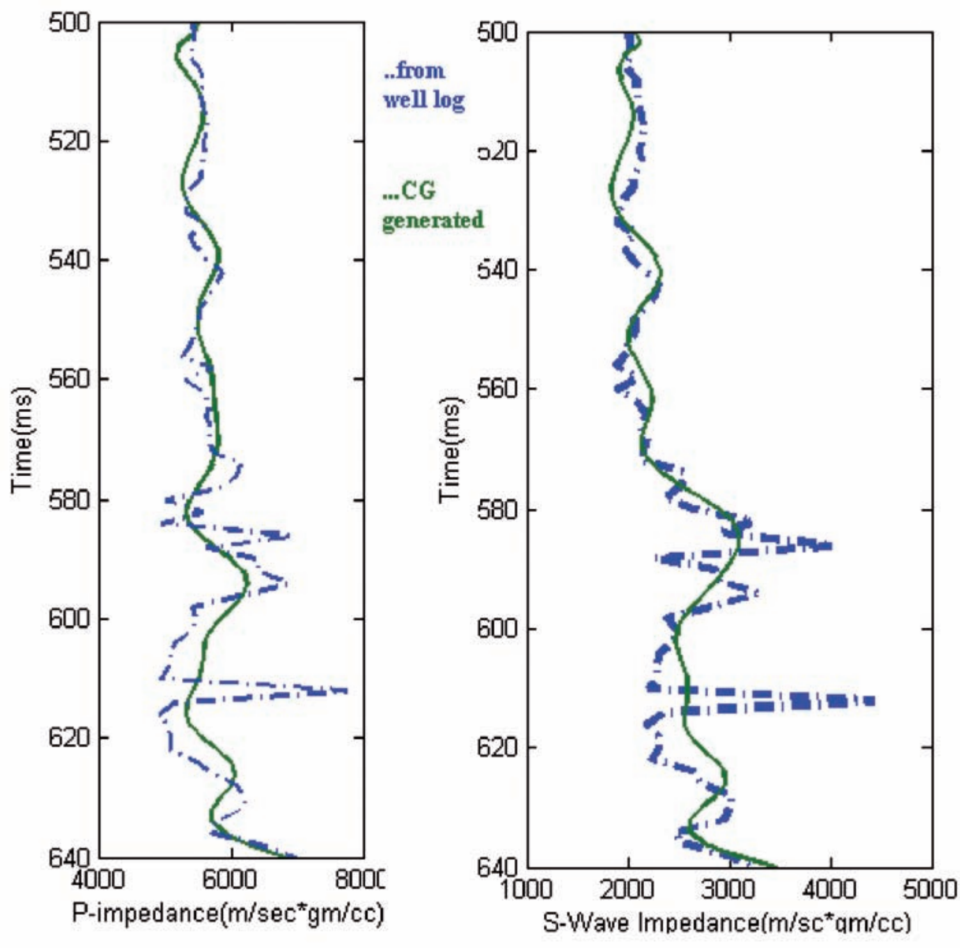

Fig. 6. P- and S-wave impedances from the conjugate gradient method (green). It is evident that models are smooth and the match with the well-log data (blue) is poor.

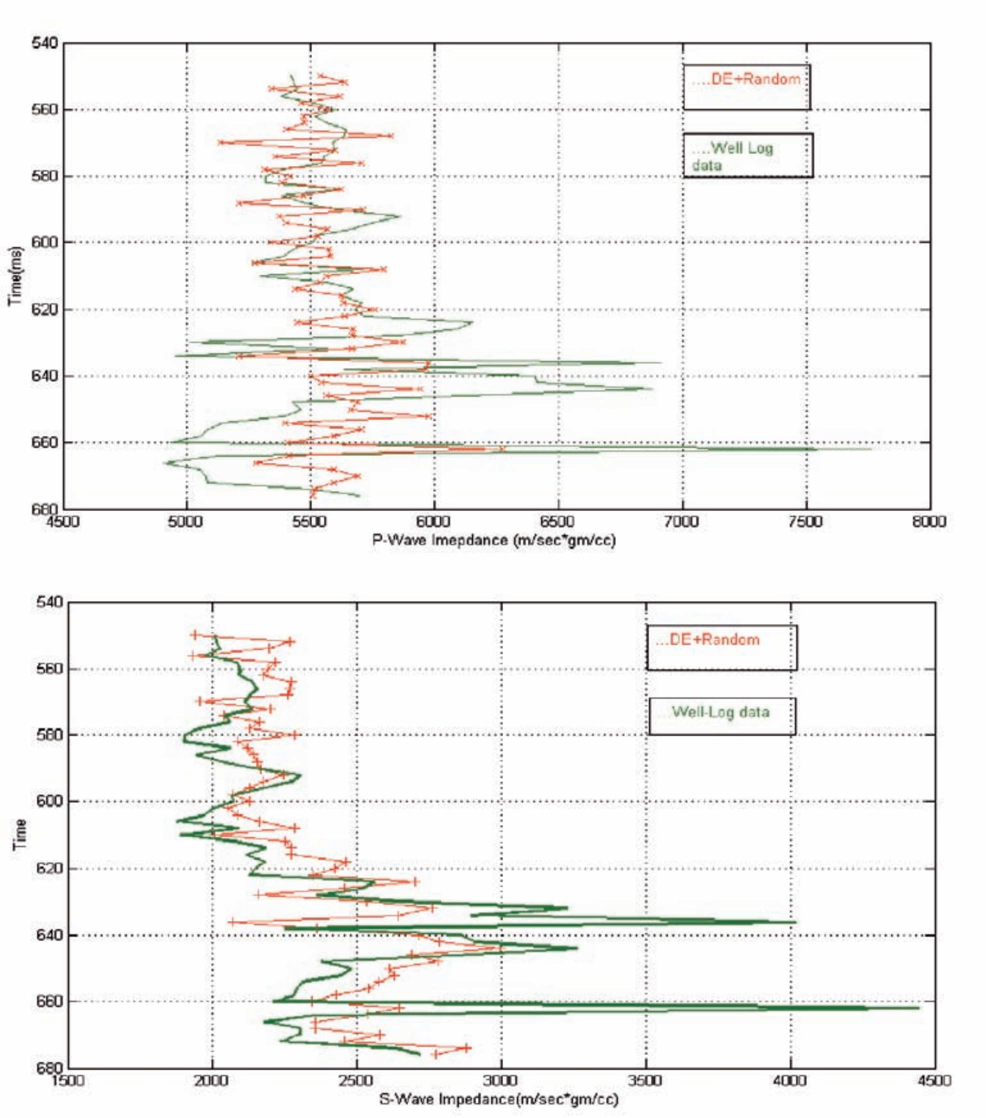

Fig. 7. Inversion results from DE (red) with Gaussian based models as starting solution. The correlation between the computed impedances (red) and well-log data (green) is poor.

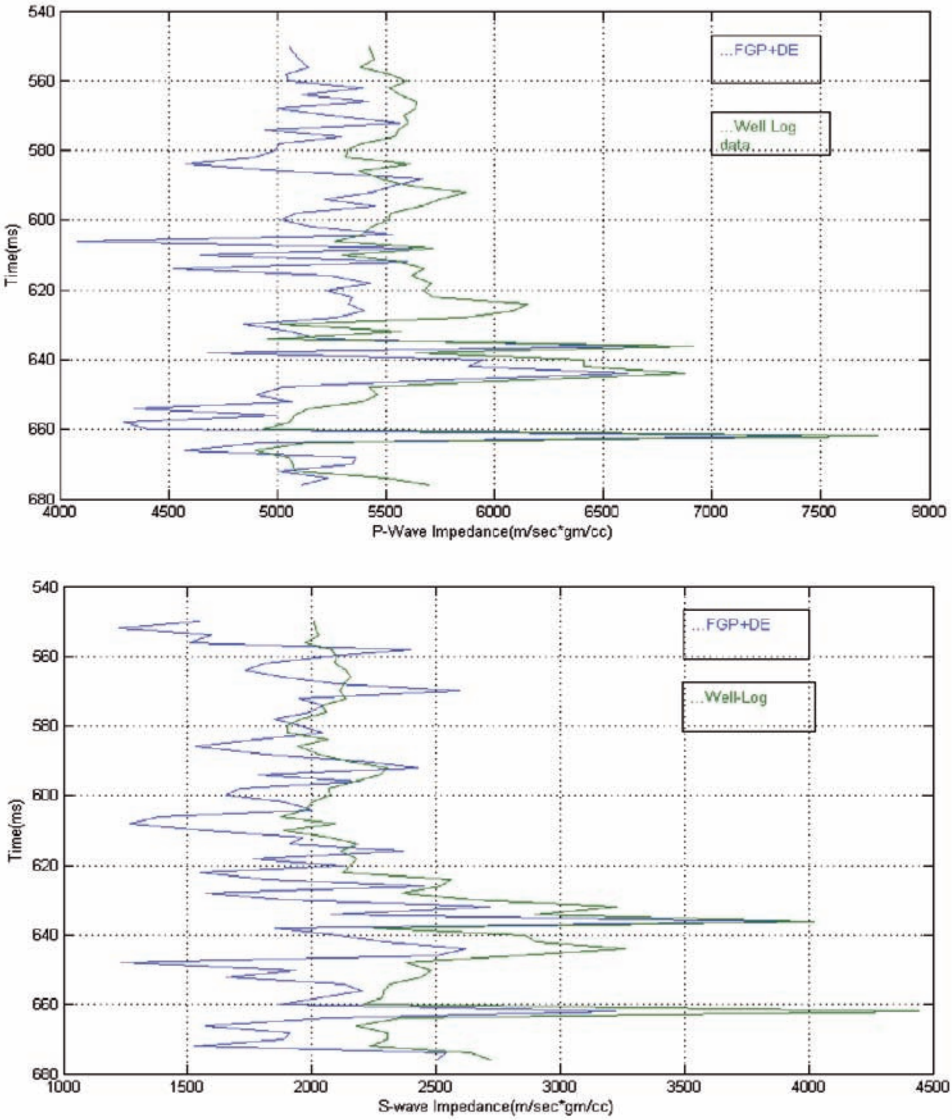

Fig. 8. DE generated P- and S-wave impedance (blue) and their match with well-log data (green). There is a good match, especially, between high frequency parameters making fractal-based priors as good starting solution.

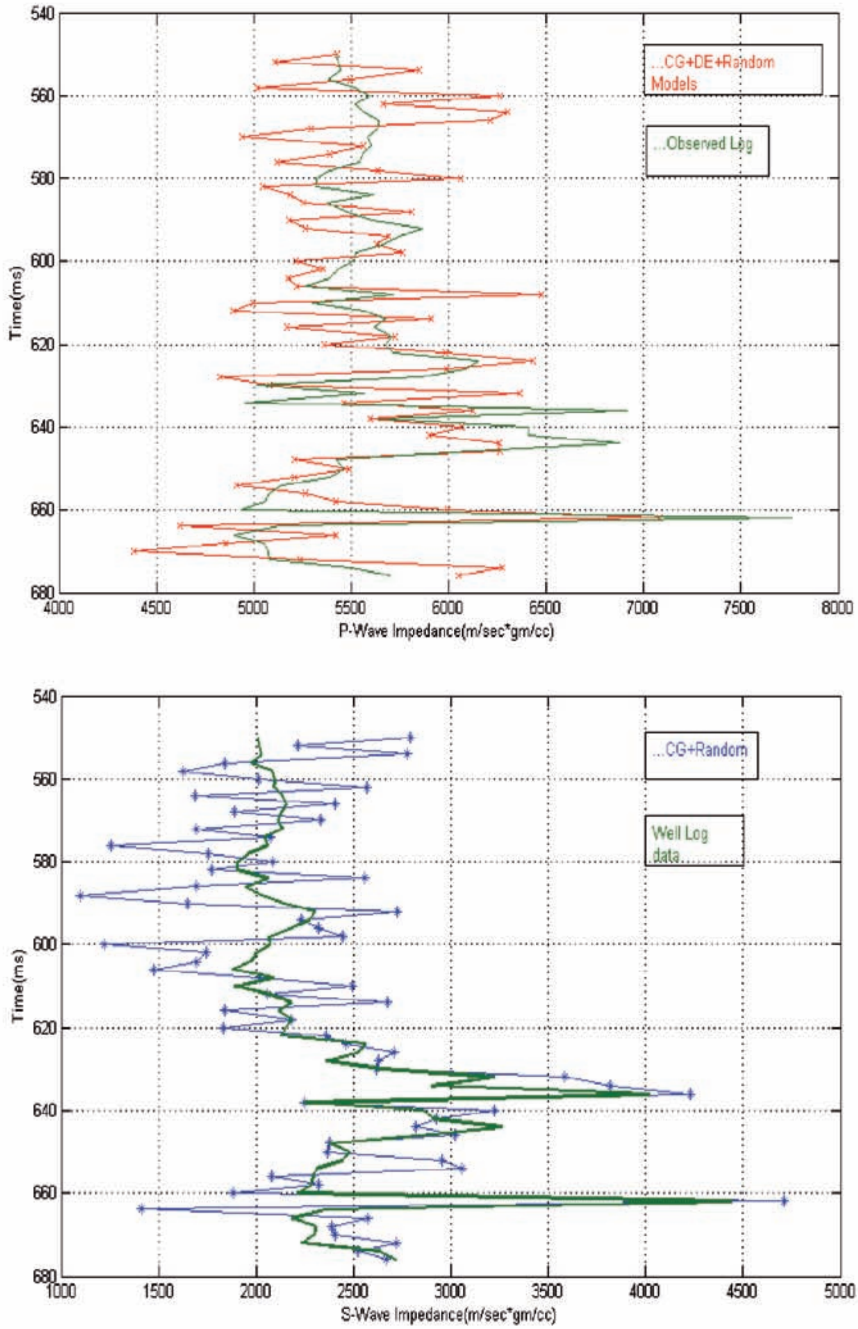

Fig. 9. In this case the result from the conjugate gradient approach is the first population member and (N-1) population members were Gaussian priors. The inversion results (red and blue) show an imperfect match with well-log data (green).

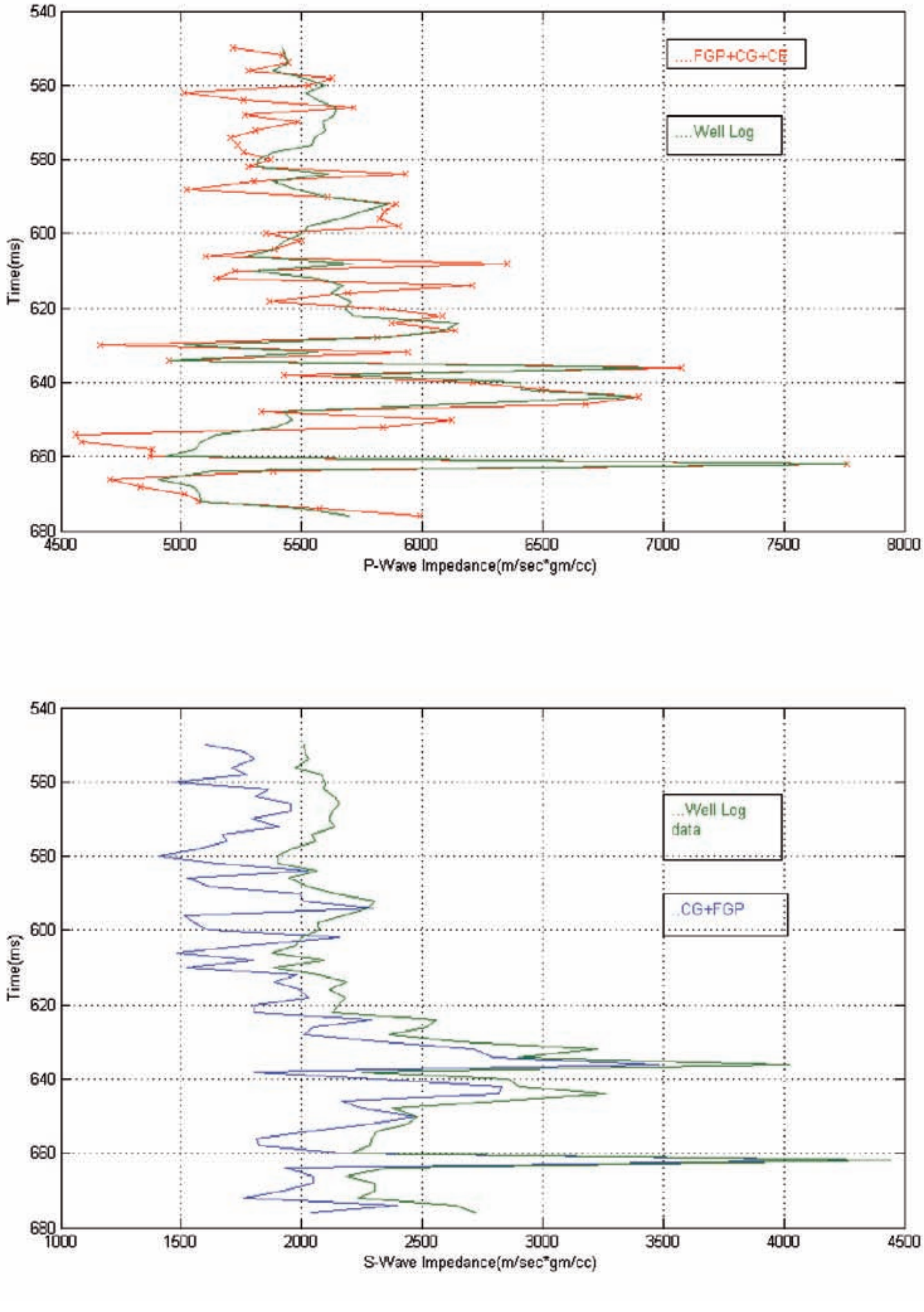

Fig. 10. Result from the proposed model, i.e., first population member as conjugate gradient generated result and remaining (N − 1) members as fractal based priors. Inversion results (red and blue) show good agreement with well-log data (green). This proves the efficacy of the proposed hybrid approach.

Results and Discussion

Figs. 7, 8, 9 and 10 show the results at the well location - they show comparisons between the well log and the inverted models of Z_P and Z_S. Careful examination of the plots indicate that method IV, which uses CG and fractal based initial models in a hybrid DE, produces the best results such that they are able to resolve the details of the well logs within the target zone. Fig. 11 shows observed and modeled gathers obtained by method IV, which are in good agreement. Based on these results, we inverted the entire 2D line using method IV; the results are shown in Fig. 12.

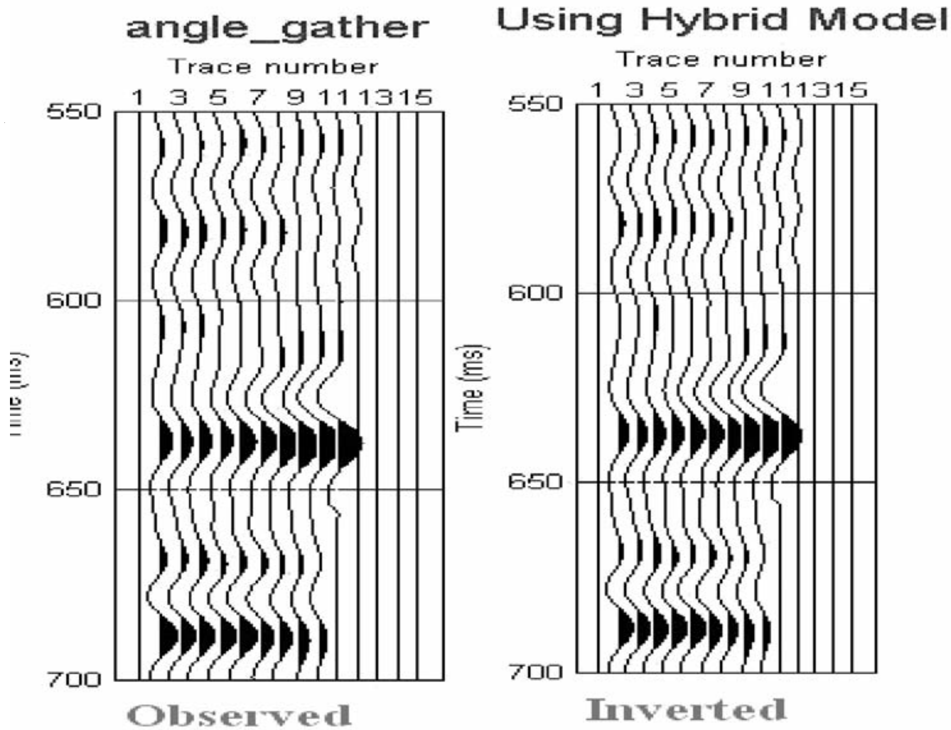

Fig. 11. Comparison of observed seismic data with the result of inversion using the proposed method. A single CMP gather is shown for evaluation.

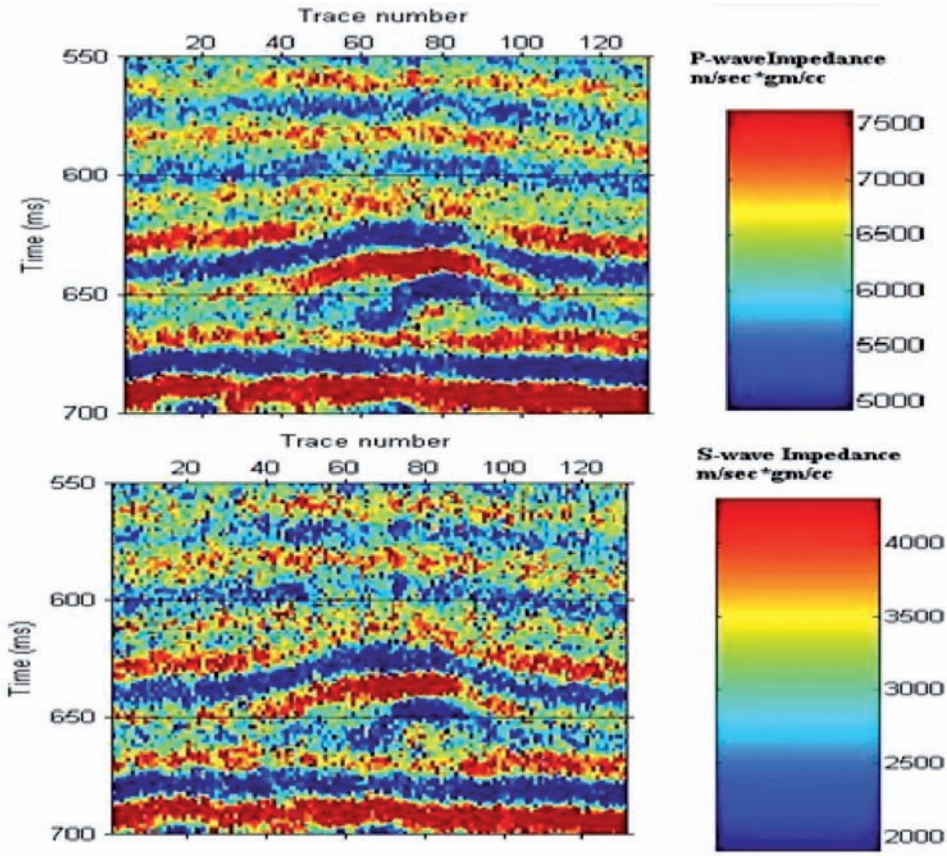

Fig. 12. 2D color plot of P- (top) and S-wave (bottom) impedance models generated using our hybrid inversion scheme. The results have high resolution and continuity along the picked horizons.

To further explore the algorithm we carried out in-depth analysis of the hybrid algorithm. The stochastic methods eventually reached the best model while accepting and evaluating various models. By analyzing the intermediate results we can derive more useful information on the final models; a plot of all the values (P- and S-wave impedance) generated in each generation are shown in Figs. 13 and 14. A confidence interval can be defined using these models; this property of DE encouraged many researchers to combine it with Monte Carlo and Markov Chain simulation (ter Braak, 2006) which can further assist in uncertainty analysis of output models. In our case we have shown the envelope of estimated models from hybrid inversion scheme around the well log data in Figs. 13 and 14.

The computational speed of our approach depends on the number of members in the population, number of generations required for convergence and minimum error attained. Fig. 15 shows error vs. generation curves with 100, 300 and 650 members in the population. The figure demonstrates that using a larger population shows better convergence properties, although, the calculation time will also increase. Tests such as the ones shown in Fig. 15 must be conducted at one of the locations to decide on the optimal choice of parameters prior to inverting a volume.

CONCLUSIONS

Our research in developing a new inversion algorithm is motivated by current research in the field of evolutionary computation and the need for precision and speed in geophysical inversion problems. The new inversion scheme based on differential evolution has proved to be successful in sampling the high frequency part of seismic data which carries most of the information of the target zone. This is improved further by the use of CG in incorporating low frequency information into the output models. Use of fractal-based starting models has contributed well in deriving geologically realistic models which corroborates well with well-log data. The pre-stack seismic data inversion scheme incorporated in this paper has been found to be satisfactory in deriving P- and S-wave impedance models. The result shows high-resolution estimates which were able to predict most of the realistic peaks available in the well log.

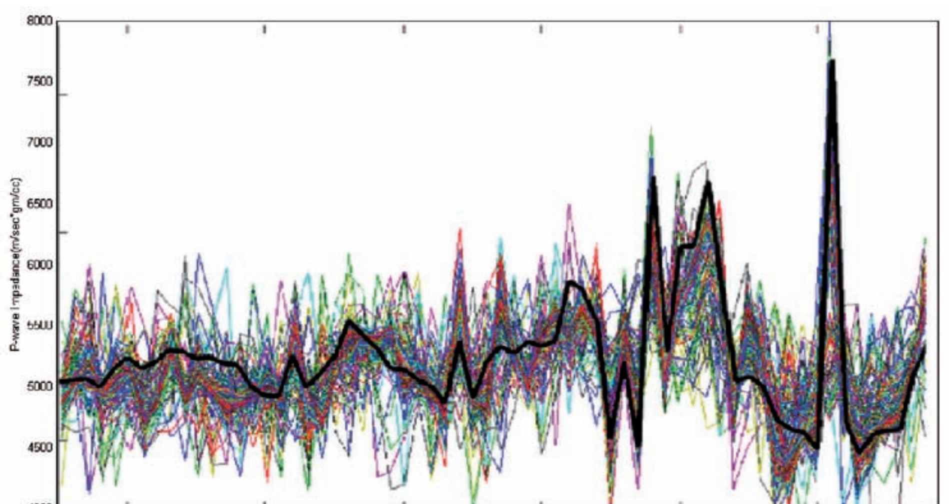

Fig. 13. Best P-wave impedance models from every generation. Note the confidence interval around well-log in black.

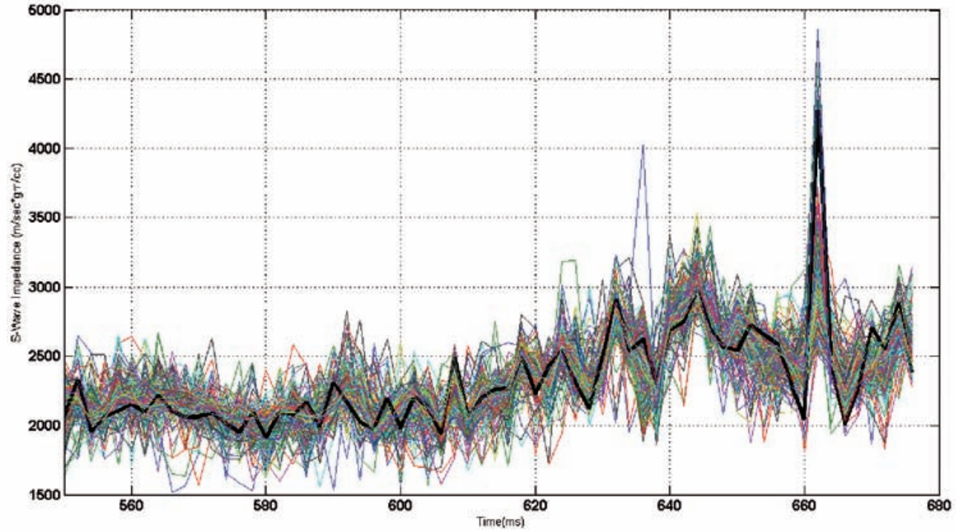

Fig. 14. Best S-wave impedance models from each generation. Note the confidence interval around the well-log data shown in black.

The most important features of the new inversion algorithm are as follows:

- A stochastic inversion using a fractal-based prior facilitates impedance modeling with a realistic frequency band similar to those observed at a well location. This is because of the fact that a fractal-based prior contains a similar frequency band as that of the real model.
- Fractal-based prior provides an intelligent initial guess to the evolutionary inversion modeling module which facilitates faster convergence.
- New hybrid optimization module which is highly efficient with DE and CG incorporating the advantages of both stochastic and deterministic methodologies.
- High-resolution estimates of rock properties with good variations over picked horizons.
- This is the first ever application of this hybrid model in geophysical inversion problem.

- The inversion process was fast and results were obtained within 800 iterations.

- The techniques of DE and Hybrid Model can be applied to a wide range of geophysical inversion problems.

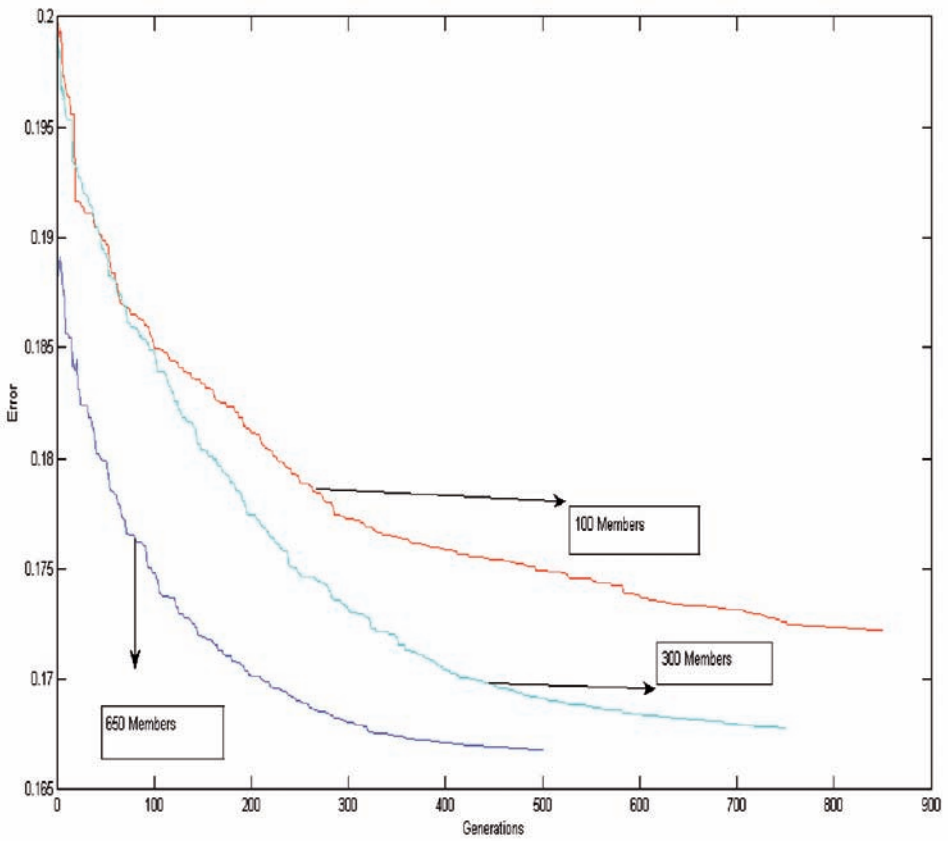

Fig. 15. Plot of the Error vs. Iterations (no. of generations), as we can see with a larger population size the convergence achieved is better compared to a smaller population size.

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